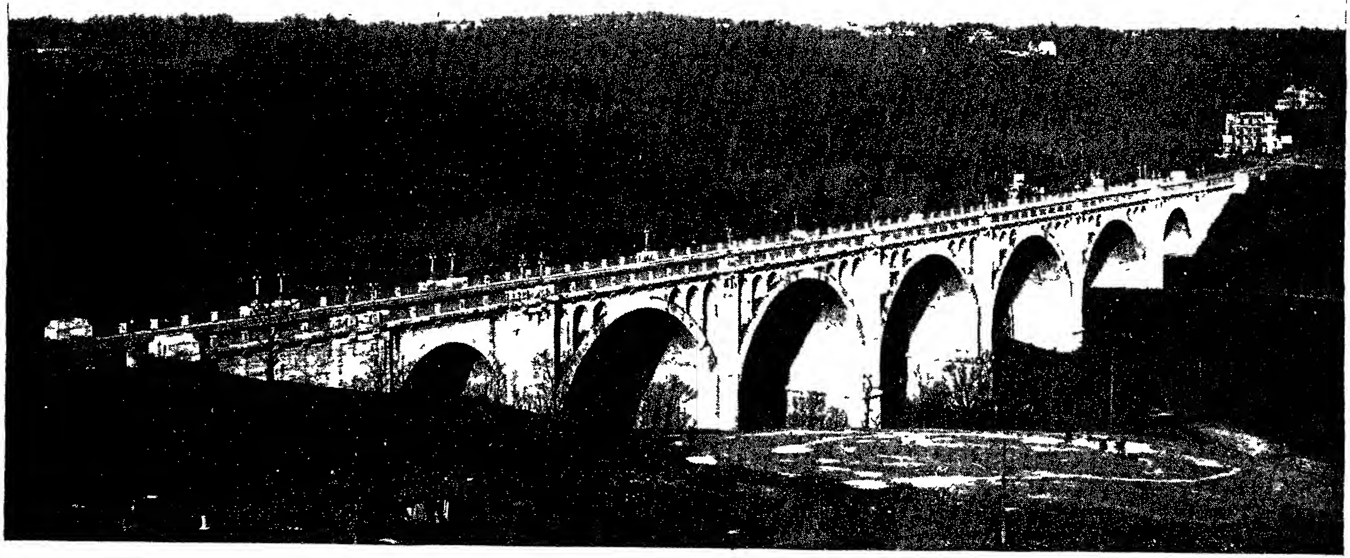

REINFORCED CONCRETE BRIDGE AT CONNECTICUT AVENUE, WASHINGTON, D C.
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Cyclopedia

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NINE VOLUMES

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Grateful acknowledgment is here made also for the invaluable co-operation of the foremost Civil, Structural, Railroad, Hydraulic, and Sanitary Engineers and Manufacturers in making these volumes thoroughly representative of the very best and latest practice in every branch of the broad field of Civil Engineering.

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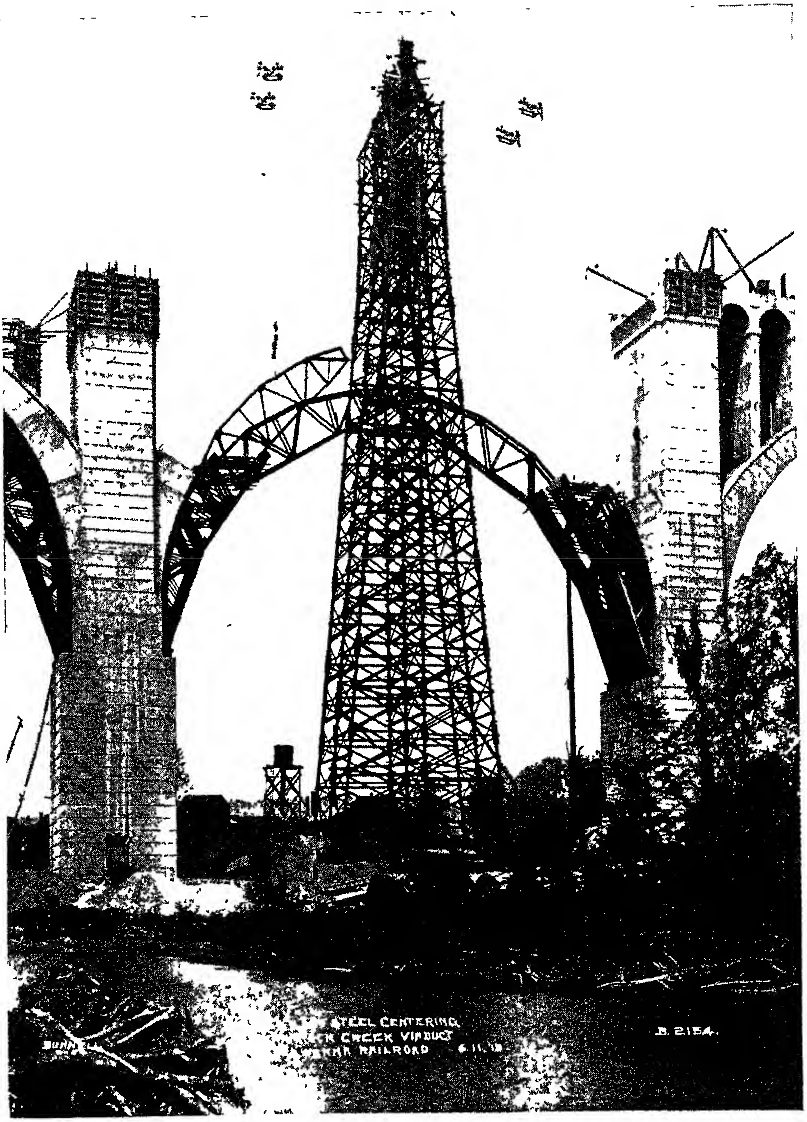

CABLEWAY TOWER AND PORTIONS OF ARCH CONSTRUCTION, TUNKHANNOCK CREEK VIADUCT

This view gives an excellent idea of the method of handling this gigantic work The last steel center is just being placed

Courtesy of Engineering Department, Delaware, Lackawanna and Western Railroad

Foreword

OF all the works of man in the various branches of engineering, none are so wonderful, so majestic, so awe-inspiring as the works of the Civil Engineer. It is the Civil Engineer who throws a great bridge across the yawning chasm which seemingly forms an impassable obstacle to further progress. He designs and builds the skeletons of steel to dizzy heights, for the architect to cover and adorn. He burrows through a great mountain and reaches the other side within a fraction of an inch of the spot located by the original survey. He scales mountain peaks, or traverses dry river beds, surveying and plotting hitherto unknown, or at least unsurveyed, regions. He builds our Panama Canals, our Arrow Rock and Roosevelt Dams, our water-works, filtration plants, and practically all of our great public works.

¶ The importance of all of these immense engineering projects and the need for a clear, non-technical presentation of the theoretical and practical developments of the broad field of Civil Engineering has led the publishers to compile this great reference work. It has been their aim to fulfill the demands of the trained engineer for authoritative material which will solve the problems in his own and allied lines in Civil Engineering, as well as to satisfy the desires of the self-taught practical man who attempts to keep up with modern engineering developments.

¶ Books on the several divisions of Civil Engineering are many and valuable, but their information is too voluminous to be of the greatest value for ready reference. The Cyclopedia of Civil Engineering offers more condensed and less technical treatments of these same subjects from which all unnecessary duplication has been eliminated; when compiled into nine handy volumes, with comprehensive indexes to facilitate the looking up of various topics, they represent a library admirably adapted to the requirements of either the technical or the practical reader.

¶ The Cyclopedia of Civil Engineering has for years occupied an enviable place in the field of technical literature as a standard reference work and the publishers have spared no expense to make this latest edition even more comprehensive and instructive.

¶ In conclusion, grateful acknowledgment is due to the staff of authors and collaborators—engineers of wide practical experience, and teachers of well recognized ability — without whose hearty co-operation this work would have been impossible.

Table of Contents

VOLUME V

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* For page numbers, see foot of pages.

† For professional standing of authors, see list of Authors and Collaborators at front of volume.

STEEL WORK AND CONCRETE ABUTMENT OF NEW HELL GATE BRIDGE

BRIDGE ENGINEERING

PART I

BRIDGE ANALYSIS

1. **Introduction.** The following treatment of the subject of Bridge Analysis, while not exhaustive, is regarded as sufficiently elaborate to develop and instill the principal theoretical considerations, to illustrate the most convenient and practical methods of analyzing the common forms of trusses and girders, and also to lay a sufficient foundation for the analysis of such other trusses as are not specifically mentioned or treated herein.

The necessary steps and operations required for a proper analysis of the several types of bridges are fully demonstrated by sketches and computations, the numerical values being mechanically obtained by the use of a *slide rule*, which is a handy instrument for quickly performing the operations of multiplication and division, and for squaring and extracting the roots of numbers. The values given may differ from the exact value by one unit in the second decimal place (seldom more) and are sufficiently accurate for the purpose of design. All bridge computers should be proficient in the use of the slide rule.

The problems given in the back of this Instruction Paper, exemplifying the practical application of the subject-matter treated in the various articles, should be solved by the student as each article is mastered.

HISTORY

2. **Early Bridges.** Early bridges were not bridges according to the present conception of the term. They were simple pile trestle bents placed at frequent intervals and connected by wooden beams on which the floor was placed. The *Pons sublicius*, built over the Tiber, at Rome, about 650 years before Christ was born, was of this trestle type. Also the famous bridge built by Cæsar across the Rhine in 55 B. C. was of the same kind of construction. As civilization progressed, the arch type was developed; and in 1390 the great

bridge at Trezzo over the River Adda was built of one span of 251 feet, which has never been eclipsed or equaled.

3. **Truss Bridge Development.** The first truss bridge is supposed to have been originated by Palladio, an Italian, who used the king-post truss (Fig. 1) about 1570. Its importance was not recognized, and it became entirely forgotten until it was rediscovered in 1798 by Theodore Burr, an American, who used it in his construction. About the same time, Burr invented the truss that bears his name, which was in reality a series of king-post trusses (see Fig. 8). This was found to be unstable under moving loads, and was therefore stiffened by the use of an arch (Fig. 2), or was built somewhat as an arch, there being considerable rise at the center of the span (Fig. 3). By 1830 the principle of the double cross-bracing in the panel was understood; and in quick succession came the patents of Long, Howe, Pratt, and Whipple on forms of trusses which bear their respective names.

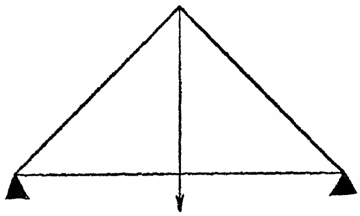

Fig 1 King-Post Truss.

It remained for Squire Whipple in 1847 to place the science of bridge building on a rational and exact mathematical basis such

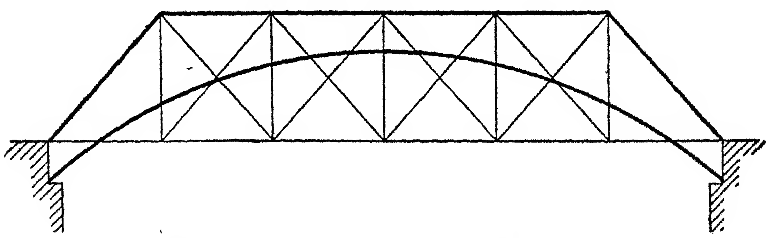

Fig. 2. King-Post Truss Bridge Stiffened by Arch.

as is now used. Previous to this time, and indeed several years afterwards—for Whipple's work did not become generally known until a much later date—bridges were built, not from previously computed strains, but by "judgment." All parts of a bridge were made of the same size, and if one started to fail it was replaced by a larger one; or small models were made and loaded proportionally, broken members being replaced by larger ones. There is no doubt

that many of the bridges built at this period were very weak as well as very strong. The failures are not remembered; but the sound judgment of many of our earlier bridge engineers is recorded in the wooden structures they left behind them, some of which have stood the demands of traffic for over a century. After 1850, bridges were built from computed stresses; wood was discarded; and the development became rapid, until about 1870, when the introduction of subdiagonal systems brought the truss system to practically what it is to-day.

DEFINITIONS AND DESCRIPTIONS

4. **Trusses.** A truss is a series of members taking stress in the direction of their length, placed together so as to form a triangle

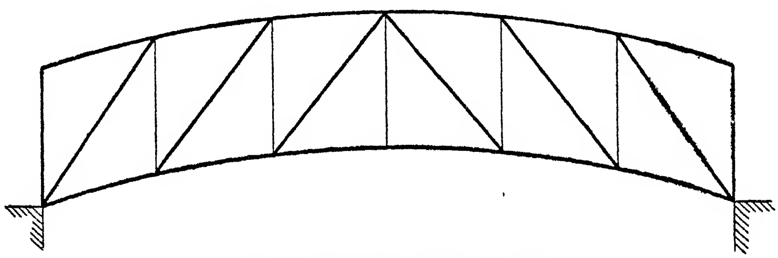

Fig 3 Burr Truss Bridge, Arched.

or system of triangles, which, when placed upon supports a certain distance apart, will, in addition to their own weight, sustain certain loads applied at the points where the members intersect. These points are called *panel points.*

5. **Bridge Trusses.** A bridge truss is one in which the members that carry the superimposed loads are in the same plane. Usually this plane is vertical.

6. **Truss Bridges.** A truss bridge is a structure consisting of two or more—usually two—bridge trusses connected by a system of beams called the *floor system,* which transfer to panel points the load for which the trusses are designed.

7. **Girders.** These are beams consisting of a wide, thin plate, called a *web plate,* with shapes, usually angles and narrow, thin plates called *flanges,* at the top and bottom edges. All are firmly riveted together. (See Part IV, "Steel Construction.")

8. **Girder Bridges.** These consist of usually two, sometimes

three, girders connected as in the case of truss bridges by a system of beams.

9. **Deck Bridges.** In cases where the floor system connects the trusses at their tops, the bridge is called a *deck bridge,* since the traffic moves on a *deck,* as it were (see Fig. 4).

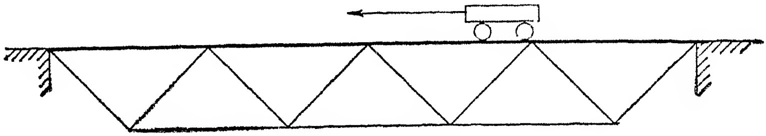

Fig. 4. Deck Bridge.

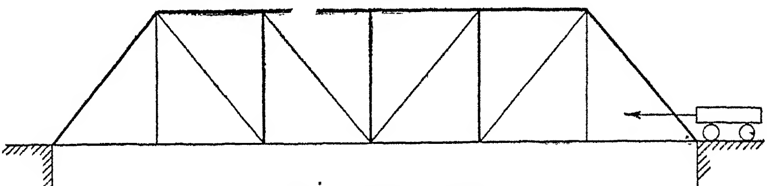

Fig 5 Through Bridge.

10. **Through Bridges.** In cases where the floor system connects the bottoms of the trusses, the bridge is called a *through bridge,* as the traffic moves *through* the space between the trusses (see Fig. 5).

11. **Members of a Truss.** Each truss consists of a *top* and *bottom chord, end-posts,* and *web members.* The web members are further divided into *hip verticals, intermediate posts,* and *diagonals.* Fig. 6 shows these various classes, *A-A* being top chord, *B-B*

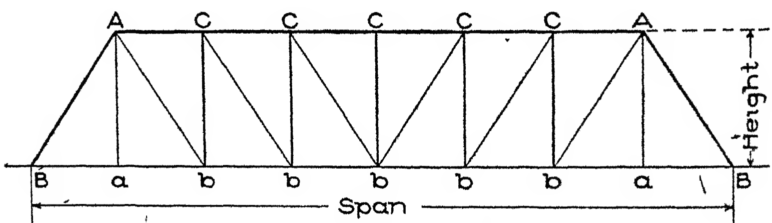

Fig. 6. Showing the Members of a Truss.

bottom chord, *A-B* end-posts, vertical members *C-b* intermediate posts, *A-a* hip verticals, and *A-b* and *C-b* diagonals.

12. **Pony-Truss Bridges.** When the height of the trusses of a through bridge is less than the height of the loads that go over them, they are called *pony trusses,* and the bridge a *pony-truss bridge.*

13. **Lateral Bracing.** In all deck bridges, and in all through bridges except pony-truss bridges, the chords which are not connected by the floor system are connected by a horizontal truss system called the *lateral bracing.* In all bridges the chords which are connected by the floor system are connected by a horizontal truss system, also called the *lateral bracing.* One of these systems is called the

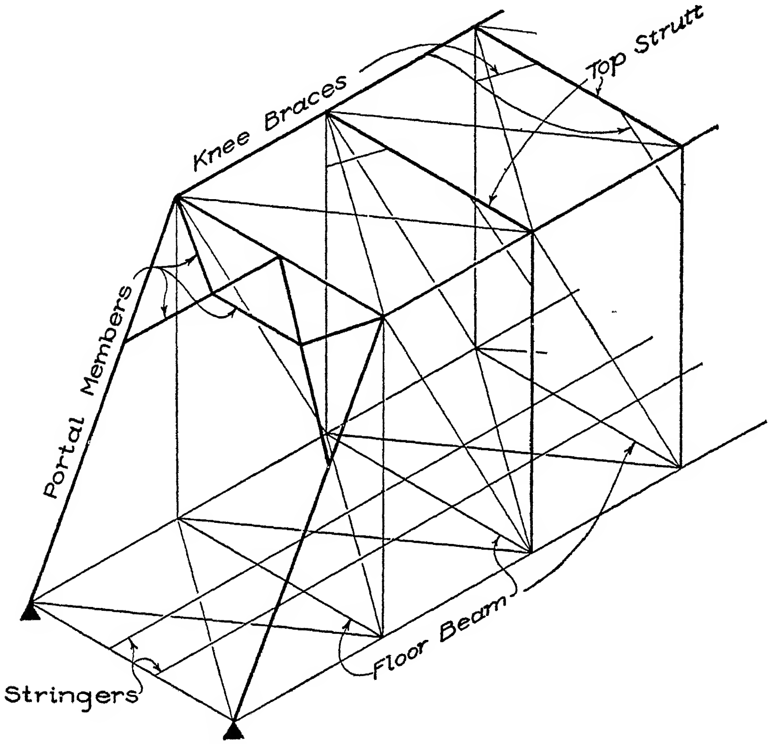

Fig. 7 Through Bridge, Showing Top and Bottom Systems of Lateral Bracing, also Portal Bracing and Floor System.

top lateral system, as it connects the top chords; and the other is called the *bottom lateral system,* as it connects the bottom chords (see Fig. 7).

14. **Portals.** In through bridges, the end-posts of the pair of trusses are connected by a system of braces in order to preserve the rectangular cross-section of the bridge. This is called the *portal bracing* (see Fig. 7).

15. **Sway Bracing and Knee-Braces.** These serve the same purpose as the portal braces, and are either small struts or systems

of cross-bracing placed at the intermediate posts. The former are called *knee-braces,* and the latter *sway bracing.*

16. **Floor Systems.** In both highway and railway bridges, there are beams running from the intermediate posts or hip verticals across to the like members opposite. These are called *floor-beams.* In highway bridges, there are smaller beams running parallel to the trusses and resting at their ends upon the floor-beams. These are called *floor-joists,* and the plank or other floor rests directly upon them. In railway bridges, two beams or girders per track run parallel to the trusses and are connected at their ends to the floor-beams. These are called *track stringers* (or simply *stringers*). The ties rest directly upon them. The various members of the floor system of a railway bridge are shown in Fig. 7. The diagonals connecting the top chords, and those connecting the bottom chords, are the *top* and *bottom laterals* respectively.

CLASSES OF TRUSSES

17. **Names.** Trusses may be classified according to their names, the character of their chords, and the system of webbing. Table I gives the classification of the more important of these according to name.

TABLE I

Chronological List of Trusses

Name	Year of Origin	Inventor	Country	Illustrated in
King-Post	1570	Palladio	Italy	Fig. 1
King-Post	1798	Theodore Burr	America	Fig. 1
Burr	1798	Theodore Burr	America	Fig. 8
Warren	1838		England	Fig. 9
Howe	1840	William Howe	America	Fig. 10
Pratt	1844	Thos & Caleb Pratt	America	Fig. 11
Whipple	1847	Squire Whipple	America	Fig 12
Bowstring	1847	Squire Whipple	America	Fig 13
Baltimore	1877	Penn. R. R.	America	Fig 14

Of the types of trusses listed in Table I, the Warren, Howe, Pratt, Bowstring, and Baltimore are now built; and of these constructions probably 90 per cent are Pratt trusses. The Baltimore truss is used for long spans only.

18. **Chord Characteristics.** In most types of bridges the

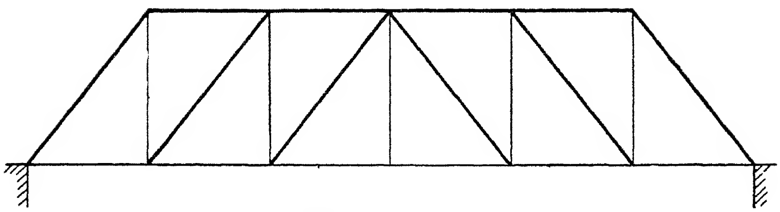

Fig. 8. Burr Truss.

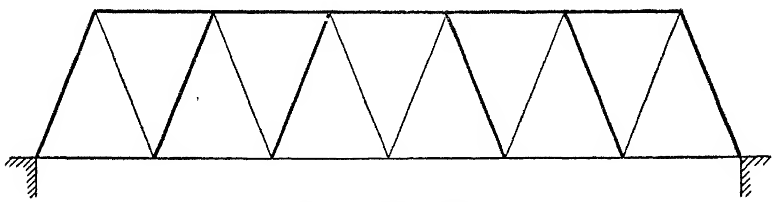

Fig 9 Warren Truss.

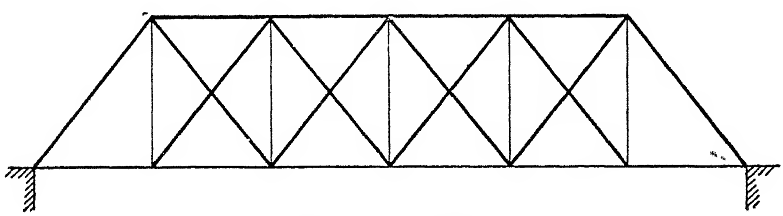

Fig. 10. Howe Truss.

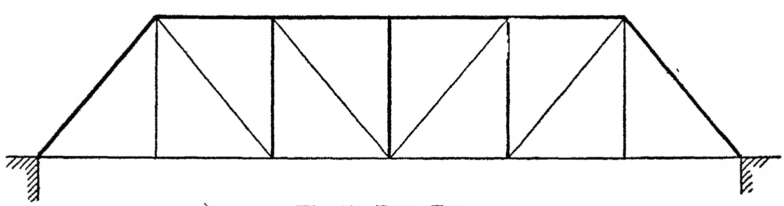

Fig 11. Pratt Truss.

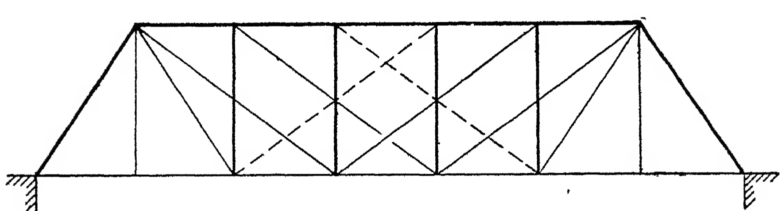

Fig. 12. Whipple Truss.

chords are parallel. When such is the case, the stresses increase from the end toward the center, and there is a considerable difference between any two adjacent panels of the same chord. This necessitates different areas for each section. When the chords are not parallel, as in the bowstring truss, the stresses in the chords are so nearly equal that the same area is used throughout or nearly throughout the entire chord. Also, the stresses in the diagonals are nearly equal. These conditions would seem to indicate that this was a very economical form of truss. Theoretically it is; but practical considerations—such as the beveled joints and the posts which must be constructed to withstand reversals of stress—customarily limit this type to the longer spans.

19. **Web Characteristics.** The web systems of the Burr, Warren, Howe, Pratt, and Bowstring trusses are called *single* systems; that of the Whipple truss is a *multiple* system; while those of the Baltimore trusses are examples of webbing with *sub-systems.* As the maximum economical panel length has been found to be about 25 feet, which makes the economical height of the truss about 30 feet, and as the length of the span should not be more than ten times the depth, the span for trusses with simple systems of webbing is limited to about 300 feet. In order to increase the limiting span, multiple systems like that of the Whipple or similar ones were introduced. Calculations of stresses in members of the Whipple truss are somewhat unreliable on account of the fact that we are unable to tell just how the effects of the loads are distributed. For this reason, that type has gone out of use, and the sub-systems are used instead. These allow spans of twice the above limit; and, indeed, trusses with this type of webbing have been built up to and over 600 feet. This style of webbing can be applied to the bowstring truss, almost all long-span bridges being of this type with sub-systems of webbing.

WEIGHTS OF BRIDGES

20. **Formulæ.** In order to obtain the stresses due to the weight of the structure, the latter quantity must be known. As this weight can be determined only after the structure has been designed, it is evident that an assumption as to the weight must be made. The best method is to use the actual weight of a similar structure of like span which has been built. As the necessary data

for this is not always available, it is customary to use formulæ to derive an approximate weight of sufficient accuracy for purposes of computation. Table II gives some of the most reliable formulæ.

TABLE II

Formulæ for Weights of Bridges

Class of Bridge	Weight of Steel per Linear Foot of Span	Author*
Heavy Interurban Riveted	$w = 600 + 1.8l + 27b + \frac{1}{12} bl\left(1 + \frac{1}{1,000} l\right)$	E. S. Shaw
First-Class Highway Riveted	$w = 300 + l + 22b + \frac{1}{15} bl\left(1 + \frac{1}{1,000} l\right)$	E. S. Shaw
First-Class Highway Pin	$w = 34 + 22b + 0.16bl + 0.7l$	J. A. L. Waddell
Light Country Highway	$w = 250 + 2.5l$	Author
Railroad Truss E 50	$w = (650 + 7l)$	F. E. Turneaure
Railroad Truss E 40	$w = \frac{7}{8} (650 + 7l)$	F. E. Turneaure
Railroad Truss E 30	$w = \frac{3}{4} (650 + 7l)$	F. E. Turneaure
Railroad Deck Girder E 50	$w = 124.0 + 12.0l$	Author
Railroad Deck Girder E 40	$w = 123.5 + 10.0l$	Author
Railroad Deck Girder E 30	$w = 111.0 + 8.8l$	Author

In the above formulæ, w = Weight of steel per linear foot of span, l = Length of span in feet; b = Breadth of roadway, including sidewalks.

In using the formulæ of Table II, remember that a span has two trusses. The weights for highway bridges do not include the weight of the wooden floor, which may be assumed as 10 pounds per square foot of floor surface. All highway bridges have steel joists. The weights of railroad spans do not include the weight of the ties and rails, which may be assumed at 400 pounds per track per linear foot of span. If solid steel floors are to be used, 700 pounds per linear foot of span are to be added to the weights computed from the table.

All the weights given for railroad spans are for single track. Double-track truss-spans are about 95 per cent heavier; and double-track girder-spans are 100 per cent heavier. Through girder spans are about 25 per cent heavier than deck girder spans; and through truss spans are about 10 per cent heavier than deck spans.

The spans on which Table II is based are of medium steel. Bridges built of soft steel or wrought iron will weigh 10 to 15 per cent more.

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* The author is indebted to the distinguished engineers whose names appear in Table II, for permission in this connection to make use of the formulæ given opposite their names.

In order to give an idea of the relative weights of steel in different classes of bridges, let it be required to compute the dead weight of a 100-foot span of each class. For heaviest highway bridges to carry heavy interurban cars:

$$w = 600 + 180 + 27 \times 16 + \frac{16 \times 100}{12}\left(1 + \frac{100}{1\,000}\right) = 1\,358 \text{ lbs per linear ft.}$$

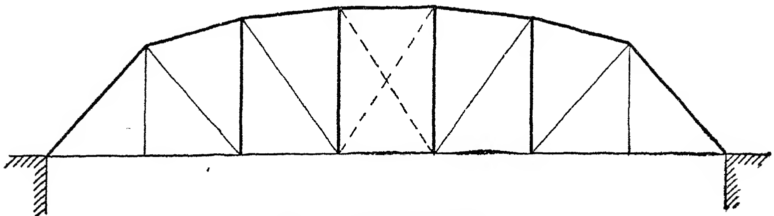


Fig 13 Bowstring Truss

For heavy riveted highway bridges to carry heavy farm engines:

$$w = 300 + 100 + 22 \times 16 + \frac{16 \times 100}{15}\left(1 + \frac{100}{1\,000}\right) = 870 \text{ lbs per linear foot.}$$

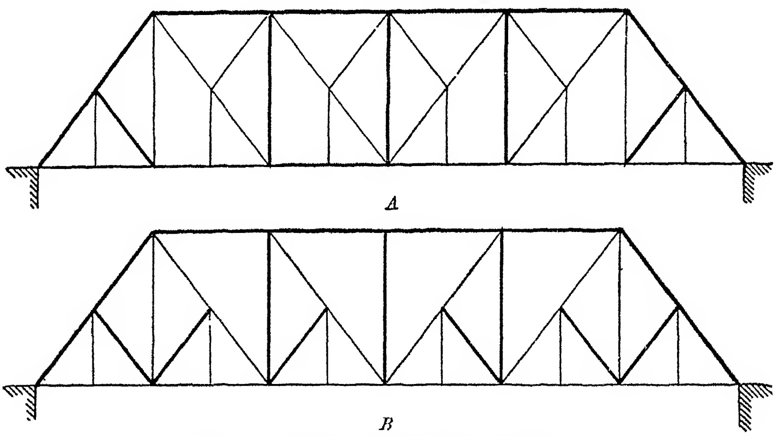


Fig 14. Two Forms of Baltimore Trusses

For heavy pin-connected highway bridges to carry heavy farm or traction engines:

$$w = 34 + 22 \times 16 + 0.16 \times 16 \times 100 + 0.7 \times 100 = 710 \text{ lbs per linear ft.}$$

For light country highway bridges to carry 100 pounds per square foot of floor surface:

$$w = 250 + 2.5 \times 100 = 500 \text{ lbs. per linear foot.}$$

If the total weight is required, the weight of the wooden floor must be added. Take, for example, the last bridge:

22733.

Weight of steel	$= 500 \times 100 =$	50 000 pounds
" " floor	$= 100 \times 16 \times 10 =$	16 000 pounds
Total dead load	$=$	66 000 pounds

The weight per linear foot for a railroad truss bridge of 100-foot span is:

$$w = 650 + 7 \times 100 = 1\,350 \text{ lbs per linear foot}$$

This is about the same as that for a heavy interurban bridge The reason for this is that in addition to the heavy rolling stock of the electric road, the heavy highway traffic must be provided for. A deck girder of 100-foot span weighs:

$$w = 124 + 12 \times 100 = 1\,324 \text{ lbs per linear foot}$$

21. **Actual Weights and Costs of Railroad Spans.** In case actual weights can be obtained, a more exact analysis can be made. Weights of bridges are given in the accompanying tables and diagrams.

The weights of through truss spans of medium steel designed for E 50 loading are given in Fig. 15. The weights include all steel and also the weight of the ties and rails which is taken at 400 pounds per linear foot of track. The weights for E 60 bridges are given in Table III.

TABLE III

Weights of Medium Steel Railroad Bridge Loading E 60
All Weights in Pounds

Span (In Ft)	I-Beam Spans	Deck Plate Girders	Through Plate Girders	Through Riveted Spans	Through Pin Spans
20	14,100				
30		16,400	30,300		
40		26,200	46,100		
50		37,800	62,300		
60		52,300	87,600		
70		75,000	116,000		
80		97,000	146,000		
90		125,000	178,000		
100		152,000	218,500	182,000	
110				204,000	
125				245,000	
140				305,000	
150				345,000	338,000
160					385,000
180					460,000
200					535,000

The weights and design of deck plate girders are given in Table XXIX, page 255, and Fig. 205 to 217, and unit prices which may be used in preliminary estimates of costs of bridges are given in Table XXX, page 276.

22. **Actual Weights of Highway Spans.** The actual weights of highway spans for heavy interurban trolley-cars and traffic, should preferably be computed from the formulæ of Shaw or Waddell (Table II). The weights of country bridges, including floor, may be taken from the diagram of Fig. 16.

LOADS

23. **Classes of Loads.** Those weights just given constitute what is called the *dead load* of the bridge. The traffic which passes

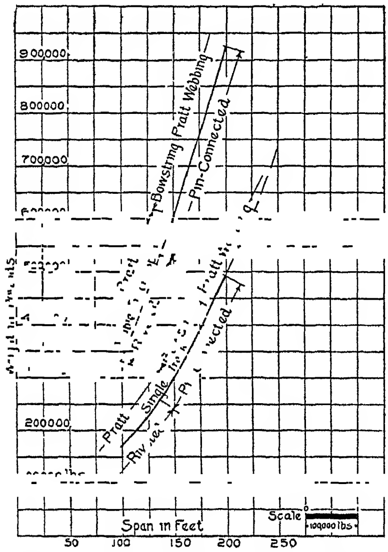


Fig. 15 Weights of Through Truss-Spans Medium Steel, E 50 Loading

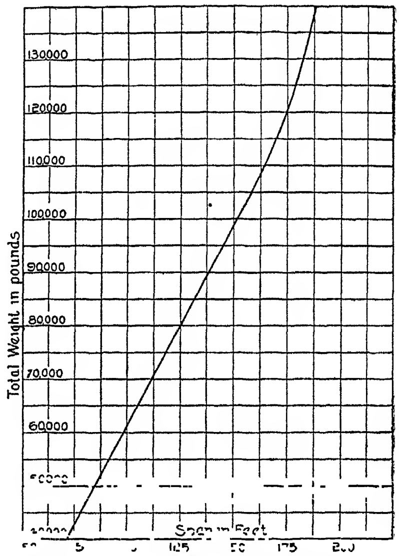


Fig 16 Weights of Country Bridges, Including Floor.

over the bridge is called the *live* or *moving load.* In addition to the two classes mentioned, is the effect of the wind, which is designated as the *wind load.* These loads vary with the class of bridge, be it highway or railway, and with the purpose for which it is intended.

24. **Live Loads for Highway Bridges.** Highway bridges are usually divided into several classes according to the traffic, which may be that of heavy interurban cars, light trolley-cars, farm engines, road rollers, teams, human beings, or some combination of these loadings. The standard specifications of J. A. L. Waddell or of Theodore Cooper are obtainable for a very small sum. Their pur-

chase is advised, and the reader is referred to them for further information.

The trusses of country highway bridges are usually designed for a live load of 100 pounds per square foot of roadway. This may be considered good practice; and it is the law in some States. The floor system of these same bridges should be of sufficient strength to sustain 100 pounds per square foot of roadway, or a 12-ton farm engine having 4 tons on the two rear wheels, which are 12 inches wide and 6 feet apart, and 2 tons on each of the front wheels, which are 6 inches wide and 5 feet apart. The axles of this engine are spaced 8 feet center to center.

25. **Live Loads for Railway Bridges.** The loads for any particular railroad bridge are not always the same, on account of the great variation in the weights and wheel spacings of engines and cars. It is customary to design the bridge for the heaviest in use at the time of construction, or for the heaviest that could reasonably be expected to be built thereafter.

As the computations with engines were formerly somewhat laborious on account of the different weights and spacing of wheels, it has been proposed by some engineers to use a uniform load, called the *equivalent load*, which would give stresses the same, or very nearly the same, as those obtained by the use of engine loads. However, as these loads are different for each weight of engine, and also different for the chord members, the web members, and the floor-beam reaction of each different length of span, and as the labor of the computations, using engine-wheel loads, has been greatly reduced by means of diagrams, it does not seem as if this method would ever come into very general favor except for long-span bridges, where the live load is much smaller than the dead load.

The equivalent loads for Cooper's E 40 (see Fig. 85) are given in Table IV.

Most railways specify that their bridges shall be computed by using two engines and tenders followed by a train. The spacing of the wheels, and the load which comes on each wheel of the engines and tenders, are fixed by the railway company. The train is represented by a uniform load. Formerly there was a great diversity of practice among the different roads in regard to the engine and train loads specified; but practice has of late years become quite uniform,

TABLE IV

Equivalent Uniform Loads

Loading E 40

Span (in feet)	Equivalent Uniform Load Chords	Web	Floor-Beam	Span (in feet)	Equivalent Uniform Load Chords	Web	Floor-Beam
10	9 000	12 000	8 200	46	6 330	7 240	5 240
11	9 310	11 640	7 960	48	6 220	7 140	5 200
12	9 340	11 330	7 830	50	6 110	7 060	5 140
13	9 340	11 080	7 600	52	6 040	6 940	5 130
14	9 210	10 860	7 460	54	5 960	6 820	5 120
15	9 030	10 670	7 330	56	5 880	6 720	5 110
16	8 850	10 500	7 120	58	5 800	6 620	5 090
17	8 650	10 350	6 940	60	5 730	6 530	5 080
18	8 430	10 240	6 780	62	5 690	6 490	5 080
19	8 220	10 100	6 630	64	5 700	6 450	5 070
20	8 000	10 000	6 500	66	5 620	6 450	5 070
21	8 040	9 780	6 390	68	5 560	6 380	5 060
22	8 040	9 580	6 290	70	5 510	6 340	5 060
23	8 010	9 400	6 200	72	5 490	6 320	5 030
24	7 960	9 230	6 120	74	5 460	6 300	5 010
25	7 890	9 080	6 040	76	5 440	6 290	4 990
26	7 780	8 930	5 970	78	5 420	6 270	4 970
27	7 660	8 790	5 900	80	5 400	6 250	4 950
28	7 540	8 660	5 830	82	5 370	6 230	4 930
29	7 420	8 540	5 770	84	5 340	6 200	4 910
30	7 300	8 430	5 720	86	5 310	6 180	4 890
31	7 220	8 320	5 680	88	5 270	6 150	4 870
32	7 140	8 190	5 650	90	5 250	6 130	4 860
33	7 050	8 080	5 620	92	5 250	6 110	4 830
34	6 960	7 980	5 600	94	5 210	6 090	4 810
35	6 870	7 890	5 570	96	5 170	6 060	4 780
36	6 820	7 820	5 530	98	5 150	6 040	4 760
37	6 760	7 750	5 500	100	5 140	6 020	4 740
38	6 700	7 690	5 460	125	5 100	5 770	4 720
39	6 630	7 630	5 430	150	5 010	5 570	4 700
40	6 560	7 570	5 400	175	4 890	5 350	4 686
42	6 530	7 450	5 340	200	4 740	5 240	4 660
44	6 470	7 340	5 300	250	4 510	5 030	4 640

with an apparent tendency to standardize in accordance with the classes of loading specified by Cooper. Cooper's Class E 50, which represents the heaviest engines now in common use, was invented by Theodore Cooper, a consulting engineer of New York City. It is given in Fig. 17.

Lighter loadings for light traffic on the same general plan are advocated by Mr. Cooper, and are given at length in his "General Specifications for Iron and Steel Railway Bridges and Viaducts" (1906 edition).

26. **Wind Loads.** Some designers require that the stresses due

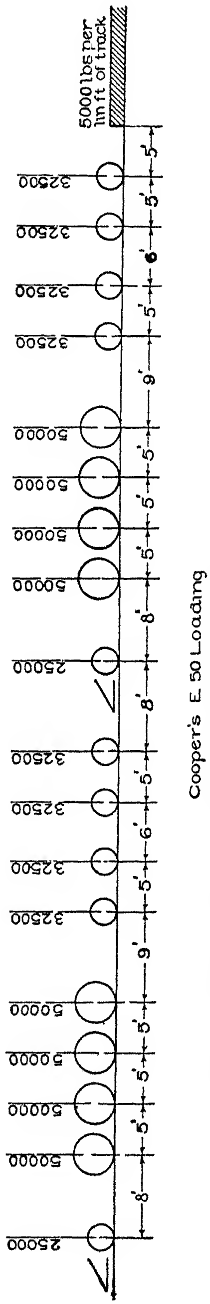


Fig. 17 Diagram of Cooper's E 50 Loading .Representing Heaviest Engines in Common Use.

to wind shall be computed by using 30 pounds per square foot of actual truss surface. This requires that you know the size of the members of the bridge before it is designed—which is evidently an impossibility; or that an assumption as to their size be made—which allows a chance for a mistake in judgment, especially in an inexperienced person. A more logical method, and one used to a great extent, is to assume a force of so many pounds per linear foot to act on the top and bottom chords and on the traffic as it moves across the bridge.

In highway through bridges, it is the usual practice to take the wind load as 150 pounds per linear foot of top and bottom chords, and 150 pounds per linear foot of the amount of live load which is on the bridge.

For railroad bridges, it is customary to use considerably higher values than those used in highway practice—not that the wind blows harder on railroad than on highway bridges, but so that the bracing designed by the use of these values may be sufficiently strong to stiffen the bridge not only against the wind, but also against the vibrations caused by the rapidly moving traffic. Good practice for through bridges is to use 150 pounds per linear foot of the top chord, 150 pounds per linear foot of the bottom chord, and 450 pounds per linear foot of live load on the bridge. This latter force is supposed to act at a line 8.5 feet above the base of the rail.

For deck bridges, for both highway and railway use, the unit-loads on the moving or live load remain the same, but the unit-loads on the top and bottom chords are reversed.

In computations involving the live load, it is always assumed that the live load moves over the bridge from right to left.

THEORY

27. Principles of Analysis. The stresses in bridge trusses may be determined by both algebraic and graphic methods. In some

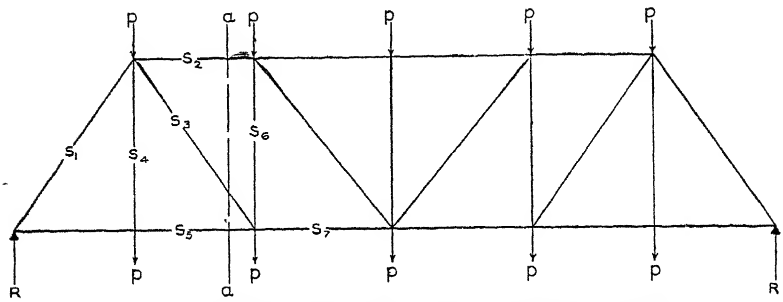


Fig 18 Truss under Loads, Illustrating Principles of Analysis

cases, one is more expeditious than the other. Algebraic methods alone will be considered in this text.

The analysis of stresses is based upon the fact that the interior stresses in a member or group of members hold in equilibrium the exterior forces. That this is a fact, can easily be understood. Consider a man pulling on a rope which is fastened at one end to an immovable object. There will be a stress in the rope equal to, and opposite in direction to, the pull exerted by the man. In order to prove this, cut the rope and apply a force equal and opposite to the pull exerted by the man, where the cut is made; and the rope and man will be in equilibrium. Also, suppose that a truss under loads, as indicated by the arrows, Fig. 18, were cut along the section *a-a*, and that forces F_2, F_3, F_5 equal to the stresses S_2, S_3, and S_5 were placed at the ends of the members as indicated in Fig. 19, then that portion of the truss to the left of the section would be in equilibrium. The interior stresses, represented by F_2, F_3, and F_5, would hold in equilibrium the exterior forces p and R.

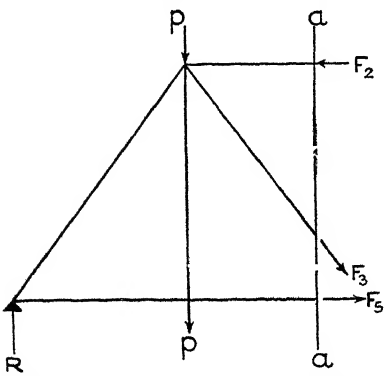


Fig. 19 Forces Substituted for Stresses in Truss of Fig. 18

From inspection of Fig. 19, it will appear evident that, as the position of the truss to the left of the section is in equilibrium, the following statements are true:

1 The algebraic sum of the moments of the exterior forces and the stresses in the members cut by the section, is equal to zero This is true of the moments taken about any or all points, for, if it were not, the portion of the truss would begin to rotate about some point, and would continue until equilibrium was established.

2 In a vertical plane, the algebraic sum of the components of the exterior forces and the stresses in the members cut by the section is equal to zero, for, if such were not the case, the portion of the truss shown would move up or down with a constant acceleration.

3. The algebraic sum of the components of the exterior forces and the stresses in the members cut by the section in a horizontal plane, is equal to zero, for, if such were not the case, the portion of the truss would move either to the right or to the left, with a constant acceleration.

4. From 2 and 3, above, it is evident that the algebraic sum of the components of the exterior forces and the stresses in the members cut by the section is equal to zero in any and all planes.

The section is not necessarily required to be a vertical line as in

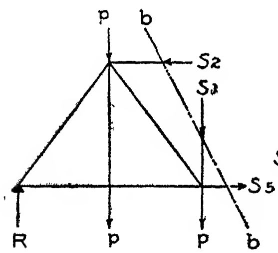


Fig 20
Oblique Section Cutting Truss.

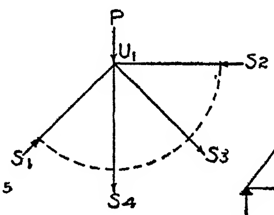


Fig 20a
Circular Section Cutting Truss.

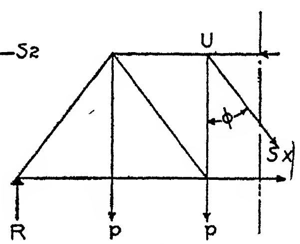


Fig 21.
Illustrating Resolution of Forces.

Fig. 19. It may be oblique, as in Fig. 20; or it may be a circular section, as shown in Fig. 20*a*. When the latter is the case, it is said that the sum of the components of the forces around the point U_1 is in equilibrium in any plane that may be taken.

It is also evident that the forces in the members cut by the section, and the exterior forces to the right, are in equilibrium. This condition is very seldom utilized in the determination of stresses, as that portion of the truss to the left of the section is almost always considered.

28. **Resolution of Forces.** This method is one of the simplest

and at the same time least laborious. The forces are generally resolved into their horizontal and vertical components, or parallel and perpendicular to some member. In cases where two unknown stresses occur, two equations can usually be formed, and these solved.

It should be assumed that the unknown stress acts *away from* the section which cuts it. It will then solve out, with the proper sign indicating the character of the stress—that is, whether it is tensile or compressive. Tensile stresses are indicated by placing

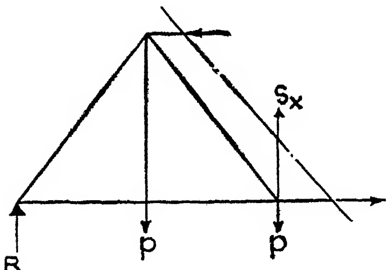


Fig. 22

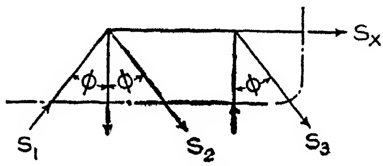


Fig. 23.

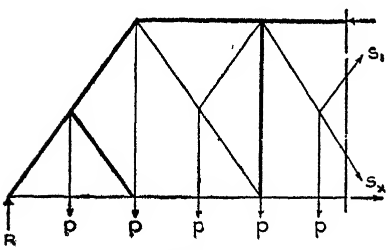


Fig. 24.

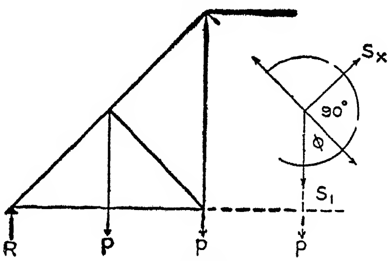


Fig. 25.

Diagrams Illustrating Application of Method of Resolution of Forces in Analysis of Trusses.

the plus (+) sign before them, while a minus (−) sign indicates compression.

A few equations showing the application of the method of the resolution of forces can be written after inspection of Figs. 21 to 25 inclusive. In all cases, S_x is the unknown stress, and is assumed to be acting away from the section. The other stresses S_1, S_2, etc., are known, and their direction given them accordingly, it being toward the section if the member is in compression, and away from the section if the member is in tension. Forces or components acting upward or to the right are considered plus; those acting down-

ward or to the left are considered minus. For a fuller explanation, see the instruction paper on Statics, Articles 17 to 23 inclusive.

In Fig 21, the sum of the vertical components is taken, and the equation is:

$$+R - p - p - S_x \cos \phi = 0;$$

whence,

$$S_x = +(+R - p - p) \sec \phi.$$

In Fig. 22, the section is oblique, and the sum of the vertical components is taken:

$$+R - p - p + S_x = 0;$$

whence,

$$S_x = -(+R - p - p).$$

In both of the above cases, it will be noted that the chord stresses do not enter into the equation, as their vertical components are zero.

In Fig. 23, the sum of the horizontal forces is used in determining the stress S_x. Note that the exterior forces R and p do not enter the equation, as they are not to the left of the section, and also their horizontal components are zero.

$$+S_1 \sin \phi + S_2 \sin \phi + S_3 \sin \phi + S_x = 0;$$

whence,

$$S_x = -(S_1 + S_2 + S_3) \sin \phi.$$

In Fig. 24, the sum of the vertical forces is again used. Here the section cuts the member with the known tensile stress S_1.

$$+R - p - p - p - p - p + S_1 \cos \phi - S_x \cos \phi = 0;$$

whence,

$$S_x = +(R - 5p) \sec \phi + S_1.$$

In Fig. 25, use is made of the fact that the sum of the components of the forces about a point is zero when they are resolved in any plane. Here they will be resolved perpendicular to the diagonal.

$$-S_1 \sin \phi + S_x = 0.$$

$$S_x = +S_1 \sin \phi.$$

These are some of the most common cases which occur in the determination of stresses in simple trusses. In all cases, follow this method of procedure:

1. Pass a section cutting as few members as possible, one of which must be the one whose stress is desired
2. The stress in all the members cut, with but one exception, must be known.
3. Write your equation, always placing it equal to zero.
4. Solve for your stress.

29. **Method of Moments.** The stresses in all members of a truss can be determined by this method. By section 1 of Art. 28, the point about which the moments are considered can be taken anywhere. Fig. 26 represents the point as taken somewhere outside of the truss at a distance a above the point U_1. The equation will then be:

$$-S_1 \times a - S_2 \times b - S_3 (a + h) + Rp \pm P_1 \times 0 + P_2 \times p = 0$$

This involves three unknown quantities, and therefore two other points should be taken, and two more equations written. By the use of the three equations, the stresses can be determined.

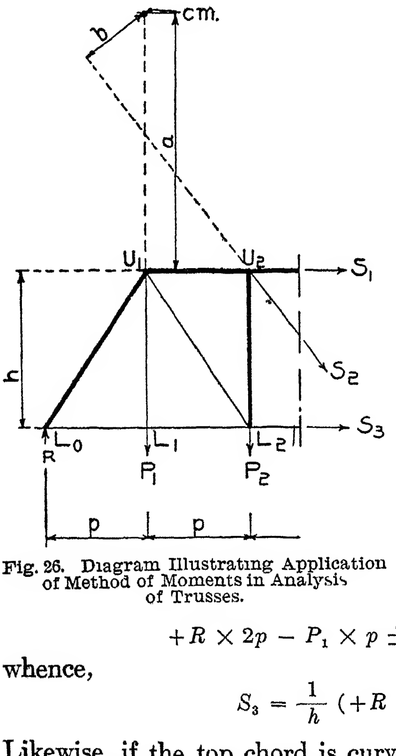


Fig. 26. Diagram Illustrating Application of Method of Moments in Analysis of Trusses.

It is customary to assume the center of moments at such a place that the moments of all the unknown stresses, with one exception, are zero. This condition requires that their lines of action pass through the center of moments. Let it be required to determine the stress S_3. If the center of moments is taken at U_2, then, as the lines of action of S_1 and S_2 pass through this point, their moments will be zero, and the following is true:

$$+R \times 2p - P_1 \times p \pm P_2 \times 0 - S_3 \times \mathrm{h} = 0.$$

whence,

$$S_3 = \frac{1}{h} (+R \times 2p - P_1 \times p).$$

Likewise, if the top chord is curved, the center of moments can be taken in such a position that only the unknown stress will enter into the equation. If it is desired to determine the stress S_2, Fig. 27, the equation would be:

$$-S_2 \times l - R \times a + P_1 (a + p) + P_2 (a + 2p) = 0,$$

the center of moment being at O, the intersection of the lines of stress of S_1 and S_3. Solving the equation just stated,

$$S_2 = \frac{1}{l} \left\{ -Ra + P_1 (a + p) + P_2 (a + 2p) \right\}.$$

30. **Stresses in Web Members.** By reference to Articles 28 and 29, it is seen that several methods are presented for the solution of stresses in web members. Each should be adapted to the case in hand. The simplest method, and the one which is commonly used in all trusses with parallel chords, is by the resolution of the vertical forces. Fig. 21 is to be referred to. The equation given on page 19 is:

$$+R - p - p - S_x \cos \phi = 0.$$

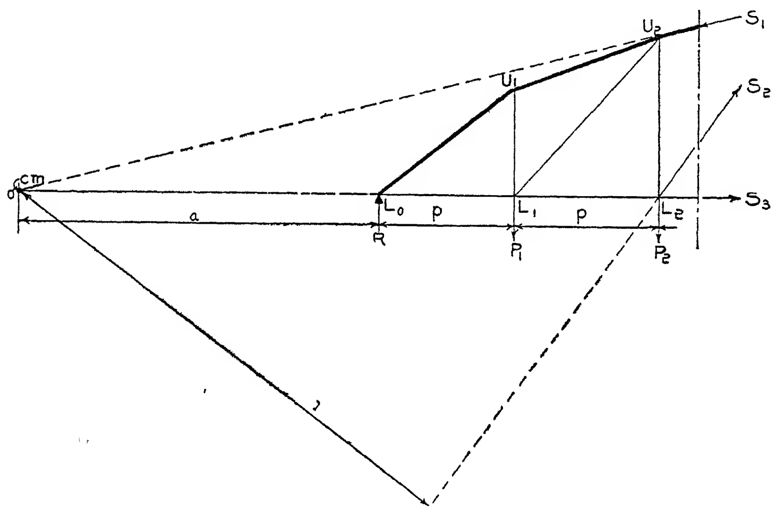


Fig 27 Diagram Illustrating Application of Method of Moments in Analysis of Trusses Top chord curved

But $R - p - p$ is equal to V, the vertical shear at the section, and so the equation may now be written:

$$V - S_x \cos \phi = 0 \qquad \ldots\ldots\ldots (1)$$

whence the following important rule is deduced:

The algebraic sum of the vertical shear at the section and the vertical components of the stress in all of the members cut by the section, is equal to zero.

In trusses with horizontal chords and a simple system of webbing, the equation may be put in the form:

$$S_x = +V \sec \phi;$$

and the statement that the stress in any web member is equal to the shear *times* the secant of the angle that it makes with the vertical is

true. The practice of using this latter statement is not to be encouraged, as it leads to confusion in the signs of the stresses. Equation (1) should be written in all cases, and the stress will then solve with its correct characteristic sign, indicating that the stress is either tensile or compressive.

As an example, let it be required to determine the stresses in the web members S_2 and S_3 of the Pratt truss shown in Fig. 28, the loads being in thousands of pounds. First, a section should be passed, cutting that member and as few others as possible. Next, the shear at that section should be computed. Then the vertical components of all the stresses cut by the section, and the vertical shear, should be

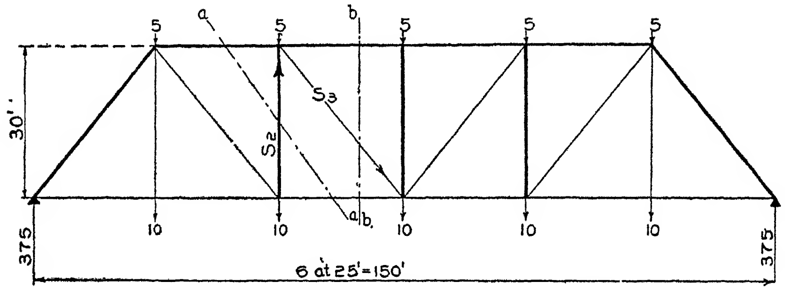


Fig. 28 Calculation of Stresses in Web Members of Pratt Truss.

equated to zero. Finally, solve the equation. Remember that the unknown stress is to be assumed as acting away from the section, and that forces or resultants acting downward are considered negative, while those acting upward are considered positive.

To determine S_2:

The vertical shear at the section $a - a$ is:

$$+37.5 - 2 \times 10 - 5 = +12.5.$$

As the chord stresses do not exert a vertical component, the equation is:

$$+12.5 + S_2 = 0$$

$S_2 = -12.5$, which is a compressive stress of 12,500 pounds.

Note that in this case the angle which the member makes with the vertical is zero, and the cosine and secant are unity.

To determine S_3:

The vertical shear at the section $b-b$ is

$$+37.5 - 2 \times 10 - 2 \times 5 = +7.5.$$

The equation is:

$$+7.5 - S_3 \cos \phi = 0$$

$$S_3 = +7.5 \sec \phi.$$

Sec ϕ is equal to $\sqrt{30^2 + 25^2} \div 30$, which is equal to 1.302; and therefore,

$$S_3 = +7\,5 \times 1\,302$$
$$= +9.765, \text{ which is a tensile stress of } 9{,}765 \text{ pounds}$$

31. **Stresses in Chord Members.** The stresses in chord may be obtained by either the method of moments or the method of resolution of forces, this latter being usually the resolution of horizontal forces.

In accordance with the text of Article 29, the following rule may be stated with regard to the solution of stresses in chord members by the method of moments:

Pass a plane section cutting the member whose stress is to be computed, and as few others as possible; then take the center of moments at such a point that the lines of action of as many forces as possible, the unknown one excepted, pass through that point, write an equation of the moments about this point of the known loads and forces to the left of the section, assuming the unknown force to act away from the section, and taking the known forces to act as given, the tensile stresses to act away from the section, and the compressive stresses to act towards the section; place the equation equal to zero, and solve.

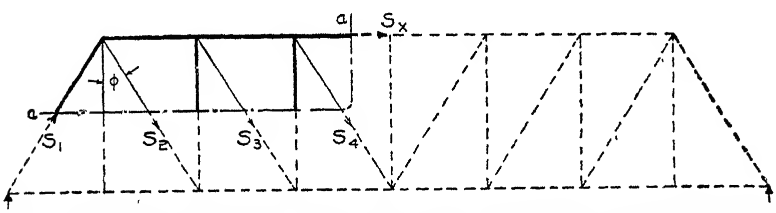


Fig. 29. Calculation of Stresses in Chord Members by "Tangent" or "Chord-Increment" Method.

The stress will solve out with its correct characteristic sign.

In the majority of cases a section can be made to cut three members only, one of the three being the one whose unknown stress is desired. In such cases, take the center of moments at the intersection of the other two, and proceed as before. As examples of this latter case, note the centers of moments at U_2, Fig. 26, and O, Fig. 27, and also the equations resulting therefrom.

When the method of resolution of forces is used, it is usually designated as the *tangent method* or the *chord increment method.* The simplest application of this method is to trusses with horizontal chords and vertical posts in the web members. Then the stress in any chord member is equal to the product of the sum of the shears

in the panels up to that section, and the tangent of the angle which the diagonals make with the vertical.

This can readily be proved by reference to Fig. 29. Let it be required to determine the stress in the chord member S_x. Pass the section $a-a$. The stresses S_1, S_2, S_3, and S_4 are now computed, and are $S_1 = -V_1 \sec\phi$; $S_2 = +V_2 \sec\phi$; $S_3 = +V_3 \sec\phi$; and $S_4 = +V_4 \sec\phi$. Now noting the directions of the known stresses and

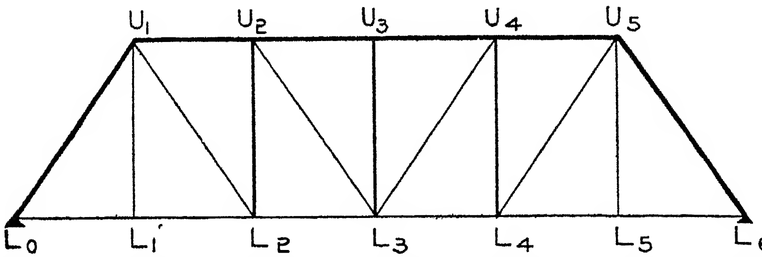


Fig. 30 Illustrating Method of Notation of Stresses and Members in a Through Bridge

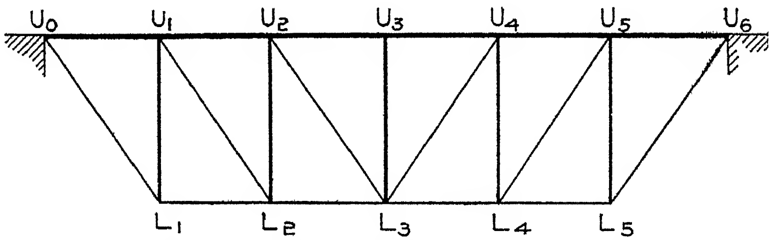


Fig. 31. Illustrating Method of Notation of Stresses and Members in a Deck Bridge.

assuming S_x to act away from the section, the equation of the horizontal component is:

$$+S_1 \sin\phi + S_2 \sin\phi + S_3 \sin\phi + S_4 \sin\phi + S_x = 0.$$

Now, substituting the values of S_1, S_2, etc., and remembering that $\sec\phi = \dfrac{1}{\cos\phi}$, the equation becomes:

$$V_1 \frac{\sin\phi}{\cos\phi} + V_2 \frac{\sin\phi}{\cos\phi} + V_3 \frac{\sin\phi}{\cos\phi} + V_4 \frac{\sin\phi}{\cos\phi} + S_x = 0,$$

from which,

$$S_x = -(V_1 + V_2 + V_3 + V_4) \tan\phi;$$
$$S_x = -\Sigma V \tan\phi.$$

From inspection of Fig. 29, it will be noticed that the stress in any section of the chord is equal to that in the section to the left of

it, *plus* the increment (horizontal component) of the diagonal; hence the name *chord increment method*.

32. **Notation.** The practice hitherto used in designating stresses by S_1, S_2, etc., will now be discontinued, as it is inconvenient in the extreme; moreover, it is not the method used in practical work. The notation to be used is that given in Figs. 30 and 31, the former being for a through and the latter for a deck truss.

The practical advantages of this system are very great. When $U_1 U_2$ is noted, it is at once known to be the top chord of the second panel; $U_2 L_2$ is known to be the second vertical; while $U_2 L_3$ is at once recognized as the diagonal in the third panel. A stress in a member, as well as the member itself, is designated by the subscript letters at its ends. Thus $U_1 L_2$ may mean the member

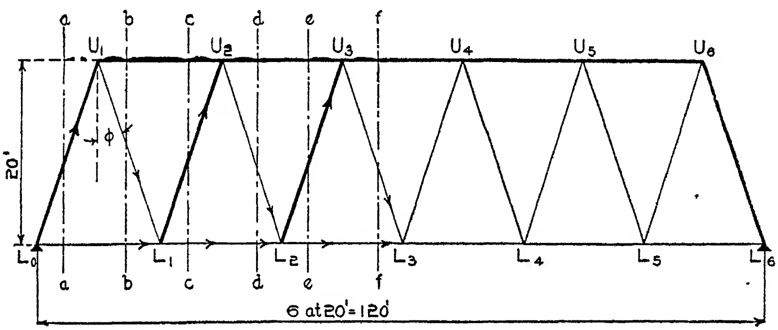


Fig 32. Calculation of Stresses in a Six-Panel Warren Truss Through Bridge.

itself or the stress in the member. The text will clear this up. In analysis, the stress would be implied, while in design the member itself would be intended.

33. **Warren Truss under Dead Loads.** The Warren truss has its web members so built of angles and plates or of channels, that they can take either tension or compression. The top chord is of structural shapes, while the lower chord may be of built-up shapes or simply of bars.

Let it be required to determine all of the stresses in the six-panel truss of a through Warren highway 120-foot span for country traffic. The height is to be 20 feet. The outline is given in Fig. 32. According to Fig. 16, the total weight of the span, including wooden floor, is 76 000 pounds. Each truss carries one-half of this, or 76 000

$\div 2 = 38\ 000$ pounds. As there are six panels, each panel load is $38\ 000 \div 6 = 6\ 333$ pounds. This means that we must compute the stresses in the above truss by considering that a load of 6 333 pounds is at points L_1, L_2, L_3, L_4, and L_5. Of course there is some weight at L_0 and L_6; but this does not stress the bridge, as it is directed over the abutments or supports. The reactions at L_0 and L_6 are each equal to $(5 \times 6\ 333) \div 2 = 15\ 833$ pounds.

The shears are next computed, and are:

$$V_1 = +15\ 833 - 0 = +15\ 833$$
$$V_2 = +15\ 833 - 6\ 333 = +\ 9\ 500$$
$$V_3 = +15\ 833 - 2 \times 6\ 333 = +\ 3\ 167$$

It is unnecessary to go past the center of the bridge, as it is symmetrical. The V_1 represents the shear on any section between L_0 and L_1; V_2 represents the shear on any section between L_1 and L_2; and V_3 represents the shear on any section between L_2 and L_3. The secant of the angle ϕ is:

$$\left(\sqrt{(\overline{20}^2 + \overline{10})^2}\right) \div 20 = 1\ 12$$

The stresses in the web members are computed as follows:

For L_0U_1. Pass section $a-a$. Assume stress acting away from the section, as shown. Then,

$$V_1 + L_0U_1 \cos \phi = 0,$$
$$L_0U_1 = -V_1 \sec \phi,$$
$$L_0U_1 = -\ 15\ 833 \times 1.12 = -\ 17\ 700 \text{ pounds},$$

which shows that L_0U_1 has a compressive stress of 17 700 pounds.

For U_1L_1. Pass section $b-b$. Assume stress acting away from the section, as shown. Then,

$$V_1 - U_1L_1 \cos \phi = 0;$$
$$U_1L_1 = +V_1 \sec \phi;$$
$$U_1L_1 = +15\ 833 \times 1.12 = +\ 17\ 700 \text{ pounds},$$

which shows that U_1L_1 has a tensile stress of 17 700 pounds.

For L_1U_2. Pass section $c-c$. Then,

$$V_2 + L_1U_2 \cos \phi = 0;$$
$$L_1U_2 = -V_2 \sec \phi;$$
$$L_1U_2 = -\ 9\ 500 \times 1.12 = -10\ 640 \text{ pounds}.$$

For U_2L_2. Pass section $d-d$. Then,

$$+9\ 500 - U_2L_2 \cos \phi = 0;$$
$$U_2L_2 = +9\ 500 \times 1.12 = +10\ 640.$$

For L_2U_3. Pass section $e - e$. Then,

$$+3\,167 + L_2U_3 \cos\phi = 0;$$
$$L_2U_3 = -3\,167 \times 1\,12 = -3\,540.$$

For U_3L_3. Pass section $f - f$. Then,

$$+3\,167 - U_3L_3 \cos\phi = 0,$$
$$U_3L_3 = +3\,167 \times 1\,12 = +3\,540.$$

The computation of the stresses in the chords is made by the method of moments, and is as follows:

For L_0L_1. Section $b - b$ cuts U_1L_1 and U_1U_2, besides the member whose stress is desired, and therefore the center of moments will be taken at their intersection U_1. The equation is:

$$+15\,833 \times 10 - L_0L_1 \times 20 = 0,$$

whence,

$$L_0L_1 = (+15\,833 \times 10) - 20,$$
$$= +7\,917 = \text{a tension of } 7\,917 \text{ pounds}$$

For L_1L_2. Either section $c - c$ or $d - d$ may be used, and each shows the center of moments to be at U_2. The equation is:

$$+15\,833 \times 30 - 6\,333 \times 10 - L_1L_2 \times 20 = 0,$$
$$L_1L_2 = (+15\,833 \times 30 - 6\,333 \times 10) - 20,$$
$$= +20\,583 = \text{a tensile stress of } 20\,583 \text{ pounds.}$$

For L_2L_3. Either section $e - e$ or $f - f$ may be used, and each shows the center of moments to be at U_3. The equation is:

$$+15\,833 \times 50 - 6\,333 \times 30 - 6\,333 \times 10 - L_2L_3 \times 20 = 0;$$

whence,

$$L_2L_3 = +26\,917.$$

The center of moments for U_1U_2 is at L_1; for U_2U_3, it is at L_2; and for U_3U_4, it is at L_3. The following equations can now be written:

$+20 \times U_1U_2 + 20 \times 15\,833 = 0$, whence $U_1U_2 = -15\,833$;
$+20 \times U_2U_3 + 40 \times 15\,833 - 20 \times 6\,333 = 0$, whence $U_2U_3 = -25\,333$;
$+20 \times U_3U_4 + 60 \times 15\,833 - 40 \times 6\,333 - 20 \times 6\,333 = 0$; whence $U_3U_4 = -28\,500$.

A diagram of half of the truss should now be made, and all the stresses placed upon it. The dimensions should also be put upon this diagram. The student should cultivate this habit, as it shows him at a glance the general relation of stresses and the general rules of their variations. Fig. 33 gives the half-truss, together with the stresses and dimensions. The stresses in the members of the right half of the truss are the same as those in the corresponding members of the left half.

From inspection of the above diagram, it is seen that the chord stresses increase from the end toward the center; that the web stresses decrease from the end toward the center; and that all members slanting the same way as the end-post L_0U_1 have stresses of that sign,

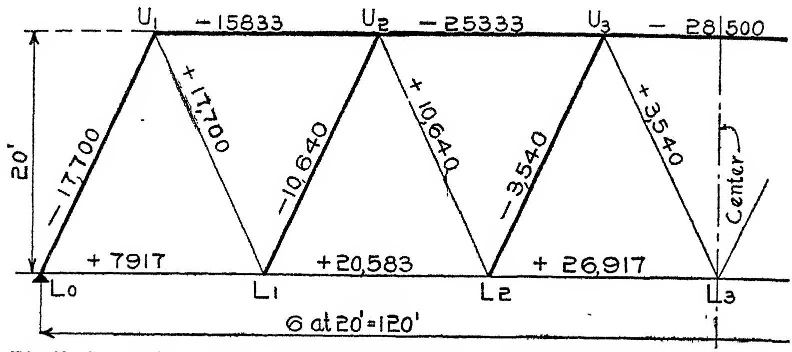


Fig. 33. Dimensions and Stress Diagram of Half a Six-Panel Warren Through Truss

while all that slant a different way have an opposite sign. These relations are true of all trusses with parallel chords and simple systems of webbings.

34. **Position of Live Load for Maximum Positive and Negative Shears.** The dead load, by reason of its nature, is an unchangeable load. The stresses due to it are the same at any and at all times.

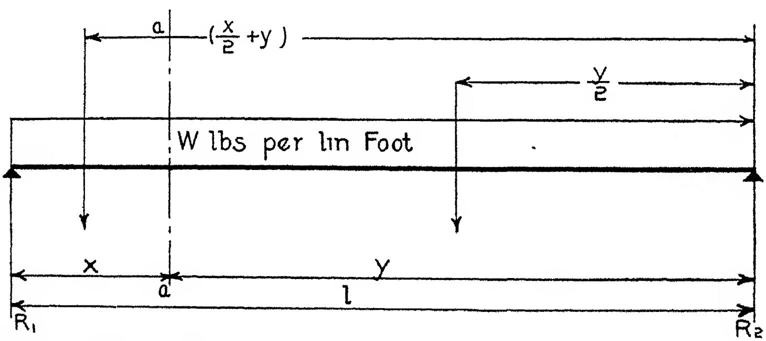


Fig 34. Calculating Maximum Positive and Negative Shears in Simple Beam under Live Load. Conventional Method

With the live load, the case is different. The live load represents the movement of traffic upon the bridge. At certain times there may be none on the bridge, while at other times it may fill the bridge partially or entirely. In such cases the shears due to live load will vary.

Conventional Method. It has been found that *the maximum positive shear at any section of a simple beam occurs when the beam is loaded from that section to the right support,* and that *the maximum negative shear occurs at the same section when this beam is loaded from the section to the left support.* This can be proved as follows:

Let a beam be as in Fig. 34, and let $a - a$ be the section under consideration. The reaction R_1 is due to the load wy on the part y, and to the load wx on the part x. That is,

$$R_1 = \left\{ wx\left(\frac{x}{2} + y\right) + wy\frac{y}{2} \right\} \div l.$$

Now the shear at the section $a - a$ is $R_1 - wx$; or,

$$\left\{ wx\left(\frac{x}{2}+y\right) + wy\frac{y}{2} \right\} \frac{1}{l} - wx = V_{a-a}$$

$$wx\left(\frac{\frac{x}{2}+y}{l}\right) + \frac{wy^2}{2l} - wx = V_{a-a}$$

$$\left\{ wx\left(\frac{\frac{x}{2}+y}{l}\right) - wx \right\} + \frac{wy^2}{2l} = V_{a-a}$$

From inspection of this last equation, it is seen that $wx\left(\frac{\frac{x}{2}+y}{l}\right)$ is the amount that is added to the reaction by loading the part x. Also, that $\frac{\frac{x}{2}+y}{l}$ is less than unity, is evident. The amount in brackets in the last equation represents the effect of the loading of the segment x of the beam. As this is negative and will only reduce the positive valued term $\frac{wy^2}{2l}$, it is therefore proved that to get the largest positive shear the beam should be loaded from the section to the right support.

From further inspection of the equation, it will be seen that the term in brackets, which represents the effect of the load on the segment x on the shear, is always negative; and that the term $\frac{wy^2}{2l}$, which represents the effect of the load on the segment y on the shear, is always positive. Hence, to get the largest negative shear at the section, the load should be on the segment x. That is, the loading should be from the section to the left support.

In a truss, the loads are placed at the panel points: and the above rules in application, should be formulated as follows:

To get the maximum positive shear at a section or in a panel, load all panel points to the right of it

To get the maximum negative shear at a section or in a panel, load all panel points to the left of it.

Example. Determine the maximum positive and the maximum negative shears in the panels of the 7-panel Pratt truss shown in Fig. 35, the

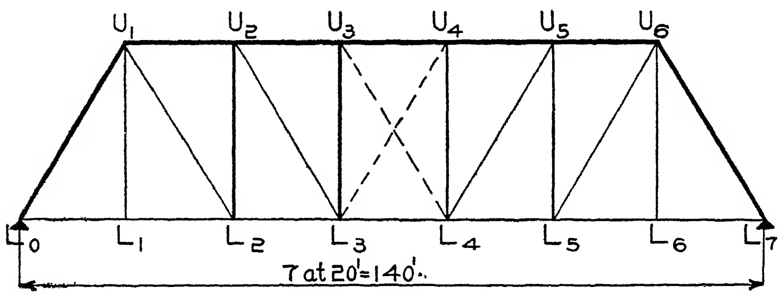


Fig. 35 Calculation of Shears in Panels of 7-Panel Pratt Truss.

live panel load being 40 000 pounds. (It will be noticed that the height of the truss is not required)

For maximum $+V$ in 1st panel, load L_1, L_2, L_3, L_4, L_5 and L_6.
" " +V " 2d " " L_2, L_3, L_4, L_5, and L_6.
" " +V " 3d " " L_3, L_4, L_5, and L_6.
" " +V " 4th " " L_4, L_5, and L_6.
" " +V " 5th " " L_5 and L_6
" " +V " 6th " " L_6.
" " +V " 7th " " no panel points at all.

The reaction produced by each of the loadings is equal to the shear for that particular case, since the shear at any section or in any panel is equal to the reaction *minus* the loads to the left of the section or panel, and, according to the method of loading, there are no loads to the left of the section; therefore the reaction is equal to the shear.

For the first panel, the computation is made as follows, the center of moments being, of course, at L_7:

$$(R_1 = +V_1) \times 7 \times 20 = 40 \times 20 + 40 \times 2 \times 20 + 40 \times 3 \times 20 + 40 \times 4 \times 20 + 40 \times 5 \times 20 + 40 \times 6 \times 20.$$

It will be seen that as 20 occurs in all terms of this equation, it can be factored out by dividing both sides by 20, and the result will be the same. The equation can now be written:

$$+V_1 \times 7 = 40 + 40 \times 2 + 40 \times 3 + 40 \times 4 + 40 \times 5 + 40 \times 6,$$

and can still be simplified by writing:

$$+V_1 = \frac{40}{7}(1 + 2 + 3 + 4 + 5 + 6) = +120\ 00,$$

which is the form customarily used, the panel length being taken as a unit of measurement. The other shears are now easily computed in a similar manner:

$$+V_2 = \frac{40}{7}(1 + 2 + 3 + 4 + 5) = +85.71$$

$$+V_3 = \frac{40}{7}(1 + 2 + 3 + 4) = +57.14$$

$$+V_4 = \frac{40}{7}(1 + 2 + 3) = +34\ 28$$

$$+V_5 = \frac{40}{7}(1 + 2) = +17.14$$

$$+V_6 = \frac{40}{7}(1) = +5\ 71$$

$$+V_7 = \frac{40}{7}(0) = +0$$

In computing the maximum negative shears, sometimes called the *minimum shears*, the reaction is not the same as the shear, as there are loads to the left of the section, and these must be subtracted. The loadings are:

For maximum	$-V$	in 1st	panel,	load	no points.
"	" $-V$	" 2d	"	"	L_1.
"	" $-V$	" 3d	"	"	L_1 and L_2.
"	" $-V$	" 4th	"	"	L_1, L_2, and L_3.
"	" $-V$	" 5th	"	"	L_1, L_2, L_3, and L_4.
"	" $-V$	" 6th	"	"	L_1, L_2, L_3, L_4, and L_5.
"	" $-V$	" 7th	"	"	L_1, L_2, L_3, L_4, L_5, and L_6

It is evident that the maximum $-V_1$ is equal to zero, there being no loads on the span. The maximum negative shear in the second panel is equal to the reaction produced by loading the panel point L_1, and the load at L_1. Thus,

$$7R_1 = 40 \times 6$$

$$R_1 = \frac{40}{7}(6)$$

$$-V_2 = R_1 - \text{load at } L_1$$

$$= \frac{40}{7}(6) - 40$$

$$= -5.71$$

The other shears are next computed as follows:

$$-V_3 = \frac{40}{7}(6 + 5) - 2 \times 40 = -17\ 14$$

$$-V_4 = \frac{40}{7}(6 + 5 + 4) - 3 \times 40 = -34.28$$

$$-V_5 = \frac{40}{7}(6 + 5 + 4 + 3) - 4 \times 40 = -57\ 14$$

$$-V_6 = \frac{40}{7}(6 + 5 + 4 + 3 + 2) - 5 \times 40 = -85.71$$

$$-V_7 = \frac{40}{7}(6 + 5 + 4 + 3 + 2 + 1) - 6 \times 40 = -120\ 00$$

The maximum positive and the maximum negative live-load shears should now be written side by side, and inspected, in order to observe any existing relations which might help to lessen the labor of future computations. The values are given in thousands of pounds below:

Location	Max + Live-Load Shear	Max − Live-Load Shear
V_1	+120 00	− 0 00
V_2	+85.71	− 5 71
V_3	+57.14	−17.14
V_4	+34.28	−34 28
V_5	+17.14	−57 14
V_6	+5.71	−85.71
V_7	+0.00	−120.00

It is at once seen that the negative shears are numerically equal in value to the positive ones, but that they occur in reverse order. This simplifies the labor required in the derivation of the negative shears; for, after computing the maximum positive shears, these may be written in reverse order, and the negative sign prefixed; the result will be the maximum negative shears.

The above method for maximum live-load shears is called the *conventional method.* It is the one that is almost universally used, and its use will be continued throughout this text.

Exact Method. On account of the fact that the floor stringers or joists transfer the loads to the panel points, it would be impossible to have a full panel live load at one panel point and no load at the panel point ahead or behind. In order to have a full panel load at one point, the stringers in the panels on both sides of the point must

be full-loaded, and this would give a load at the panel point ahead, provided the bridge was fully loaded up to and not beyond the panel point ahead, equal in value to one-half of a full panel load (see Fig.

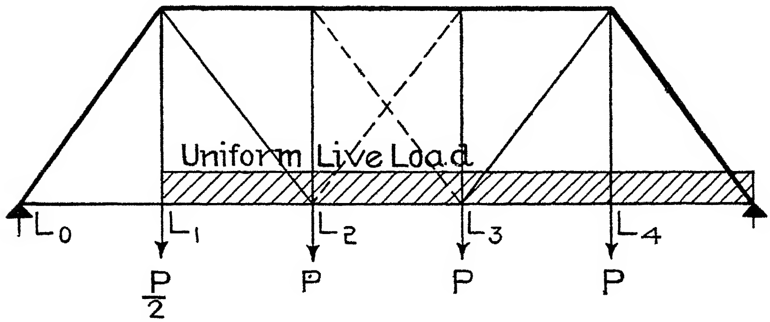


Fig 36 Illustrating 'Exact" Method of Calculating Live-Load Shears in Panels.

36). The uniform live load, in order to produce full panel loads at L_2, L_3 and L_4, will also produce one-half a panel load at L_1.

By the methods of differential calculus, it can be proved that the true maximum positive live-load shear occurs in a panel when the

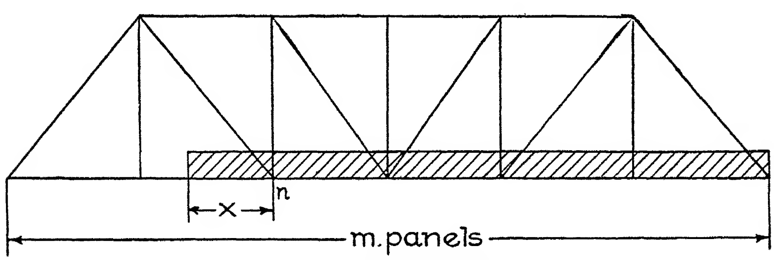


Fig. 37. Calculating Maximum Positive Live-Load Shear in Panel.

live load extends from the panel point to the right into that panel an amount (see Fig. 37) equal to

$$x = \frac{n}{m-1} p,$$

in which,

n = Number of the panel point to the right of the panel under consideration, counting from the left;

m = Total number of panels in the bridge;

p = Panel length.

Let the truss of Fig. 35 be considered, the live load being 2 000

pounds per linear foot of truss, and let it be required to determine the true maximum positive live-load shear in the 5th panel *from the right end.*

$$x = \frac{4}{7-1} \times 20 = 13\ 333 \text{ feet.}$$

There will now be $(4 \times 20 + 13.333) \times 2\ 000 = 186\ 666$ pounds on the truss; and the left reaction will be $\{186\ 666 \times (4 \times 20 + 13.333) \div 2\} \div 140 = 62\ 200$ pounds. From this must be subtracted the amount of the load on the 13.333 feet, which is transferred to the point L_1. This is equal to the reaction of a beam of a span equal to the panel length, loaded for a distance of 13.333 feet from the right support with a uniform load of 2 000 pounds per linear foot. This amounts to $(13.333 \times 2\ 000 \times \frac{13.333}{2}) \div 20 = 8\ 890$ pounds. The true shear is now:

$$+V_3 = +62\ 200 - 8\ 890 = +53\ 310 \text{ pounds.}$$

The $+V_3$, as computed by the conventional method, was $+\ 57\ 140$, making a difference of 3 730 pounds between the two. If the true shears were computed and compared with the conventional shears, it would be found that the V_1 would be the same, and that the remainder of the conventional shears would be greater than the corresponding true shears. The difference between any two corresponding shears would increase from the left to the right end; that is, the difference between the conventional and exact shears would be greatest in the panel L_5L_6.

To get the maximum negative shear in any panel, load from the left support and out into the panel under consideration an amount $p - x$, and proceed in a manner similar to that above described.

As this method of exact or true shears is seldom employed, problems illustrating its application will here be omitted.

35. **Position of Live Load for Maximum Moments.** *In order to obtain the maximum moment at any point, the live load must cover the entire bridge.* Let the beam of Fig. 34 be considered, and let it be required to obtain the maximum moment at the section $a - a$. The reaction, as before computed, is:

$$R_1 = \left\{ wx \left(\frac{x}{2} + y \right) + \frac{wy^2}{2} \right\} \frac{1}{l},$$

all terms of which are positive. The moment at the section is:

$$M = R_1 \times x - wx\frac{x}{2};$$

and substituting for R_1 its value,

$$M = \frac{wx^2}{l}\left(\frac{x}{2} + y\right) + \frac{wy^2x}{2l} - \frac{wx^2}{2};$$

$$= \frac{wx^2}{l}\left(\frac{x}{2} + y\right) - \frac{wx^2}{2} + \frac{wy^2x}{2l};$$

$$= wx^2\left(\frac{x}{2l} + \frac{y}{l} - \frac{1}{2}\right) + \frac{wy^2x}{2l}.$$

But $y = l - x$; therefore,

$$M = wx^2\left(\frac{x}{2l} + \frac{l-x}{l} - \frac{1}{2}\right) + \frac{wy^2x}{2l};$$

$$= wx^2\left(\frac{2l-x}{2l} - \frac{1}{2}\right) + \frac{wy^2x}{2l}.$$

The first term of this equation represents the effect of the load on the portion x, and the second term represents the effect of the load on the portion y. The value of M will always be positive. The quantity x varies between 0 and l. When $x = 0$, M is equal to 0. When $x = l$, $y = 0$ and the moment is equal to zero. For all values of x between 0 and l, the first term is positive; and the second term being positive in all cases, it is therefore proved that for maximum live-load moments at any point, the entire span should be loaded, as loads on both segments add positive values to the moment value.

36. **Warren Truss under Live Load.** In order to analyze a truss intelligently, it is necessary to know its physical structure; that is, it must be known what character of stress can be withstood by the different members. The top chords of all trusses are built to take only compression, and the bottom chords are built to take only tension; while some web members of some trusses are built for tension stresses, some for compression stresses, and some for both. The characteristic of the Warren truss is that the web members are built so as to be able to withstand either tension or compression.

Let it be required to determine the live-load stresses in the Warren truss of Fig. 32. Let the live load per square foot of roadway, which is assumed to be 15 feet wide, be 100 pounds. The live panel load is then $100 \times 15 \times 20 \div 2 = 15\,000$ pounds, and the live-load reaction under full load is $2\frac{1}{2} \times 15\,000 = 37\,500$ pounds.

As the live load must cover the entire bridge to give maximum moments—and therefore maximum chord stresses, as the chord stress is equal to the moment divided by the height of the truss—a simple method for the determination of live-load chord stresses presents itself. The live load and the dead load being applied at the same points, and being different in intensity, the stresses produced will be proportional to the panel loads. The maximum live-load chord stresses (see Fig. 33) will then be equal to the dead-load chord stresses multiplied by 15 000 ÷ 6 333 = 2.371, and they are as follows:

$$
\begin{aligned}
L_0U_1 &= -2\,371 \times 17\,700 = -42\,000 \\
U_1U_2 &= -2\,371 \times 15\,833 = -37\,530 \\
U_2U_3 &= -2.371 \times 25\,333 = -60\,050 \\
U_3U_4 &= -2.371 \times 28\,500 = -67\,600 \\
L_0L_1 &= +2\,371 \times 7\,917 = +18\,770 \\
L_1L_2 &= +2\,371 \times 20\,583 = +48\,800 \\
L_2L_3 &= +2.371 \times 26\,917 = +63\,850
\end{aligned}
$$

The next step in order is to determine the maximum positive shears, and from these write the maximum negative shears. This is done as follows:

	+ Live-Load V	− Live-Load V
$V_1 = \frac{15\,000}{6}(1+2+3+4+5)$	$= +37\,500$	0
$V_2 = \frac{15\,000}{6}(1+2+3+4)$	$= +25\,000$	$-2\,500$
$V_3 = \frac{15\,000}{6}(1+2+3)$	$= +15\,000$	$-7\,500$
$V_4 = \frac{15\,000}{6}(1+2)$	$= +\ 7\,500$	$-15\,000$
$V_5 = \frac{15\,000}{6}$	$= +\ 2\,500$	$-25\,000$
$V_6 =$	$+\ 0$	$-37\,500$

The stresses produced by the positive shears are called the *maximum live-load stresses*, and are:

$$
\begin{aligned}
+L_0U_1 \cos\phi + 37\,500 &= 0 & \therefore L_0U_1 &= -37\,500 \times 1.12 = -42\,000 \\
-U_1L_1 \cos\phi + 37\,500 &= 0 & \therefore U_1L_1 &= +37\,500 \times 1\ 12 = +42\,000 \\
+L_1U_2 \cos\phi + 25\,000 &= 0 & \therefore L_1U_2 &= -25\,000 \times 1.12 = -28\,000 \\
-U_2L_2 \cos\phi + 25\,000 &= 0 & \therefore U_2L_2 &= +25\,000 \times 1.12 = +28\,000 \\
+L_2U_3 \cos\phi + 15\,000 &= 0 & \therefore L_2U_3 &= -15\,000 \times 1\ 12 = -16\,800 \\
-U_3L_3 \cos\phi + 15\,000 &= 0 & \therefore U_3L_3 &= +15\,000 \times 1.12 = +16\,800
\end{aligned}
$$

The stresses produced by the negative shears are called the *minimum live-load stresses*, and are:

$$
\begin{array}{ll}
+L_0U_1 \cos \phi + 0 \quad = 0 & \therefore L_0U_1 = 0 \\
-U_1L_1 \cos \phi + 0 \quad = 0 & \therefore U_1L_1 = 0 \\
+L_1U_2 \cos \phi - 2\,500 = 0 & L_1U_2 = +2\,500 \times 1.12 = +2\,800 \\
-U_2L_2 \cos \phi - 2\,500 = 0 & U_2L_2 = -2\,500 \times 1.12 = -2\,800 \\
+L_2U_3 \cos \phi - 7\,500 = 0 & \therefore L_2U_3 = +7\,500 \times 1.12 = +8\,400 \\
-U_3L_3 \cos \phi - 7\,500 = 0 & \therefore U_3L_3 = -7\,500 \times 1.12 = -8\,400
\end{array}
$$

These stresses, together with the dead-load stresses, should now be placed together as a half-diagram, as is done in Fig. 38, the stresses being rounded off to the nearest ten pounds and then expressed in thousands of pounds. No minimum live-load stress is given for the chords, as this will evidently be zero in all cases, since no position of the live load will cause a reversal of stress. It will be seen that the stresses produced by the negative shears are of opposite

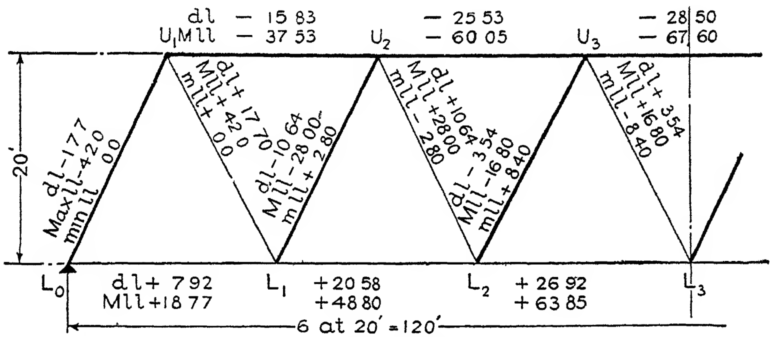
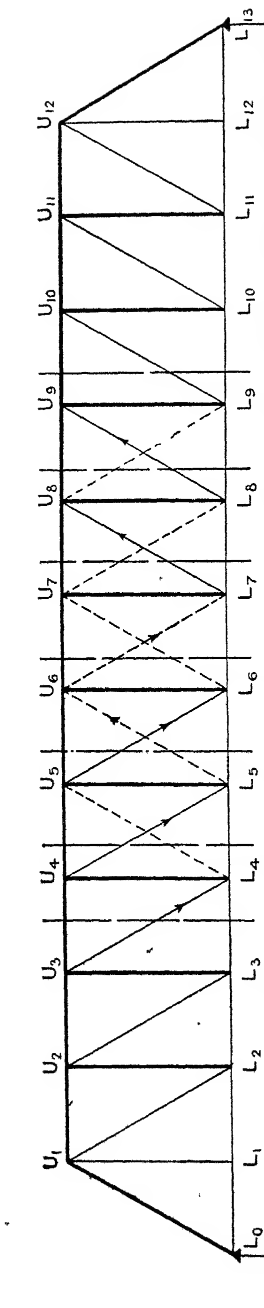


Fig. 38. Dimension and Stress Diagram of Warren Half-Truss under Live Load.

sign from the stress produced by the dead load, and these tend to decrease the dead-load stress by that amount; and in some cases (see L_2U_3 and U_3L_3, Fig. 38) it will be so large as to overcome the dead-load stress and therefore change the total stress from one kind to another. Do not forget, in considering any combination of the above stresses, that the dead load occurs with either the maximum or the minimum live load, but *not with both at the same time.*

37. **Counters.** By reference to U_3L_3 (Fig. 38), it is seen that when the live load is on the panel points L_1 and L_2 the total stress in the member is $+3.54 + (-8.40) = -4.86$, a compressive stress of 4 860 pounds; whereas, under dead load alone, the stress was $+3.54$, a tensile stress of 3 540 pounds. If the member U_3L_3 had been built of long, thin bars which could take only tension, and which consequently would have doubled up under the resultant compression



	L0–L1	L1–L2	L2–L3	L3–L4	L4–L5	L5–L6	L6–L7	L7–L8	L8–L9	L9–L10	L10–L11	L11–L12	L12–L13
d.l.	+ 132.0	+ 110.0	+ 88.0	+ 66.0	+ 44.0	+ 22.0	0	− 22.0	− 44.0	− 66.0	− 88.0	− 110.0	− 132.0
+ l.l.	+ 351.0	+ 297.0	+ 247.5	+ 202.5	+ 162.0	+ 126.0	+ 94.5	+ 67.5	+ 45.0	+ 27.0	+ 13.5	+ 4.5	+ 0.0
− l.l.	− 0.0	− 4.5	− 13.5	− 27.0	− 45.0	− 67.5	− 94.5	− 126.0	− 162.0	− 202.5	− 247.5	− 297.0	− 351.0

Fig. 39. Load Diagram of 13-Panel Pratt Truss.

brought upon them by the combined action of the dead and minimum live-load stresses, then this member could not be used in this case, but some other arrangement would be necessary in order to insure the stability of the truss.

In the Warren truss, no special arrangement is necessary, as the web members are built so as to take either tension or compression; but with the Pratt and Howe trusses some special arrangement is necessary, as the diagonals are built to take one kind of stress only. The case of the Pratt will be considered first.

The Pratt truss has the diagonals made of long bars which take tension only, and the intermediate posts are constructed so as to be able to take compression only. It is not necessary to consider the intermediate posts, for the action of the web members is such that the total resulting stresses are always compressive.

Let the 13-panel Pratt truss of Fig. 39 be considered. The panel length is 18 feet, the height 25 feet, the dead panel load 22 000 pounds, and the live panel load 58 500 pounds. The secant is $(18^2 + 25^2)^{\frac{1}{2}} \div 25 = 1.231$. The dead-load shears and the maximum and minimum live-load shears are placed directly below their respective panels. Only those members are shown full-lined in Fig. 39 which act under the dead load. Note that the dead-load shears in the center

panel being zero, the dead-load stress in the diagonals in the center panel would be $0 \times \sec \phi = 0$.

In the first four panels from either end, the live-load shear, which is of a different sign from that of the dead-load shear, is smaller than the dead-load shear, and therefore will not cause a reversal of stress in the member in that panel. For example, take $U_3 L_4$; then, for dead-load stress,

$$-U_3L_4 \cos \phi + 66.0 = 0 \qquad U_3L_4 = +66.0 \times 1.231 = +81.20$$

For live-load stress,

$$-U_3L_4 \cos \phi - 27.0 = 0 \qquad \therefore U_3L_4 = -27.0 \times 1.231 = -33.25$$

The total stress $= +81.20 - 33.25 = +47.95$, which is still tension and can be taken by the thin bar U_3L_4.

Considering $L_9 U_{10}$, the stress equations are:

For dead-load stress,

$$+L_9U_{10} \cos \phi - 66.0 = 0 \qquad \therefore L_9U_{10} = +81.20$$

For live-load stress,

$$+L_9U_{10} \cos \phi + 27.0 = 0 \qquad \therefore L_9U_{10} = -33.25$$

The total stress, as before, is $+ 47.95$, or a tension of 47 950 pounds.

An inspection of the center panel and the two panels on each side of it, shows that the live-load shear is of a different sign from the dead-load shear, and is also greater in value than the dead-load shear. If the members shown in full lines in Fig. 39 were the only ones in the panels, then the dead-load stresses would be:

$$\begin{aligned}
-U_4L_5 \cos \phi + 44.0 &= 0 \qquad & U_4L_5 &= +54.20 \\
-U_5L_6 \cos \phi + 22.0 &= 0 \qquad & U_5L_6 &= +27.10 \\
+L_7U_8 \cos \phi - 22.0 &= 0 \qquad & L_7U_8 &= +27.10 \\
+L_8U_9 \cos \phi - 44.0 &= 0 \qquad & L_8U_9 &= +54.20
\end{aligned}$$

and the live-load stresses caused by the shear of opposite sign from that of the dead-load shear, are:

$$\begin{aligned}
-U_4L_5 \cos \phi - 45.0 &= 0 \qquad & U_4L_5 &= -55.40 \\
-U_5L_6 \cos \phi - 67.5 &= 0 \qquad & U_5L_6 &= -83.10 \\
+L_7U_8 \cos \phi + 67.5 &= 0 \qquad & L_7U_8 &= -83.10 \\
+L_8U_9 \cos \phi + 45.0 &= 0 \qquad & L_8U_9 &= -55.40
\end{aligned}$$

As no diagonal acts under dead load in the center panel, we may assume that $U_6 L_7$ acts under live load. The stresses which occur in this are:

$$\begin{aligned}
-U_6L_7 \cos \phi + 94.5 &= 0 \qquad & U_6L_7 &= +116.30 \\
-U_6L_7 \cos \phi - 94.5 &= 0 \qquad & U_6L_7 &= -116.30
\end{aligned}$$

The above shows that compressive stresses will occur in the diagonals which were built for tension only. These stresses are:

$$
\begin{array}{lllll}
U_4L_5 = & +54\ 20 - & 55\ 40 = & -\ \ 1\ 200 & \text{pounds} \\
U_5L_6 = & +27.10 - & 83.10 = & -\ 56\ 000 & \text{"} \\
L_7U_8 = & +27.10 - & 83\ 10 = & -\ 56\ 000 & \text{"} \\
L_8U_9 = & +54\ 20 - & 55\ 40 = & -\ \ 1\ 200 & \text{"} \\
U_6L_7 = & 0 - & 116\ 30 = & -116\ 300 & \text{"}
\end{array}
$$

If some provision were not made for these stresses, they would cause the members to crumple up and the truss to fail. In order to allow for them, diagonals are placed in the panels, as shown by the dashed lines. These members will take up the above stress; and moreover, as they slope the opposite way from the main members, they will be in tension.

In order to prove this, assume L_5U_6 to act when the live load is on points L_5, L_4, L_3, L_2, and L_1. Now, U_5L_6 will not be regarded, as its stress will be zero. Then the stresses will be:

For dead load,

$$+L_5U_6 \cos \phi + 22.0 = 0 \qquad L_5U_6 = -27.10.$$

For live load,

$$+L_5U_6 \cos \phi - 67.5 = 0 \qquad L_5U_6 = +83\ 10;$$

and the total stress in L_5U_6 will be $-\ 27.10 + 83.10 = +\ 56.00$.

In a similar manner, the stresses in the other members are: $L_4U_5 = +1.2$; $L_6U_7 = +\ 116.30$; $U_7L_8 = +\ 56.00$; and $U_8L_9 = +\ 1.2$. These diagonals are called *counters* or *counter-bracing*.

From a consideration of the foregoing, it is evident that:

(*a*) *If the live-load shear in any panel is of opposite sign and greater than the dead-load shear in the same panel, then a counter is required.*

(*b*) *The stress in a counter is equal to the algebraic sum of the dead-load shear and the live-load shear of opposite sign times the secant of the angle it makes with the vertical.*

This is true for any truss with horizontal chords and a simple system of webbing with diagonals and verticals.

38. **Maximum and Minimum Stresses.** Some specifications require the member to be designed for the maximum stress, while others take into account the range of stress. In this latter case it is necessary to determine the minimum as well as the maximum stress. Except where a reversal of stress occurs—and this does not happen in trusses with horizontal chords—few specifications require any

but the maximum stresses to be computed. For that reason, little space will here be devoted to the minimum stresses, their computation in succeeding articles being thought to illustrate them sufficiently.

(*a*) *The maximum stress in a member is equal to the sum of the dead-load stress and the live-load stress of the same sign*

(*b*) *The minimum stress is equal to the sum of the dead-load stress and the live-load stress of the opposite sign*, or *to the dead-load stress alone*, according to which gives the smallest value *algebraically*. By this latter statement it should be seen that if the maximum stress is -58.60, then 0 or $+18.00$ would be smaller than -3.00

(*c*) It is evident that the minimum in all counters and in all main members in panels where counters are employed will be zero, for when the counter is acting the main member is not, and therefore its stress is zero. The reverse is also true

(*d*) An exception to *a* is seen in the case of the counters. Here it is evident that the maximum stress is equal to the algebraic sum of the dead-load shear and the live-load shear of opposite sign *times* the secant of the angle which the counter makes with the vertical

While it is true that in trusses with horizontal chords the loading for maximum shears will give the maximum live-load stress to be added to the dead load for *the maximum stress*, it is not always true that the loading for minimum live-load shears will give the stress to add to the dead-load stress to get the *minimum stress*. However, the loading for the minimum live-load shears will give the live-load stress to be added to the dead-load stress for the minimum stress, *except* in the case

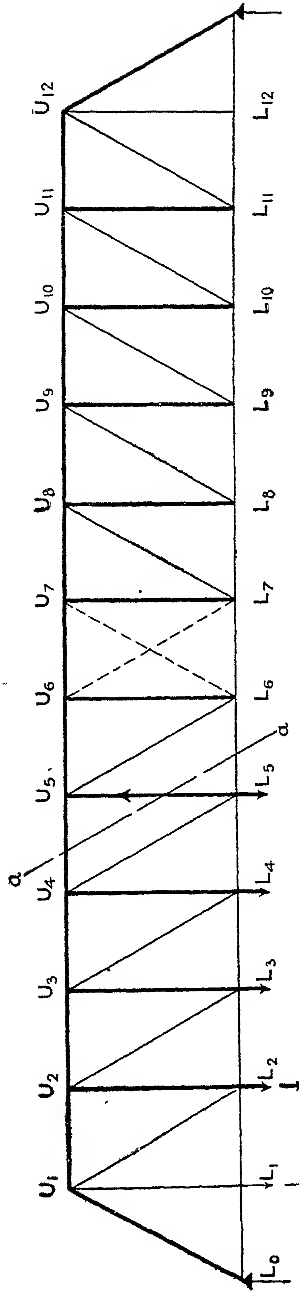
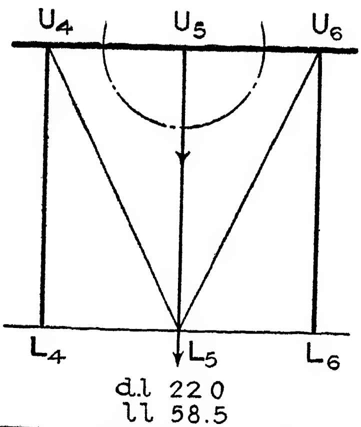


Fig. 19 Calculation of Stresses in Vertical of 13-Panel Pratt Truss

of verticals placed between panels each of which contains counters, and in that case it may or may not do so. In such cases a loading must be assumed—preferably the one for minimum shears—and the shears in the panels on each side of the vertical must be computed *for the loading assumed.* If the resultant shear is the same sign as the dead load, then the main diagonal acts; if it is of different sign, then the counter acts.



dlv	+ 440	+ 22.0
llv	− 90	− 67 5
Totalv	+	−

Fig. 41 Stress Diagram for Vertical in Truss of Fig. 40

As an example, let it be required to find the minimum stress in the vertical U_5L_5 of the truss of Figs. 39 and 40. It is assumed that the loading for minimum shears will give the result. The section $a-a$ is then passed, and the live load placed on L_5 and all points to the left. The shears will then be as shown in Fig. 41. To obtain the shear in the panel L_4L_5, *under this loading,* it must be remembered that a load is at L_5; and so the shear is the shear in the panel L_5L_6 with the panel load at L_5 added, or, $-67.5 + 58.5 = -9.0$. The diagonals now act as indicated by Fig. 41, and the total stress in U_5L_5 is determined by passing a circular section around U_5, and it is :

$$-\text{Load at } U_5 - U_5L_5 = 0.$$

As there is no load at U_5, the stress in U_5L_5 is $= 0$. The same result will occur if points L_4 or L_3 and to the left are loaded; but if points L_2 and to the left are loaded, the members U_4L_5 and U_5L_6 will act, and the stress in U_5L_5 will then be equal to the shear on the section $a-a$. The stresses are: Dead-load, -22.0; and live-load, $+13.5$, which gives a total of -8.5; but as the maximum stress is $-22.0 - 126.0 = -148.0$, it is evident that 0 and not -8.5 is the minimum.

The computation of the maximum stress is as follows:

Load points L_6 and to the right. The shear on $a-a$ is, for

dead load, $+22.0$; and for + live load, $+126.0$; and the equations of the stresses are:

$$+\ 22\ 0 + U_5L_5 = 0 \qquad U_5L_5 = -\ 22\ 0$$
$$+126\ 0 + U_5L_5 = 0 \qquad U_5L_5 = -126\ 0$$
$$\text{Max.} = -148\ 0$$

TRUSSES UNDER DEAD AND LIVE LOADS

39. **The Pratt Truss.** The Pratt truss is used to perhaps a greater extent than any other form; probably 90 per cent of all simple truss spans are of this kind

Let it be desired to determine the stresses in the 8-panel 200-foot single-track span shown in Fig. 42, the height being 30 feet, the dead panel load being 30 000 pounds, and the live panel load 62 400 pounds. The secant is $\left(\overline{25}^2 + \overline{30}^2\right)^{\frac{1}{2}} \div 30 = 1\ 302$, and the cosine is 0.7685. The dead-load reaction is $3\frac{1}{2} \times 30.0 = 105.0$.

The dead-load shears are:

$$V_1 = +105.0$$
$$V_2 = +\ 75\ 0$$
$$V_3 = +\ 45\ 0$$
$$V_4 = +\ 15\ 0$$
$$V_5 = -\ 15\ 0$$

The dead-load chord stresses may be tabulated as follows (see Articles 27 and 29):

Dead-Load Chord Stresses

Member	Section	Center of Moments	Stress Equation	Stress
$L_0L_1 = L_1L_2$	$a-a$	U_1	$+105\ 0 \times 25 - L_1L_2 \times 30 = 0$	$+\ 87\ 5$
L_2L_3	$b-b$	U_2	$+105\ 0 \times 50 - 30\ 0 \times 25 - L_2L_3 \times 30 = 0$	$+150.0$
L_3L_4	$c-c$	U_3	$+105\ 0 \times 75 - 30\ 0\ (25 + 50) - L_3L_4 \times 30 = 0$	$+187\ 5$
U_1U_2	$a-a$	L_2	$+105\ 0 \times 50 - 30\ 0 \times 25 + U_1U_2 \times 30 = 0$	-150.0
U_2U_3	$b-b$	L_3	$+105\ 0 \times 75 - 30\ 0\ (25 + 50) + U_2U_3 \times 30 = 0$	-187.5
U_3U_4	$c-c$	L_4	$+105.0 \times 100 - 30\ 0\ (25 + 50 + 75) + U_3U_4 \times 30 = 0$	-200.0

In determining dead-load stresses in web members, it is customary to assume one-third of the dead panel loads as applied at the

upper chord points. This, as will be seen, makes no difference in the stresses in the chords or in the diagonals, the stresses in the verticals only being different from what is the case when all the dead load is taken on the lower chord.

The stresses in the diagonals (see Articles 27, 28, and 30) are:

Dead-Load Stresses in Diagonals

Member	Section	Shear on Section	Stress Equation	Stress
L_0U_1	$o-o$	$+105\ 0$	$+105\ 0 + L_0U_1 \times 0\ 7685 = 0$	$-136\ 70$
U_1L_2	$a-a$	$+\ 75.0$	$+\ 75\ 0 - U_1L_2 \times 0\ 7685 = 0$	$+\ 97\ 60$
U_2L_3	$b-b$	$+\ 45\ 0$	$+\ 45\ 0 - U_2L_3 \times 0\ 7685 = 0$	$+\ 58\ 60$
U_3L_4	$c-c$	$+\ 15\ 0$	$+\ 15\ 0 - U_3L_4 \times 0\ 7685 = 0$	$+\ 19\ 53$

In determining the stresses in the verticals, it is to be remembered that one-third the dead panel load (or 10.0) is at the panel

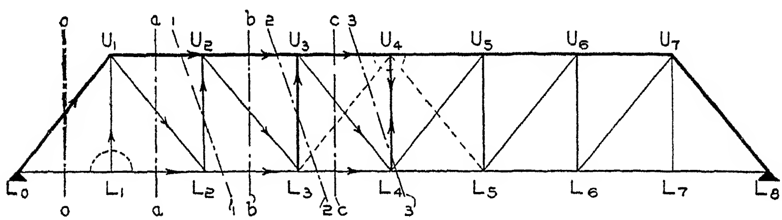


Fig 42 Outline Diagram of 8-Panel Single-Track Pratt Truss Span.

points of the upper chord, and two-thirds (or 20.0) is at the lower chord. The stress in the hip vertical U_1L_1 is determined by passing a circular section around L_1. It is solved thus:

$$-20\ 0 + U_1L_1 = 0 \qquad U_1L_1 = +20.0$$

In a similar manner the stress in U_4L_4 is found to be:

$$-10 - U_4L_4 = 0 \qquad U_4L_4 = -10.0$$

In order to find the stress in the remaining verticals, sections 1 — 1 and 2 — 2 are passed, cutting them, and the shears on these sections computed. The shears are:

$$V_{1-1} = +\ 105\ 0 - 2 \times 20 - 1 \times 10 = +55\ 0$$
$$V_{2-2} = +\ 105\ 0 - 3 \times 20 - 2 \times 10 = +25\ 0$$

The stress equations are written, remembering that as the verticals

make an angle of zero with the vertical, their cosine is equal to unity. These equations are:

$$+U_2L_2 + 55.0 = 0 \qquad U_2L_2 = -55.0$$
$$+U_3L_3 + 25.0 = 0 \qquad U_3L_3 = -25.0$$

The live-load chord stresses will be proportional to the dead-load chord stresses, as both loads cover the entire truss in exactly the same manner. The ratio of the panel loads by which the dead-load chord stresses are multiplied in order to get the live-load chord stresses, is $\frac{62\ 400}{30\ 000} = 2.08$, and the chord stresses are:

$$\begin{aligned}
L_0L_1 = L_1L_2 &= +\ 87.5 \times 2.08 = +182.0\\
L_2L_3 &= +150.0 \times 2.08 = +312.0\\
L_3L_4 &= +187.5 \times 2.08 = +390.0\\
U_1U_2 &= -150.0 \times 2.08 = \quad 312.0\\
U_2U_3 &= -187.5 \times 2.08 = -390.0\\
U_3U_4 &= -200.0 \times 2.08 = -416.0
\end{aligned}$$

As the entire bridge is to be loaded to get the maximum stress in L_0U_1, it is therefore equal to the dead-load stress *times* the above ratio; or $L_0U_1 = -136.70 \times 2.08 = -284.20$.

The maximum live-load stress in U_1L_1 is determined by passing a circular section around L_1, and is solved (see Fig. 43) from the equation:

$$+U_1L_1 - 62.4 = 0 \qquad \therefore U_1L_1 = +62.4$$

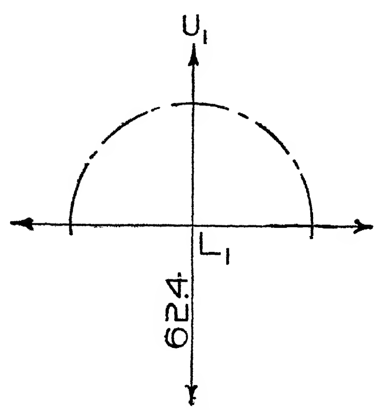


Fig 43 Calculation of Maximum Live-Load Stress in a Vertical of Span of Fig 42

For U_1L_2, the section $a - a$ is passed, and the points L_2 and to the right are loaded. The maximum shear is:

$$+V_2 = +\frac{62.4}{8}(1 + 2 + 3 + 4 + 5 + 6) = +163.8;$$

and the stress equation is:

$$+163.8 - U_1L_2 \times 0.7685 = 0 \qquad \therefore U_1L_2 = +213.2.$$

In a similar manner, pass section $b - b$, and load points L_3 and to the right, and the shear and the stress equations for U_2L_3 are:

$$+V_3 = +\frac{62.4}{8}(1 + 2 + 3 + 4 + 5) = +117.0$$

$$+117.0 - U_2L_3 \times 0.7685 = 0 \qquad \therefore U_2L_3 = +152.4$$

For U_3L_4, the section $c - c$ is passed, and the panel points to the right are loaded. The shear and stress equations are:

$$+V_4 = +\frac{62\ 4}{8}(1 + 2 + 3 + 4) = +78\ 0$$

$$+78\ 0 - U_3L_4 \times 0\ 7685 = 0 \qquad \therefore U_3L_4 = +101\ 6$$

For the maximum stresses in the verticals, sections $1-1$, $2-2$, and $3-3$ are passed, and in each case the panel points to the right of these loaded. The shears are

$$V_{1-1} = \frac{62.4}{8}(1 + 2 + 3 + 4 + 5) = +117.0$$

$$V_{2-2} = \frac{62.4}{8}(1 + 2 + 3 + 4) = +78.0$$

$$V_{3-3} = \frac{62.4}{8}(1 + 2 + 3) = +46\ 8$$

The stress equations for U_2L_2 and U_3L_3 are simple, as only three main members are cut. They are:

$$+117.0 + U_2L_2 = 0 \qquad \therefore U_2L_2 = -117.0$$
$$+78\ 0 + U_3L_3 = 0 \qquad \therefore U_3L_3 = -78.0$$

It is seen that the section $3-3$ cuts the member L_4U_5, and therefore the stress in this must be determined before the stress equation can be written, as its vertical component will enter into it. However, by comparing the dead-load shear in that panel, which is -15.0, and the live-load shear V_{3-3}, which is $+46.8$, it is seen that the resultant shear is $+$; and, as this is of opposite sign from the dead-load shear, a counter is required

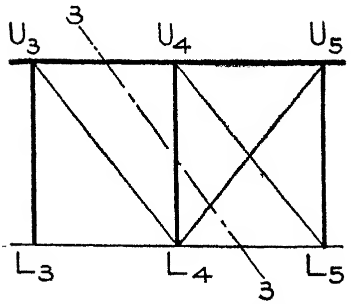


Fig. 44. Calculation of Stress in Diagonal of Span of Fig. 42

and is acting. The stress in U_5L_4 is zero, and the diagonals act as in Fig. 44, the section $3 - 3$ then cutting three members. The stress equation is $+ 46.8 + U_4L_4 = 0$, from which $U_4L_4 = - 46.8$.

Care should be taken not to add to this -46.8 the -10.0 derived as dead-load stress on page 44, in order to get the maximum stress, as the -10.0 previously derived was the dead-load stress in U_4L_4 when U_3L_4 and L_4U_5 were acting. The dead-load stress which goes with the live-load stress of -46.8 acts simultaneously with it,

and is the dead-load stress in U_4L_4 when the members U_3L_4 and U_4L_5 are acting as in Fig. 44. The dead-load shear on the section 3 – 3 would then be the left reaction *minus* the loads at points U_1, U_2, U_3, L_1, L_2, L_3 and L_4; or,

$$V_{3-3} = +105\ 0 - 3 \times 10 - 4 \times 20 = -5\ 0,$$

and

$$-5\ 0 + U_4L_4 = 0 \qquad \therefore U_4L_4 = +5.0$$

Remember that this +5.0 can act only when the live load tends to produce a stress of −46.8; and thus the total stress in U_4L_4 with live load in that position is −46.8 + 5.0 = −41.8, while with dead load only in the truss the stress is −10.0.

The dead-load shears and the maximum + and − live-load shears should now be written for inspection, in order to investigate for counters and then for the minimum stresses. Those whose derivation has not been given should be easily computed by the student at this time. The shears are

	Dead-Load	+ Live-Load	− Live-Load
V_1	+105 0	+218 4	0
V_2	+ 75 0	+163 8	− 7 8
V_3	+ 45 0	+117 0	−23 4
V_4	+ 15 0	+ 78 0	−46 8

From a study of these it is seen that a counter is required in the 4th panel according to rule *a*, Article 37; and according to rule *b* of the same article, the maximum stress is (−46.8 + 15.0) × 1.302 = +41.4, the minimum stress for it and also U_3L_4 being zero according to the same article. A counter is also required in panel 5, as the truss is symmetrical.

The minimum live-load stress in U_1L_1 is zero, and occurs when no live load is at the point L_1.

The minimum live-load stresses in the diagonals U_1L_2 and U_2L_3 occur when the truss is loaded successively to the left of the sections $a - a$ and $b - b$, in which case the shears are −7.8 and −23.4 respectively. The stress equations are

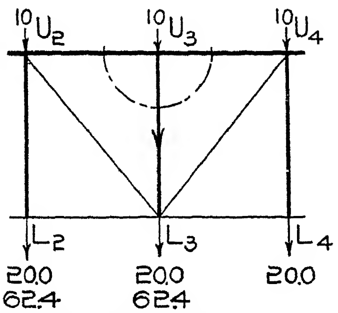
$$-\ 7\ 8 - U_1L_2 \times 0\ 7685 = 0 \qquad \therefore U_1L_2 = -10\ 16$$
$$-23.4 - U_2L_3 \times 0\ 7685 = 0 \qquad \therefore U_2L_3 = -29\ 15$$

The minimum live-load stress in U_2L_2 is obtained by passing

section 1 – 1 and loading the panel points to the left. The live-load shear is the same at this section as it is at the section $b - b$ —namely, -23.4. The stress equation is

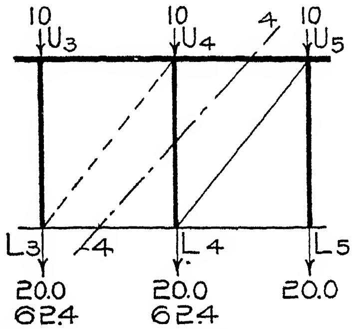
$$+U_2L_2 - 23.4 = 0 \qquad \therefore U_2L_2 = +23.4$$

To determine the minimum live-load stress in U_3L_3, proceed as indicated on page 42. By loading points L_3 and to the left, the live-load shear in the 4th panel will be -46.8, and in the 3d panel *under this same loading* it will be $-46.8 + 62.4 = +15.6$. The sign of the total shear in the two adjacent panels, and the members acting, are shown in Fig. 45. The stress in U_3L_3 is then determined by using a



d.l.v.	+45.0	+15.0
l.l.v.	+15.6	−46.8
Total v.	+	—

Fig 45



d.l.v.	+15.0	−15.0
l.l.v.	−15.6	−78.0
Total v	—	—

Fig. 46.

Stress Diagrams for Verticals in Span of Fig. 42

circular section around U_3, and is simply the dead load at U_3, there being no live-load stress in the member when the bridge is loaded as has been done.

In finding the minimum live-load stress and also the minimum stress in U_4L_4, the same method of procedure will be followed. Let L_4 and to the left be loaded. Then the shear in the 5th panel is -78.0, and *under this same loading* the shear in the 4th panel is $-78.0 + 62.4 = -15.6$. The sign of the total shear in each of the adjacent panels is given in Fig. 46. It should be remembered that a resultant shear with the same sign as the dead-load shear causes the main diagonal to act, while a resultant shear of opposite sign to that of the dead-load shear causes the counter to act. The members

acting are shown, and a section $4 - 4$ can be passed. The dead-load shear at this section is $105 - 3 \times 20 - 4 \times 10 = +5.0$; and accordingly,

$$-U_4L_4 + 5.0 = 0$$

Therefore,

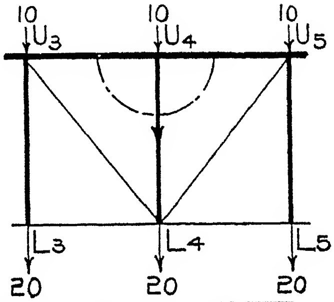
$$U_4L_4 = +5.0 = \text{Dead-load stress in this case.}$$

The live-load stress which acts at the same time is:

$$-U_4L_4 - 15.6 = 0 \qquad U_4L_4 = -15.6,$$

the term -15.6 representing the live-load shear on the section $4 - 4$. This is not the minimum stress, as will next be shown, but it illustrates the fact that the loading for minimum live-load shears does not always give the minimum live-load stress.

By loading L_1, the live-load shear in the second panel, and likewise all others from this to the right support, will be -7.8. The total shears, together with their sign, and also the members they cause to act, are given in Fig. 47. The minimum live-load stress in U_4L_4 is found to be zero, and the dead-load stress is -10, as is derived by passing a circular section around U_4, the equation being as follows:



d.l.v.	+ 15.0	− 15.0
l.l.v	− 7.8	− 7.8
Total v	+	−

Fig 47 Stress Diagram for Vertical in Span of Fig. 42

$$-\text{Live load at } U_4 - U_4L_4 = 0 \qquad \therefore U_4L_4 = 0 \text{ for live load}$$
$$-\text{Dead load at } U_4 - U_4L_4 = 0 \qquad U_4L_4 = -10.0 \text{ for dead load.}$$

A diagram of half the truss should now be made, and all dead and live load stresses placed upon it, and these should be combined so as to form the maximum and the minimum stresses. Such a diagram, together with all stresses, is given in Fig. 48.

The stresses are written in the following order: Dead load, maximum live load, minimum live load, the maximum, and the minimum. In the chord and end-post stresses, there is no minimum live-load stress recorded, it being zero. Where pairs of stresses occur simultaneously, a bent arrow connects them.

40. The Howe Truss. The physical make-up of the Howe truss

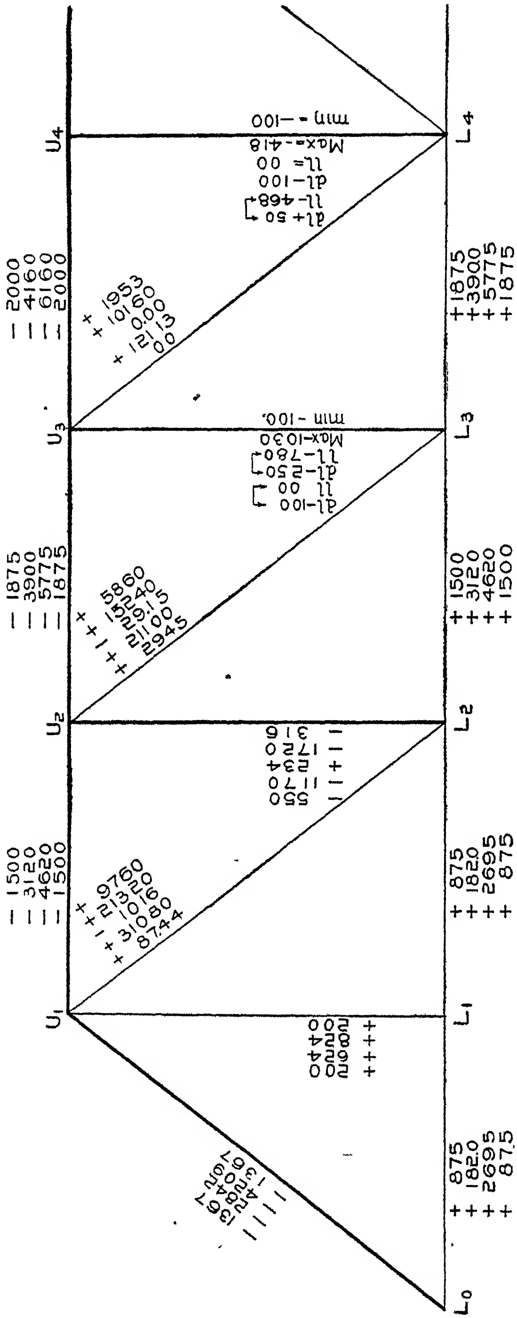


Fig. 48. Stress Diagram of Half-Span of Fig. 42, Pratt Truss, Single Track

differs from that of the Pratt in that the diagonals are made to stand compression only, and the verticals can stand tension only. In the Pratt truss it was found that none of the intermediate posts could be brought into tension by any loading. In the Howe truss it will be found that none of the verticals can be brought into compression.

Let it be required to determine the stresses in a Howe truss of the same span, height, and loading as the Pratt truss of Article 39. An outline diagram is given in Fig. 49.

The dead-load shears and the maximum and minimum live-load shears will be the same as for the Pratt truss, and they are:

Dead-Load V		+ Live-Load V	− Live-Load V
V_1	+105 0	+218 4	− 0
V_2	+ 75 0	+163 8	− 7 8
V_3	+ 45 0	+117 0	−23 4
V_4	+ 15 0	+ 78 0	−46 8
V_5	− 15 0	+ 46 8	−78 0

Inspection of these shows that counters are required in the 4th and 5th panels (see Article 37).

The dead-load lower chord stresses will be computed by the

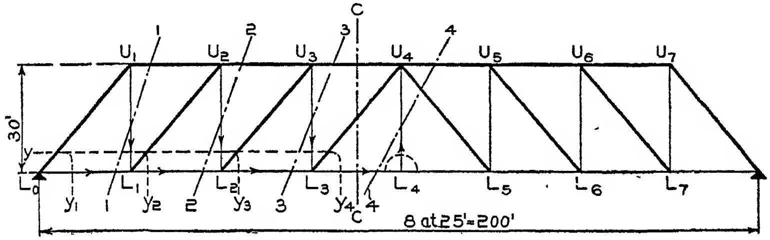


Fig. 49 Outline Diagram of 8-Panel Single-Track Howe Truss Span

tangent method (see Article 31), the section being $y - y_1$, etc. The tangent of ϕ is $25 \div 30 = 0.8333$. The stresses may be conveniently tabulated as follows:

Dead-Load Chord Stresses (Lower Chord)

Member	Section	Stress Equation	Stress
L_0L_1	$y - y_1$	$-105\ 0 \times 0\ 8333 + L_0L_1 = 0$	+ 87.5
L_1L_2	$y - y_2$	$-(105.0 + 75.0)\ 0.8333 + L_1L_2 = 0$	+150.0
L_2L_3	$y - y_3$	$-(105\ 0 + 75\ 0 + 45)\ 0\ 8333 + L_2L_3 = 0$	+187.5
L_3L_4	$y - y_4$	$-(105\ 0 + 75.0 + 45.0 + 15\ 0)\ 0.8333 + L_3L_4 = 0$	+200 0

A simple method for the determination of the upper chord stresses, is to pass a section and to equate the sum of the horizontal forces. Pass section 1 − 1. The only horizontal forces are the stresses in L_0L_1 and U_1U_2; and as these are parallel, one must be equal and opposite to the other. In a like manner the stresses in the other sections of the top chord are found. The stresses are:

$$U_1U_2 = -L_0L_1 = -(+\ 87\ 5) = -\ 87.5$$
$$U_2U_3 = -L_1L_2 = -(+150.0) = -150\ 0$$
$$U_3U_4 = -L_2L_3 = -(+187.5) = -187.5$$

A consideration of the Pratt truss shows that this method can be applied to it in determining the chord stresses.

As it is known that the diagonal web members are in compression under the dead load which produces a positive shear in the left half of the truss, it is evident that positive live-load shears will produce compressive stresses, and negative live-load shears tensile stresses, in the diagonals in the left half of the truss. Also, from Article 30, the stress in a diagonal is $V \sec \phi$. The stresses can now be written directly without the aid of the stress equation:

$$
\begin{aligned}
L_0U_1 &= -105\ 0 \times 1\ 302 = -136\ 70 \\
L_1U_2 &= -\ \ 75\ 0 \times 1\ 302 = -\ \ 97\ 60 \\
L_2U_3 &= -\ \ 45\ 0 \times 1\ 302 = -\ \ 58\ 60 \\
L_3U_4 &= -\ \ 15.0 \times 1\ 302 = -\ \ 19\ 53
\end{aligned}
$$

Likewise the stresses in the verticals can be written directly, remembering that here the secant is unity, and that the shear at the section cutting the member is to be used, not forgetting that $\frac{1}{3}$ of the dead panel load is applied at the top panel points. The shears and stresses are:

$$
\begin{aligned}
V_{1-1} &= +105\ 0 - 10 = +95.0 & U_1L_1 &= +95.0 \\
V_{2-2} &= +105\ 0 - 20 - 2 \times 10 = +65\ 0 & U_2L_2 &= +65\ 0 \\
V_{3-3} &= +105\ 0 - 2 \times 20 - 3 \times 10 = +35.0 & U_3L_3 &= +35.0
\end{aligned}
$$

The member U_4L_4 cannot be easily determined by passing a section 4 — 4, for this cuts four members. It is determined by passing a circular section about the point L_4, the equation being $+U_4L_4 - 20.0 = 0$, from which $U_4L_4 = +20.0$, which is equal to the dead panel load at the point L_4.

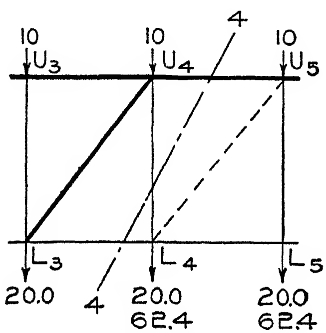
The live-load chord stresses are determined by multiplying the dead-load chord stresses by the ratio of the live to the dead loads. This has been found to be equal to 2.08. The live-load chord stresses are found to be:

$$
\begin{aligned}
L_0L_1 &= +182.0 & U_1U_2 &= -182.0 \\
L_1L_2 &= +312\ 0 & U_2U_3 &= -312\ 0 \\
L_2L_3 &= +390\ 0 & U_3U_4 &= -390.0 \\
L_3L_4 &= +416\ 0
\end{aligned}
$$

As the character of the stresses which can be taken by the diagonals and the verticals is known, the maximum and minimum live-load stresses can be written without first writing the stress equations. The maximum live-load stresses are:

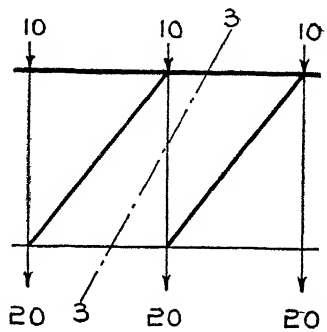
$$
\begin{array}{llll}
L_0U_1 = -218\ 4 \times 1\ 302 = -284\ 36 & U_1L_1 = +218\ 4 \\
L_1U_2 = -163\ 8 \times 1\ 302 = -213\ 27 & U_2L_2 = +163\ 8 \\
L_2U_3 = -117\ 0 \times 1\ 302 = -152\ 33 & U_3L_3 = +117\ 0 \\
L_3U_4 = -\ \ 78\ 0 \times 1\ 302 = -101.56 & U_4L_4 = +\ 78\ 0
\end{array}
$$

It should be noted that when L_4 and all panel points to the right are loaded, the shears and the members acting are as shown in Fig. 50. The dead-load shear on the section $4-4$ is $+15.0$, less the load at U_4, or $+15.0 - 10.0 = +5.0$; and the equation of stress is $-U_4L_4 + 5.0 = 0$, from which $U_4L_4 = +5.0$. Thus it is seen that in this



d.l.v.	+ 15 0	−15.0
l.l.v.	+78.0	+15 6
Total v.	+	+

Fig 50.



d.l.v.	+ 45 0	+ 15.0
l.l.v	— 7.8	— 7.8
Total v.	+	+

Fig. 51.

Stress Diagrams for Members of Howe Truss Span of Fig 49.

case the dead-load stress is $+5.0$ when the live-load stress is $+78.0$.

The maximum stresses in the counters (see Article 37) are:

$$(-46.8 + 15\ 0)\ 1\ 302 = -41.4.$$

The minimum live-load stresses are now written as follows:

$$
\begin{array}{ll}
L_1U_2 = +\ 7\ 8 \times 1\ 302 = +10.16 & U_1L_1 = 0 \\
L_2U_3 = +23\ 4 \times 1\ 302 = +29.15 & U_2L_2 = -7.8 \\
L_3U_4 = 0 & U_3L_3 \\
U_3L_4 = 0 & U_4L_4
\end{array}
\left.\right\}\ \text{See discussion following.}
$$

If live panel loads were placed at points L_1, L_2, and L_3 the live-load shear in $c - c$ would be -46.8; and the dead-load shear being $+15$, the counter would act, and the stress in U_3L_3 would be tensile and equal to the sum of the dead and live panel loads which are at its lower end L_3. If points L_1 and L_2 had live panel loads on them,

the resultant shear in $c - c$ would be $-23.4 + 15.0 = -8.4$; the counter would act, and the stress in U_3L_3 would be tensile and equal to the dead panel load which is at L_3. There being no live panel load at L_3, the live-load stress in U_3L_3 would be zero under this loading. If a live panel load be placed at L_1 only, then the shears and the members acting will be as shown in Fig. 51, and V_{3-3} for dead load $=$ $+45.0$ $-$ the load at U_3, or $= 45 - 10 = +35.0$. The V_{3-3} for live load $= -7.8$, and the stress equation $- U_3L_3 - 7.8 = 0$, from which $U_3L_3 = -7.8$. So this live-load compression stress of 7 800 pounds occurs at the same time as the dead-load tensile stress of 35 000 pounds.

By loading various groups of panel points in succession and determining the resulting live-load stresses in U_4L_4, it will be found that under no loading can a negative live-load stress be produced. The minimum live-load stress is therefore zero, and occurs when there is no live load on the bridge.

The stresses should now be placed on an outline diagram similar to that of Fig. 48, and the stresses in corresponding members compared with those in that figure. This is left for the student.

41. **Bowstring and Parabolic Trusses.** A bowstring truss is shown in Fig. 13, the full lines representing the main members, which are the members under stress by the dead load. The dotted members represent counters which may be stressed by the action of the live load.

As before mentioned, the stresses in the chords and also in the webbing are quite uniform. When the end supports and the panel points lie on the arc of a certain curve, called a parabola, then, under full load, the stresses in all panels of the lower chord are equal; the stress in all verticals is tensile and is equal to the panel load at the lower end; and the stress in all diagonals is zero. Under partial load, the stresses in the webbing are exceedingly small, and the chord stresses remain almost equal.

If it is desired to have a parabolic truss, first decide upon the length of span, the number of panels, and the height at the center. The height of any vertical post is given by the formula:

$$h = H - \frac{4Hd^2}{l^2},$$

in which,

H = Approximate height at center;
d = Distance of vertical post from center;
l = Span;
h = Height of vertical post sought

All distances are in feet. Suppose, as an example, that it was desired to determine the heights of the vertical posts in an 8-panel parabolic truss of a height approximately equal to 24 feet. One-half

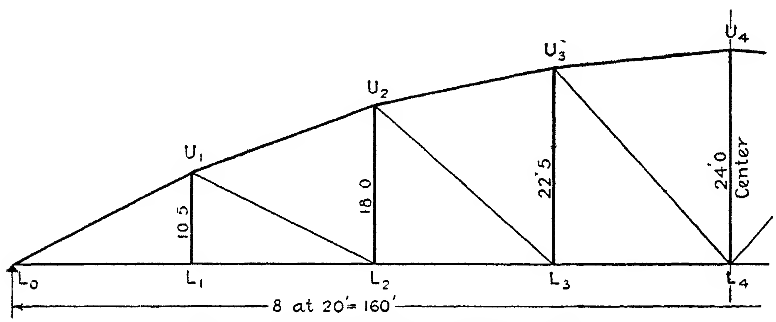


Fig 52 One-Half of 8-Panel Parabolic Truss

the truss is shown in Fig. 52. At the center, $d = 0$, and the equation reduces to $h = H$, which is 24 feet. For U_3L_3, $d = 20$; then,

$$h = 24 - \frac{4 \times 24 \times \overline{20}^2}{\overline{160}^2},$$

from which,

$$h = 22\ 5 \text{ feet.}$$

For U_2L_2,

$$d = 40$$

$$h = 24 - \frac{4 \times 24 \times \overline{40}^2}{\overline{160}^2} \qquad h = 18.0 \text{ feet.}$$

For U_1L_1,

$$d = 60$$

$$h = 24 - \frac{4 \times 24 \times \overline{60}^2}{\overline{160}^2} \qquad h = 10.5 \text{ feet.}$$

Inspection of the above results shows that the span or the center height must become quite great before the clearance at U_1L_1 will be sufficient to allow the traffic to pass under a portal bracing at this point. For this reason these trusses are usually built as through trusses with bracing on the outside of the truss, which connects to the floor-beams extended.

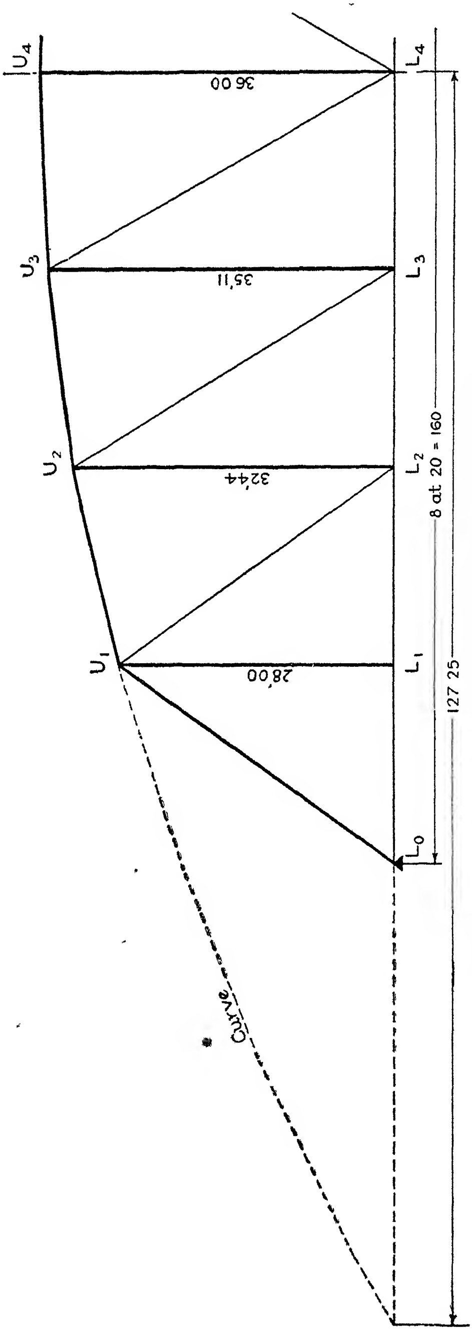


Fig 53. Calculation of Heights of Vertical Posts in Parabolic Truss

In the bowstring truss, the panel points of the top chord usually lie on the arc of a parabola which does not pass through the supports. For example, suppose that it was decided to have the span and panels the same as shown in Fig. 52, but the height at L_1 was to be 28 feet, and at L_4 36 feet. By substituting these values in the equation just given, and solving for l, the place will be determined where the parabolic curve cuts the lower chord extended, and the lengths of the vertical posts may be computed as before. Substituting these results:

$$28 = 36 - \frac{4 \times 36 \times \overline{60}^2}{l^2},$$

$$(-36 + 28)\, l^2 = -4 \times 36 \times \overline{60}^2$$

$$l = \sqrt{\frac{4 \times 36 \times \overline{60}^2}{8}}$$

$$= 254.5,$$

which shows that the arc cuts the lower chord extended at a point $254.5 \div 2 = 127.25$ feet from the center of the span (see Fig. 53).

The other vertical posts are:

$U_3L_3 \qquad h = 36 - \dfrac{4 \times 36 \times \overline{20}^2}{\overline{254.5}^2} = 35.11 \text{ feet};$

$U_2L_2 \qquad h = 36 - \dfrac{4 \times 36 \times \overline{40}^2}{\overline{254.5}^2} = 32.44 \text{ feet};$

$U_1L_1 \qquad h = 36 - \dfrac{4 \times 36 \times \overline{60}^2}{\overline{254.5}^2} = 28.00 \text{ feet, which checks.}$

The analysis of a bowstring truss will now be given. Both the maximum and minimum stresses will be determined, as reversal of

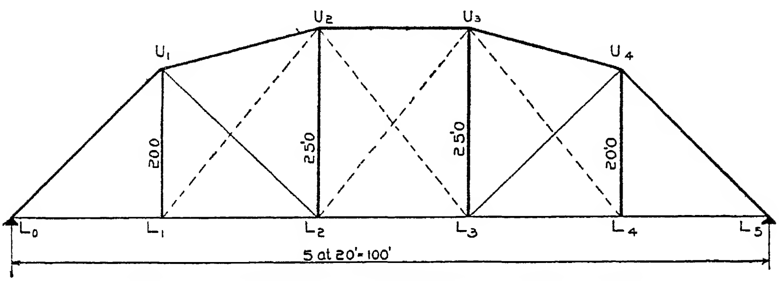


Fig 54 Outline Diagram of 5-Panel Bowstring Truss Span

stresses is liable to occur in the intermediate posts The loading for minimum live-load stresses can be ascertained only by trial, care being taken to compute the dead-load stresses for the arrangement of web members caused by that particular live loading.

Let it be required to determine the maximum stresses in the 5-panel 100-foot bowstring truss shown in Fig. 54, remembering that the diagonals take only tension. The height of U_1L_1 is 20 feet, and of U_2L_2 25 feet. The dead panel load is 17 200 pounds, and the live panel load is 50 000 pounds. The full lines show the main members which act under dead-load stress, and the dotted lines show the counters which may act under the action of the live load. One-third of the dead panel load, or 5 730 pounds, is taken as acting at the upper panel points, while the remainder, 11 470 pounds, acts at the lower

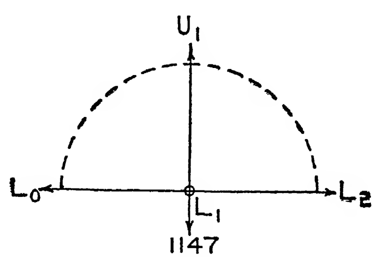


Fig 55 Resolution of Forces around Panel Point in Bowstring Truss of Fig 54.

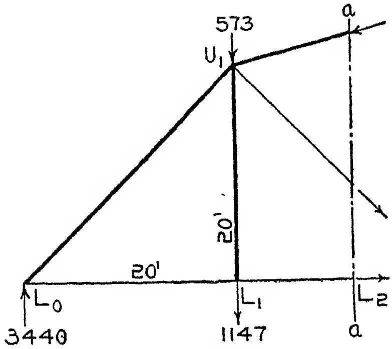


Fig 56

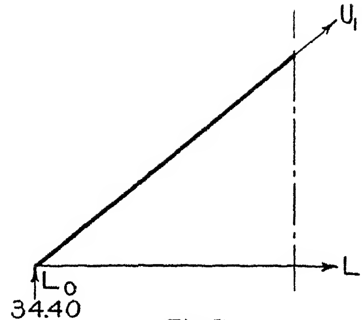


Fig. 58

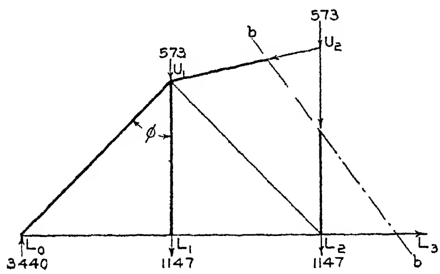


Fig 57

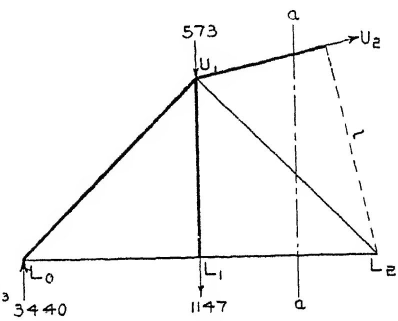


Fig 59

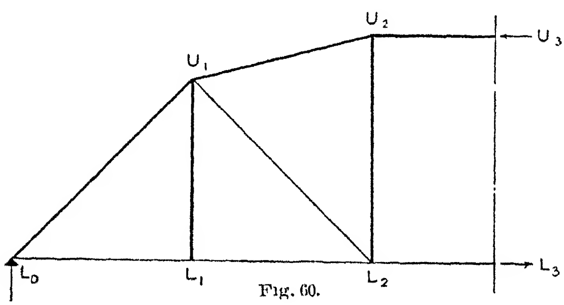


Fig. 60.

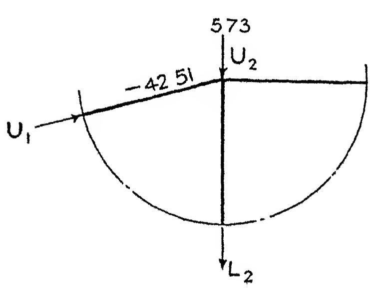


Fig 61

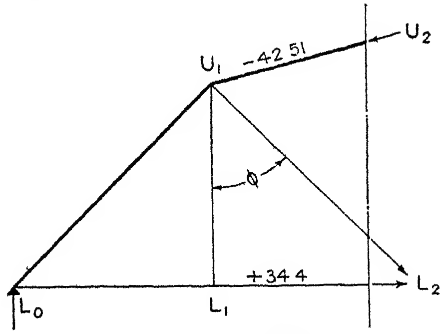


Fig. 62.

Analysis of Stresses in Various Members of the Bowstring Truss of Fig 54.

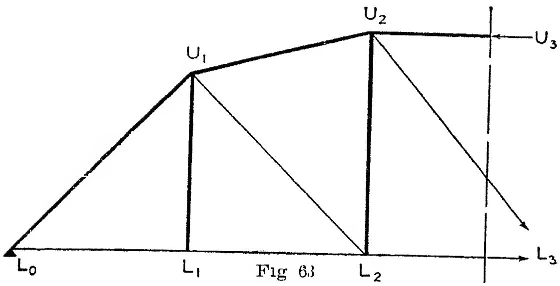


Fig 63

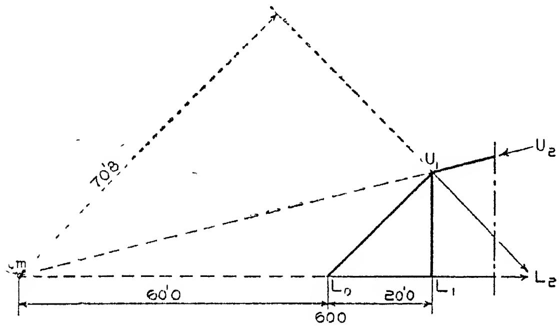


Fig 64

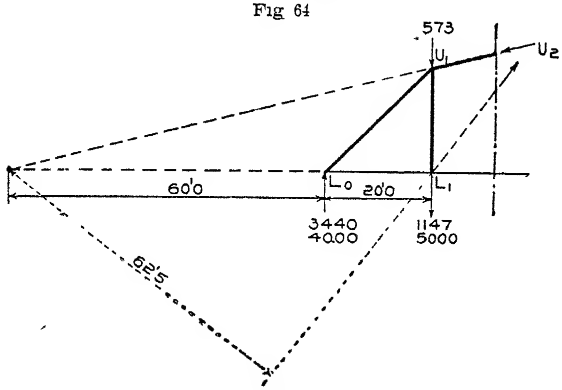


Fig 65

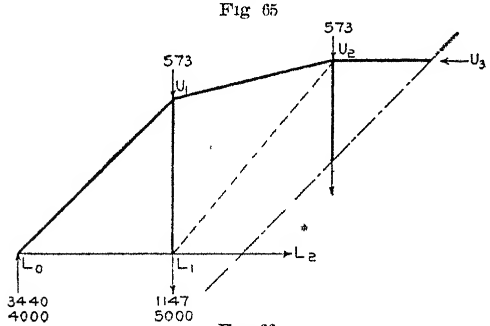


Fig 66.

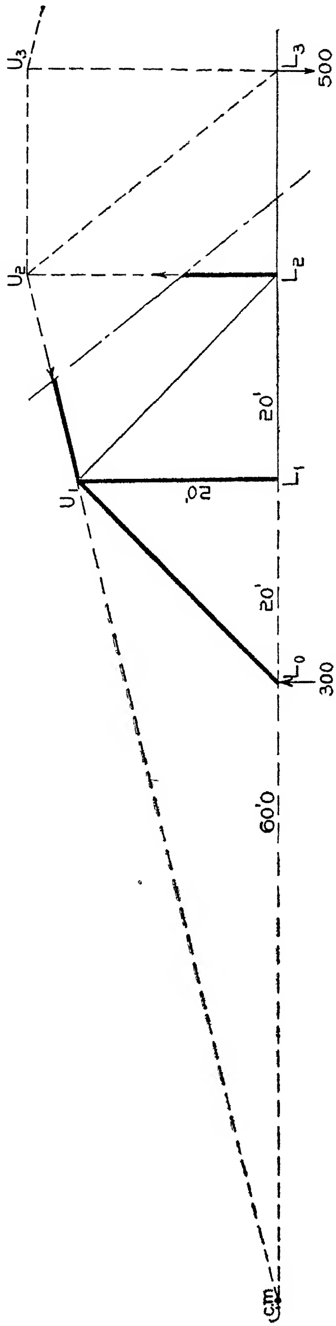


Fig 67

Analysis of Stresses in Various Members of the Bowstring Truss of Fig. 54.

ones Articles 27, 28, and 29 should be carefully reviewed before going further. The shear times the secant method cannot be conveniently employed for the live-load stresses in the members U_1L_2 and L_1U_2, as the section will cut the member U_1U_2, and the vertical component of its stress must be reckoned with in the stress equation The method of moments as illustrated in Fig. 27, Article 29, will be used for these members.

The dead-load reaction is $2 \times 17.2 = +34.4$. The dead-load chord stresses should first be computed.

By resolving the horizontal forces around L_1, it is seen that $L_0L_1 = L_1L_2$ (see Fig. 55). Passing the section $a - a$, taking the center of moments, at U_1, and stating the equation of the moments to the left of the section, there results (see Fig. 56):

$$+34.4 \times 20 - L_1L_2 \times 20 = 0 \qquad \therefore L_1L_2 = +34\ 4$$

For L_2L_3, the section $b - b$ is passed; the center of moments is at U_2; and the equation of the moments to the left of this section (see Fig. 57) is:

$$+34\ 4 \times 2 \times 20 - (11\ 47 + 5.73)\ 20 - L_2L_3 \times 25 = 0 \quad \therefore L_2L_3 = +41.26.$$

By passing a vertical section cutting L_0L_1 and L_0U_1, the stress in L_0U_1 can be determined by taking the sum of the vertical forces to the left and equating them to the vertical component of the stress (see Fig. 58). The equation is:

$$+34.4 + L_0U_1 \times 0.707 = 0, \text{ from which } L_0U_1 = -34.4 \times 1.414 = -48\ 7.$$

A section $a - a$ (Fig. 59) shows that the center of moments for U_1U_2 is at L_2; and stating the moments of the stress, and the forces to the left of the section, there results an equation in which an unknown lever arm enters. This lever arm l is readily computed to be 24.28 feet, and the equation can now be written:

$$+34.4 \times 2 \times 20 - (11.47 + 5.73)\ 20 + U_1U_2 \times 24.28 = 0$$
$$\therefore U_1U_2 = -42.51.$$

The stress in U_2U_3 is determined by passing a vertical section in the 3d panel, and taking the sum of the horizontal forces. As there is no dead-load stress in the members L_2U_3 and U_2L_3, their components will be zero. Therefore (see Fig. 60) it is evident that U_2U_3 must be equal and opposite to L_2L_3 and will be equal to -41.26.

By reference to Fig. 55, the stress in L_1U_1 is seen to be tensile and equal to $+11.47$.

Pass a circular section around U_2 and take the sum of the vertical components, assuming that the stress in U_2L_2 acts away from the section. The length of U_1U_2 is $\sqrt{5^2 + 20^2} = 20.6$, and therefore the vertical component of U_1U_2 will be $(42.51 \div 20.6) \times 5 = 10.32$, which acts upward. The stress equation of U_2L_2 (see Fig. 61) is:

$$+10\ 32 - 5\ 73 - U_2L_2 = 0 \qquad \therefore U_2L_2 = +4.59,$$

showing that a tensile stress occurs in U_2L_2 when all panel points are loaded.

The simplest method of ascertaining the stress in U_1L_2 is to pass a vertical section cutting members as shown in Fig. 62, and to equate the horizontal forces and stresses. The horizontal component of U_1U_2 is:

$$\frac{42\ 51}{20\ 6} \times 20 = 41\ 30, \text{ which acts toward the left.}$$

The equation of stress is, then:

$$-41\ 30 + 34\ 40 + U_1L_2 \sin\phi = 0, \text{ but } \sin\phi = 0\ 707;$$
$$U_1L_2 = +6\ 90 \times 1\ 414$$
$$= +9\ 76$$

All the dead-load stresses being computed, the next operation will be to determine the live-load chord stresses. These are proportional to the dead-load stresses in the same ratio as the live panel load is to the dead panel load. This ratio is $50 \div 17.2 = 2.907$, and the chord and end-post live-load stresses are:

$$L_0U_1 = -48\ 71 \times 2\ 907 = -141.7$$
$$U_1U_2 = -42\ 51 \times 2\ 907 = -123\ 6$$
$$U_2U_3 = -41.26 \times 2\ 907 = -120.3$$
$$L_0L_2 = +34\ 40 \times 2\ 907 = +100\ 2$$
$$L_2L_3 = +41\ 26 \times 2\ 907 = +120\ 3$$

Also, the stress in U_2L_2 when the live load covers the entire bridge is *not* 2.907×4.59, as it must be remembered that part of the dead load is at the panel points of the upper chord. Taking a circular section around U_2 (see Fig. 61), and noting that there is no load at U_2, it is seen that the stress in U_2L_2 due to live load is simply equal to the vertical component of the live-load stress of U_1U_2 and will be tensile. It is:

$$U_2L_2 = (123.6 \div 20.6) \times 5 = +30\ 0.$$

The maximum live-load stress in U_1L_1 is tensile, and equal to the live panel load at L_1 (see Fig. 55).

To obtain the maximum stress in U_2L_3, load L_3 and L_4. The shear V_3 will then be $\frac{50}{5}$ (1 + 2) = +30.0. The section will cut the members as shown in Fig. 63, and the equation of stress will be:

$$+30.0 - U_2L_3 \cos\phi = 0; \text{ but } \cos\phi = \frac{25}{\sqrt{20^2 + 25^2}} = 0.782,$$

$$\therefore U_2L_3 = +38.4.$$

If panel points L_1 and L_2 were loaded, it is evident that the stress in L_2U_3 would be +38.4.

To obtain the maximum live-load stress in U_1L_2, a section is passed cutting U_1U_2, L_1L_2, and U_1L_2 (Fig. 64). The center of moments will be at the intersection of U_1U_2 and L_1L_2, and this point lies some place to the left of the support L_0. The lever arm of U_1L_2 will be the perpendicular distance from this point to the line U_1L_2 extended. The panel points L_2, L_3, and L_4 are loaded. The left reaction is then (1 + 2 + 3) $\frac{50}{5}$ = + 60.0. The lever arms are easily computed, and these, together with the members cut, are shown in Fig. 64. The equation of stress is:

$$-60.0 \times 60\,0 + U_1L_2 \times 70.8 = 0 \qquad \therefore U_1L_2 = +50.80$$

If a load were put on L_1 only, then the reaction at L_0 would be $\frac{4}{5} \times 50 = 40$; and the equation of stress would then be:

$$-40.0 \times 60.0 + 50 \times (60.0 + 20\,0) + U_1L_2 \times 70.8 = 0 \quad \therefore U_1L_2 = -22.6.$$

As this is compression and greater than the dead-load stress, +9.76, a counter is required in that panel. In order to get the stress in the counter, it must be inserted, U_1L_2 being left out, and the dead and live load stresses computed and their difference taken. Fig. 65 gives the lever arms, center of moments, and the forces acting in this case.

The dead-load stress is:

$$-34.4 \times 60.0 + (11.47 + 5.73)(60 + 20) - L_1U_2 \times 62.5 = 0$$
$$\therefore L_1U_2 = -11.02,$$

and the live-load stress is:

$$-40 \times 60 + 50\,(60 + 20) - L_1U_2 \times 62.5 = 0 \qquad \therefore L_1U_2 = +25.60,$$

and the stress in the counter is the algebraic sum of these two, or $-11.02 + 25.60 = +14.58$.

When a live panel load is at L_1, L_1U_2 is acting, as has just been proved. As this load at L_1 causes a negative shear in all panels to the right, this negative shear in the center panel will cause L_2U_3 to act. A section may now be passed as shown in Fig. 66, and the stress equations for U_2L_2 written:

For dead load, $+34.4 - 11\ 47 - 2 \times 5\ 73 - U_2L_2 = 0 \quad \therefore U_2L_2 = +11\ 47$
For live load, $+40\ 0 - 50.0 - U_2L_2 = 0 \quad \therefore U_2L_2 = -10\ 00$
Total $= +\ 1.47$

This is evidently not a maximum for U_2L_2, for when a full live load was on the span, the stress was $+30.0$ due to live load and $+4.59$ due to dead load.

It might be well to consider what effect is produced by loading L_3 and L_4. The loading of L_2 and L_1 need not be considered, since it is evident that, as this causes the total shear in panel 2 to be positive and the total shear in panel 3 to be negative, therefore U_1L_2 and L_2U_3

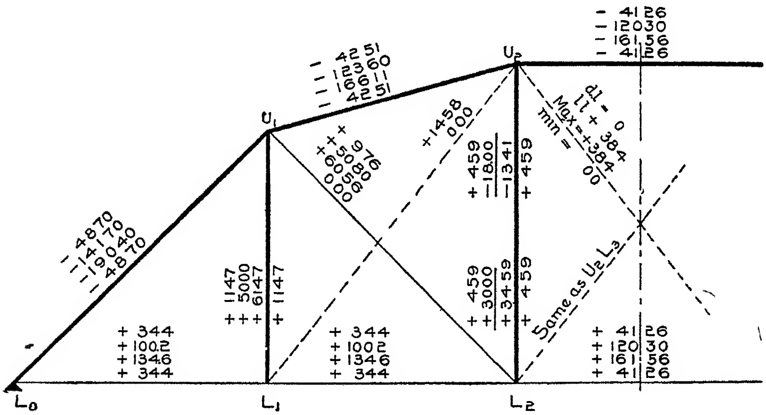


Fig. 68. Stress Diagram of Half-Span of Parabolic Truss of Fig 54

will act, and this causes a tensile stress in U_2L_2 equal to the vertical components of the dead and live load stresses in U_1U_2 less the dead panel load at U_2. With a live panel load at L_3 and L_4, the left reaction is $\frac{50}{5}(1 + 2) = +30.0$. The section, the live-load forces, the center of moments, and the members acting are shown in Fig. 67. The dead-load stress in U_2L_2 will be the same as when the truss has no live load on it. The stress equation for the live load is:

$$-60 \times 30 - (60 + 20 + 20) \times L_2U_2 = 0 \qquad \therefore L_2U_2 = -18.0.$$

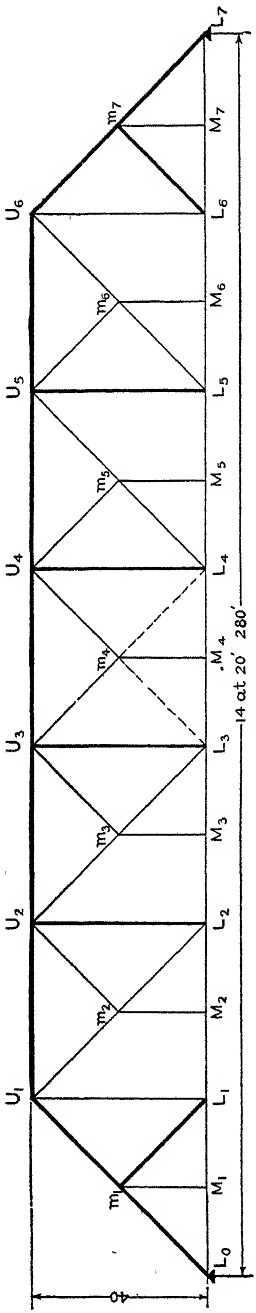


Fig 69 Outline Diagram of 14-Panel Baltimore Truss Span

The dead-load stress being $+4.59$, this stress of -18.0 causes a reversal of stress in the vertical. For this reason the verticals of bowstring trusses are, like web members of Warren trusses, built so as to take either tension or compression. The minimum stresses in the diagonals will be zero, for when one diagonal in a panel is acting, the other is not.

The diagram of half of the truss in Fig. 68 gives all the stresses.

It is to be noted by the student, that in some cases one method for the determination of stresses is preferable to others in that it saves labor of computation. The analysis of the truss of Fig. 68 illustrates this fact.

42. **The Baltimore Truss.** Baltimore trusses are of two classes—those in which the half-diagonals, called *sub-diagonals,* are in compression, and those in which the sub-diagonals are in tension. The latter class is the one most usually built, as it is more economical on account of many of its members being in tension, in which case these members are cheaper and easier to build than if they were compression members. Fig. 14 shows both types of truss. The Baltimore truss does not have a simple system of webbing, and for that reason the analysis is here presented. As the tension sub-diagonal truss is the type in most common use, its analysis will be given.

Let it be required to compute the maximum stresses in the 14-panel 280-foot span of Fig. 69. The height is 40 feet, the dead panel load 24 000 pounds,

and the live panel load 40 000 pounds. One-third of the dead panel load is applied at the upper ends of the long verticals and also of the half-verticals. These half-verticals are designated as *sub-verticals*. Attention is called to the system of notation used for the ends of the sub-verticals. The full lines in Fig. 69 represent the main members, being stressed by dead load only. The heavy lines indicate those members that take compression, the light lines those that take tension, and the broken lines the counter-braces. In this, as in nearly all Baltimore trusses, the diagonals make an angle of 45 degrees with the vertical.

The dead and the positive live-load shears in the various panels should be computed. They are:

DEAD-LOAD V		+ LIVE-LOAD V
V_1	$+156.00$	$V_1 = (1 + \ldots 13)\frac{40}{14} = +260.00$
V_2	$+132.00$	$V_2 = (1 + \ldots 12)\frac{40}{14} = +223.00$
V_3	$+108\ 00$	$V_3 = (1 + \quad 11)\frac{40}{14} = +188\ 50$
V_4	$+\ 84\ 00$	$V_4 = (1 + \quad 10)\frac{40}{14} = +157.20$
V_5	$+\ 60\ 00$	$V_5 = (1 + \quad 9)\frac{40}{14} = +128.50$
V_6	$+\ 36\ 00$	$V_6 = (1 + \quad 8)\frac{40}{14} = +102\ 80$
V_7	$+\ 12.00$	$V_7 = (1 + \quad 7)\frac{40}{14} = +\ 80\ 00$

It is only necessary to determine the negative live-load shear in panels 5 and 7, in order to ascertain if there is a counter required. These shears are:

$$-V_5 = (10 + 11 + 12 + 13)\frac{40}{14} - 4 \times 40 = -28.60$$

$$-V_7 = (8 + 9 + 10 + 11 + 12 + 13)\frac{40}{14} - 6 \times 40 = -60\ 00$$

From a comparison of these with the dead-load shears, it is seen (see Article 37) that a counter is required in panel 7 only.

The dead-load stresses are first to be computed. The stress in any sub-vertical is found by passing a circular section around its lower end, and equating the sum of the vertical forces, assuming in this, as in all cases, that the unknown stress acts away from the

section. Take M_1m_1, for example. Fig. 70 gives the section, the forces acting, and the members cut. Then,

$$+M_1m_1 - 16\ 0 = 0 \qquad \therefore M_1m_1 = +16\ 0$$

As all sub-verticals have the same dead load at their lower end, it follows that the dead-load stress in all sub-verticals is the same, a tensile stress of 16 000 pounds.

The dead-load stresses in the sub-diagonals are determined by resolving the forces around the joint at their lower end. The components perpendicular to the diagonal are taken (see Fig. 71). Take

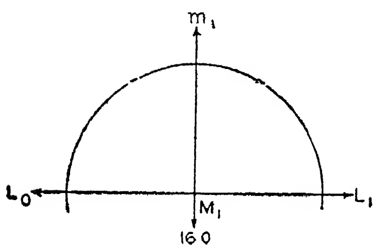


Fig. 70 Diagram for Calculating Stress in Sub-Vertical of Baltimore Truss

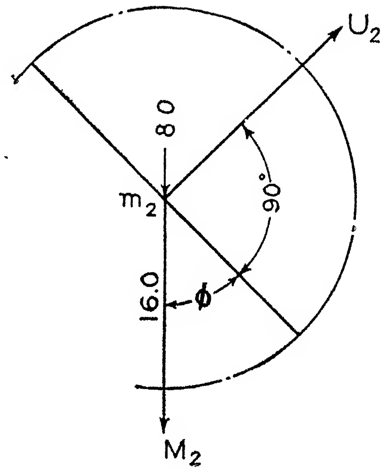


Fig. 71. Diagram for Calculating Stress in Sub-Diagonal of Baltimore Truss.

m_2U_2. The known forces or stresses are the dead panel load of 8.0 and the stress in m_2M_2, which is 16.0 and which being tensile acts away from the section. The stress equation is:

$$+m_2U_2 - 8.0 \text{ sine } \phi - 16.0 \text{ sine } \phi = 0$$
$$\phi = 45°, \text{ sine } \phi = 0.707, \text{and}$$
$$m_2U_2 - 8.0 \times 0.707 - 16.0 \times 0.707 = 0$$
$$\therefore m_2U_2 = +16.96.$$

This equation may be put in another form by multiplying and dividing the numerical values by 2:

$$m_2U_2 - \frac{(8\ 0 + 16.0)}{2} \times 1.414 = 0;$$

or,

$$m_2U_2 = +\frac{24}{2} \sec \phi,$$

which proves the well-known saying that *the stress in the sub-diagonals is equal to one-half the panel load, times the secant of the angle* ϕ. It also shows that *the vertical component of the sub-diagonal is equal to*

one-half the panel load. This fact should be remembered, as it will be frequently used further on.

In a similar manner, the stress in all the tension sub-diagonals will be found to be the same, +16.96, and the stress in the compression sub-diagonal m_1L_1 is −16.96.

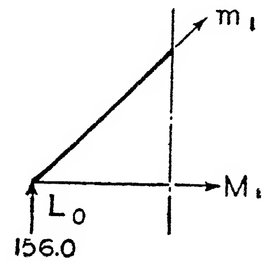


Fig. 72 Stress in Diagonal of Baltimore Truss

The stress in the member L_0m_1 and in the upper half of any main diagonal (*i. e.*, U_1m_2, U_2m_3, and U_3m_4) is determined as in the diagonals of the Pratt or Howe truss, for the section passed cuts but one member, which has a vertical component. Take L_0m_1 (see Fig. 72). Then $+156.0 + L_0m_1 \cos 45° = 0$, from which $L_0m_1 = -220.5$. For U_1m_2 the section is passed as in Fig. 73, and the equation of stress is $+V_3 - U_1m_2 \cos 45° = 0$, or $+108.0 - U_1m_2 \times 0.707 = 0$, from which $U_1m_2 = +152.9$.

In a similar manner,

$$U_2m_3 = +60.0 - 0.707 = +84.84;$$
$$U_3m_4 = +12\,0 - 0.707 = +16\,96.$$

The stresses in m_1U_1, m_2L_2, and m_3L_3 may be determined by

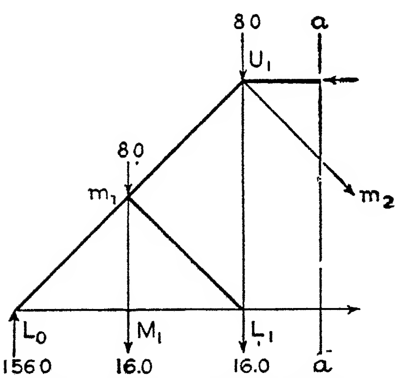


Fig. 73. Stress in Upper Half of Main Diagonal of Baltimore Truss.

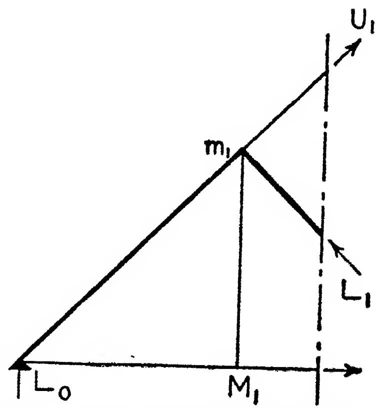


Fig. 74 Stress in Diagonal of Baltimore Truss

resolving the forces about m_1, m_2, and m_3; but a neater solution is to pass a vertical section cutting the member whose stress is desired, and to equate to zero the shear and the vertical components of all

the members cut (see Fig. 24, Article 28). The section for m_1U_1 is passed as in Fig. 74. The equation of stress is then:

$$m_1U_1 \cos 45° + m_1L_1 \cos 45° + V_2 = 0,$$

but the vertical component of m_1L_1 is $\frac{24}{2} = 12$; and therefore,

$$m_1U_1 \times 0.707 + 12 + 132 = 0$$
$$\therefore m_1U_1 = -203.6.$$

For m_2L_2, the section is as shown in Fig. 75, and the stress equation is:

$$-m_2L_2 \times 0.707 + \text{vert. component } m_2U_2 + V_4 = 0$$
$$-m_2L_2 \times 0.707 + 12 + 84 = 0$$
$$\therefore m_2L_2 = +135.6.$$

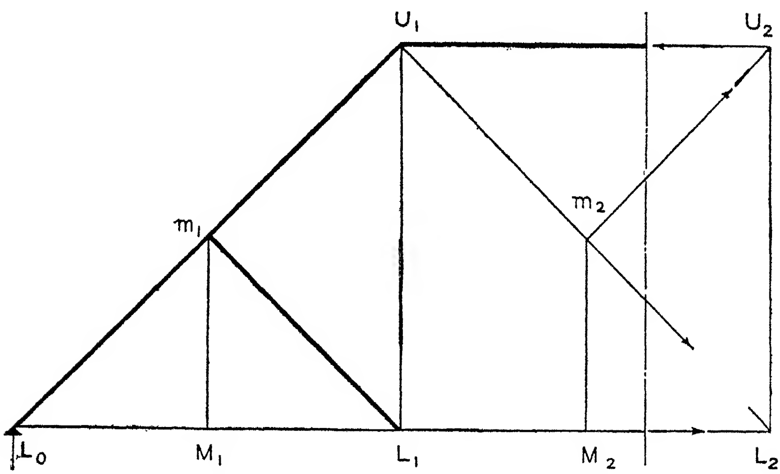


Fig. 75. Calculating Stress in Lower Half-Diagonal of Baltimore Truss

In a similar manner, passing a section cutting U_2U_3, m_3U_3, m_3L_3, and M_3L_3, the stress equation may be written:

$$-m_3L_3 \times 0.707 + 12 + 36 = 0$$
$$\therefore m_3L_3 = +67.85.$$

The stresses in the verticals are best determined by resolving the vertical forces at their lower end. Referring successively to diagrams a, b, and c of Fig. 76, the stress equations are:

$$+U_1L_1 - 16.0 - 12.0 = 0 \qquad \therefore U_1L_1 = +28.0$$
$$+U_2L_2 - 16.0 + 96.0 = 0 \qquad \therefore U_2L_2 = -80.0$$
$$+U_3L_3 - 16.0 + 48.0 = 0 \qquad \therefore U_3L_3 = -32.0$$

96 and 48 being the vertical components of m_2L_2 and m_3L_3 respectively.

The chord stresses are easiest computed by considering the resolution of horizontal forces at the panel points. As the diagonals make an angle of 45° with the vertical, their horizontal and vertical

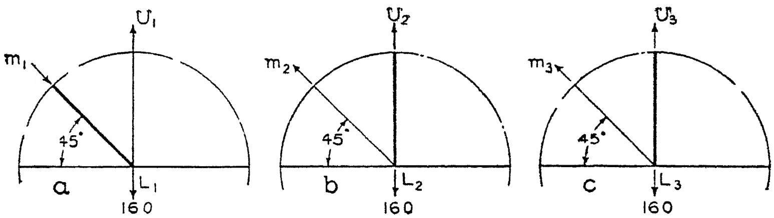


Fig 76 Calculating Stresses in Verticals of Baltimore Truss of Fig. 69.

components are equal. For instance, the horizontal component of the members L_0m_1, U_1m_2, and U_2m_3 are equal to the shear in that panel, which is their vertical component. At point L_0 (see Fig. 77), there results:

$$+L_0M_1 - \text{horizontal component of } m_1L_0 = 0, \text{ or,}$$
$$+L_0M_1 - 156 = 0$$
$$\therefore L_0M_1 = +156.0,$$

and from Fig. 70 it is evident that $L_0M_1 = M_1L_1$. At point L_1 (see Fig. 78), L_1M_2 is equal to M_1L_1, less the horizontal component of m_1L_1, and the equation is:

$$-156 + 12 + L_1M_2 = 0$$
$$\therefore L_1M_2 = +144.0, \text{ and } M_2L_2 = +144.0.$$

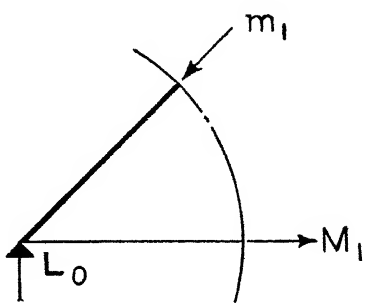


Fig. 77 Chord Stress in Baltimore Truss.

At point L_2 (see Fig. 79), L_2M_3 is equal to the sum of the horizontal components of M_2L_2 and m_2L_2; that is,

$$+L_2M_3 - 144.0 - 96.0 = 0$$
$$\therefore L_2M_3 = M_3L_3 = +240.0$$

In a similar manner, at point L_3, the stress equation is:

$$+L_3M_4 - 240.0 - 48.0 = 0$$
$$\therefore L_3M_4 = M_4L_4 = +288.0.$$

At the upper panel point U_1 (see Fig. 80), there results the equation:

$$+U_1U_2 + \text{hor. comp } U_1m_2 + \text{hor. comp } m_1U_1 = 0,$$
$$U_1U_2 + 108.0 + 144 = 0; \text{ or, } U_1U_2 = -252.0.$$

For the member U_2U_3 (see Fig. 81), the equation is:

$$+U_1U_2 + U_2U_3 - \text{hor. comp. } m_2U_2 + \text{hor. comp. } U_2m_3 = 0$$
$$+252.0 + U_2U_3 - 12 + 60 = 0$$
$$\therefore U_2U_3 = -300.0.$$

In a similar manner, by resolving the horizontal forces at U_3, it will be seen that the action of m_3U_3 will neutralize that of U_3m_4, as

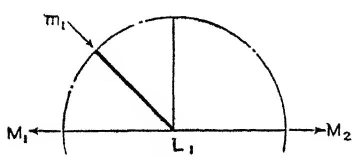


Fig. 78.

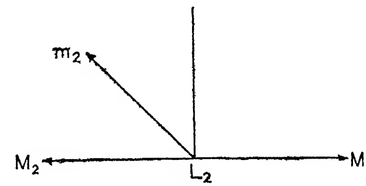


Fig. 79.

Bottom Chord Stresses in Baltimore Truss

they are equal and pull in opposite directions, and U_3U_4 is equal to $U_2U_3 = -300.0$.

The live-load stresses in the chords, the end-post, and the sub-diagonals are all proportional to the dead-load stresses in the same

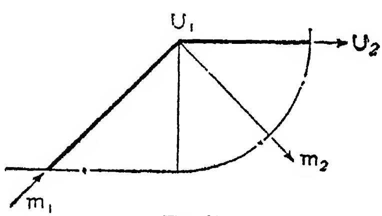


Fig 80.

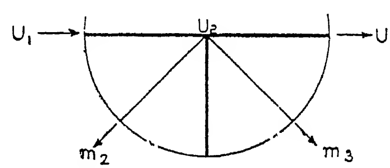


Fig 81

Top Chord Stresses in Baltimore Truss

ratio as the live panel load is to the dead panel load. This ratio is $\frac{40}{24} = 1.667$. By reference to Fig 70, it will be seen that the live-load stress in the sub-verticals is $+40.0$ for each one The following stresses can now be determined:

$$
\begin{aligned}
L_0m_1 &= -220.5 \times 1.667 = -367.5\\
m_1U_1 &= -203.6 \times 1.667 = -339.5\\
U_1U_2 &= -252.0 \times 1.667 = -420.0\\
U_2U_3 &= -300.0 \times 1.667 = -500.0\\
U_3U_4 &= -300.0 \times 1.667 = -500.0
\end{aligned}
$$

$$
\begin{aligned}
L_0L_1 &= +156.0 \times 1.667 = +260.0\\
L_1L_2 &= +144.0 \times 1.667 = +240.0\\
L_2L_3 &= +240.0 \times 1.667 = +405.0\\
L_3L_4 &= +288.0 \times 1.667 = +481.0
\end{aligned}
$$

$$m_1L_1 = -16\ 96 \times 1\ 667 = -28\ 28$$
$$m_2U_2 = m_3U_3 = +16\ 96 \times 1\ 667 = +28.28$$

The vertical U_1L_1 will have its maximum live-load stress when points M_1 and L_1 are loaded, for these are the only loads which cause a stress in that member (see Fig. 76a). The equation is:

$$-\frac{40}{2} - 40 + U_1L_1 = 0,$$

from which,

$$U_1L_1 = +60\ 0.$$

The maximum live-load stresses in U_1m_2, U_2m_3, and U_3m_4 are obtained in a manner exactly like that used in obtaining dead-load stress, only the live-load positive shear is used. The stresses are:

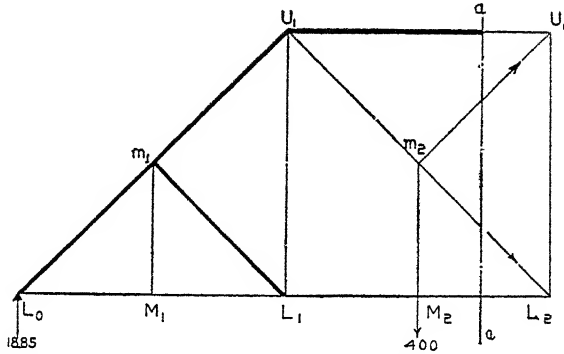


Fig. 82 Stress in Lower Half of Main Diagonal of Baltimore Truss

$$-U_1m_2 \times 0\ 707 + 188\ 5 = 0 \quad \therefore U_1m_2 = +266\ 5$$
$$-U_2m_3 \times 0\ 707 + 128.5 = 0 \quad \therefore U_2m_3 = +181\ 5$$
$$-U_3m_4 \times 0\ 707 + 80\ 0 = 0 \quad \therefore U_3m_4 = +113\ 1$$

In the determination of the maximum live-load stress in the lower halves of the main diagonals, m_2L_2, m_3L_3, and m_4L_4, one of the peculiarities of this truss becomes apparent. A section being passed as in Fig. 82, the panel point ahead of the section, and all between the section and the right support, must be loaded. This of course produces a stress in m_2U_2, and the vertical component of this enters the stress equation. The shear in the section $a - a$ *under this loading* is:

$$V_{a-a} = +188.5 - 40 = +148.5;$$

and the stress equation is:

$$-m_2L_2 \times 0\ 707 + \frac{40}{2} + 148\ 5 = 0$$
$$\therefore m_2L_2 = +238\ 0.$$

If the truss had been loaded from the section to the right, there being no load on M_2, no stress would result in m_2U_2, and the stress in m_2L_2

would have been $m_2L_2 = \frac{157.2}{0.707} = +222.2$. In a similar manner, by loading successively points M_3 and to the right, and M_4 and to the right, the stress equations of m_3L_3 and m_4L_4 are:

$$-m_3L_3 \times 0.707 + \frac{40}{2} + 128.5 - 40 = 0 \qquad \therefore m_3L_3 = +153.3$$

$$-m_4L_4 \times 0.707 + \frac{40}{2} + 80.0 - 40 = 0 \qquad \therefore m_4L_4 = +84.8$$

The maximum live-load stresses in the main verticals occur when the panel points to the right of the section which cuts the member under consideration are loaded. There being no load at the end of the sub-vertical just to the left of the section, there will be no stress in the sub-diagonal which the section cuts.

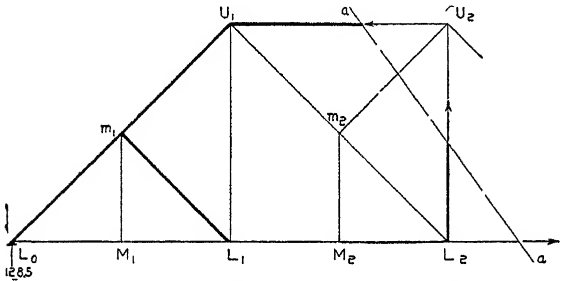


Fig 83 Stress in Main Vertical of Baltimore Truss

The chords, of course, do not exert a vertical component; and so the only unknown term of the stress equation is the stress in the member itself. Fig. 83 shows how the section should be passed when U_2L_2 is considered. The stress equation is:

$$+U_2L_2 + V_{a-a} = 0; \qquad +U_2L_2 + 128.5 = 0, \qquad \therefore U_2L_2 = -128.5.$$

In a similar manner, by passing a section cutting U_2U_3, m_3U_3, U_3L_3, L_3m_4, and loading M_4 and to the right, it is seen that the stress equation for U_3L_3 is:

$$+U_3L_3 + 80.0 = 0 \qquad \therefore U_3L_3 = -80.0$$

The components of m_3U_3 and L_3m_4 are zero, as can readily be proved by solving for them under this loading.

Fig. 84 gives all the stresses, and they are written in order of dead load, live load, and maximum.

43. **Other Trusses.** The analysis of the foregoing trusses will enable one to solve any of the trusses of modern times. For the solution of the Whipple (sometimes called the "double-intersection Pratt") and others which are not mentioned in this text, the student

is referred to the text-books of F. E. Turneaure and Mansfield Merriman.

ENGINE LOADS

44. **Use of Engine Loads.** It was formerly the custom for railroads to specify that the engine to be used in computing the stresses in their bridges should be one of their own which was in actual use. The engines of different roads were usually different both in regard to the weight on the various wheels and in regard to the number and spacing of the wheels. Of late years, considerable progress has been made towards the adoption of a typical engine loading as standard. These typical engines (see Fig. 17, Article 25) vary only in regard to the weights on the wheels, the number and spacing of wheels being the same in all engines.

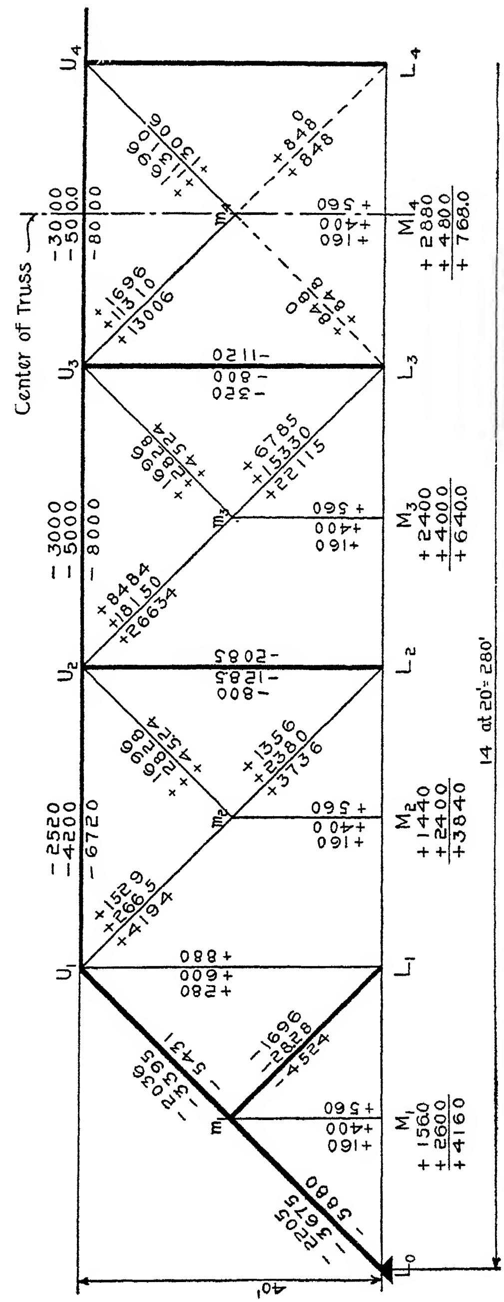


Fig. 84 Stress Diagram of Baltimore Truss of Fig 69. Stresses indicated in order of Dead-Load, Live-Load, and Maximum.

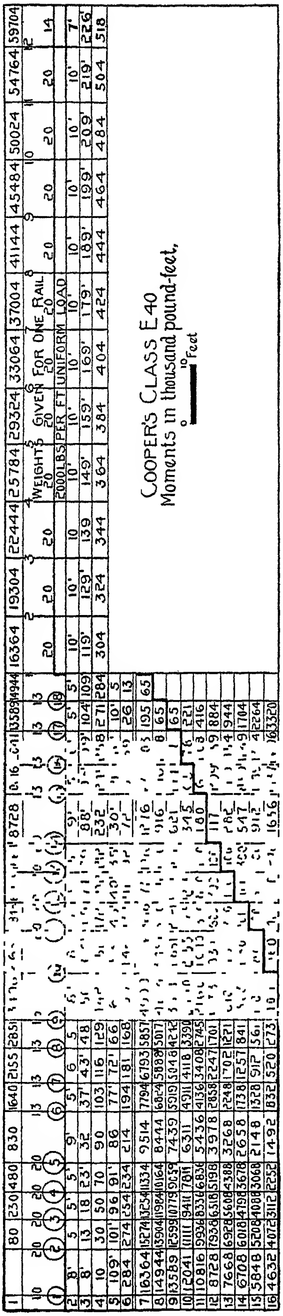


Fig 85 Diagram for Computing Stresses under Engine Loads

The distance between wheels is a whole number of feet, instead of a number of feet and inches and fractions thereof. For examples of loadings which are in almost universal use, consult the specifications of Cooper or Waddell.

The labor of computation of stresses when engine loads are used is considerably lessened by the use of the so-called *engine diagrams*. Fig. 85 gives a diagram which has been found very convenient. The first line at the top represents the bending moment of all the loads about the point to the left of it. All the loads are given in thousands of pounds, and all the moments are in thousands of pound-feet. The practice of writing results in thousands of pounds—or, in case of moments, in thousands of pound-feet or pound-inches—is to be recommended, as it saves the unnecessary labor of writing ciphers. Throughout this text this practice has been extensively followed, the stresses being written to the nearest ten pounds or one-hundred pounds as the case may be. For example, 6 433 may be written 6.43 or 6.4, the few pounds which are neglected making no appreciable difference in the design. The distances are in feet.

As an example of the use of the first line at the top, suppose that it is desired to find the moment of all the loads to the left of a certain point when wheel 6 (the numbers of the wheels are placed inside of the circles

representing the wheels) is just over the point. The moment will be 1 640 000 pound-feet, which is obtained by reading off the 1 640 just to the right of the line through wheel 6.

When using the first line for values at sections in the uniform load, the values given represent the moment of all wheel and uniform loads about the points in the line or section to the left of the value given. For example, if it is desired to find the moment about a point in line 2, it will be 19 304 000 pound-feet, the value 19 304 appearing to the right of the line.

The line of figures below the wheels indicates the distances between any two wheels.

The third line of figures indicates the distance from the first wheel to the wheel to the right. For instance, 37 is the distance from wheel 1 to wheel 7.

The values in the fourth line indicate the sum total of all the loads to the left of the value given. For example, 245 signifies that the loads 1 to 15 inclusive weigh 245 000 pounds.

The values in lines 5 and 6 are similar to those of lines 3 and 4, except that the starting point is at the head of the uniform load. For example, 40 in line 5, and 112 in line 6, indicate that it is 40 feet from the head of the uniform load to the wheel 12, and that wheels 18 to 12 inclusive weigh 112 000 pounds.

The values in lines 7 to 16 indicate the value of the moment of all the wheels from the zigzag line up to and including the one to the left or the right, according as the value is to the left or the right of the zigzag line. For example, 2 745, line 11, indicates that the moments of wheels 8 to 14 inclusive about the zigzag line just under wheel 15, is 2 745 000 pound-feet; or the value 1 704, line 14, shows that the moments of wheels 13 to 18 about the zigzag line just under wheel 12 is 1 704 000 pound-feet.

When line 4 of figures is under the uniform load, the values refer to the vertical line to the right; thus 324 is the value of all loads to the left of line 3 about that line.

For values of moments at points which fall in between wheels, or at positions in the uniform load where the value of the moment is not given, a very important principle of applied mechanics is used. It is:

$$M_a = M' + Wx + \frac{wx^2}{2}.$$

in which,

M_a = Moment at section desired;
M' = Value of moment at preceding vertical line;
W = Sum total of all loads to the left of and at the point where M' is taken,
x = Distance from section under consideration to vertical line to which M' is referred,
w = Uniform load per linear foot on the distance x.

Let it be desired, for example, to determine the moment at a point c, 3 feet to the right of wheel 13. The position of the loads is given in Fig. 86. The moment is:

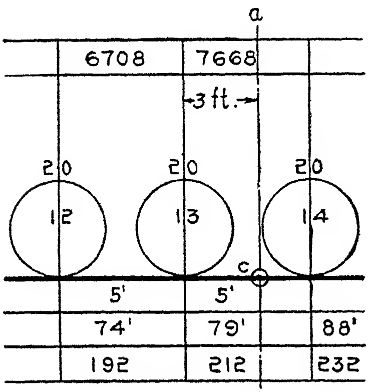


Fig. 86. Calculation of Moment at a Point under Engine Load.

$$M_{a\text{-}a} = 7\,668 + 212 \times 3$$
$$= 7\,668 + 636$$
$$= 8\,304 = 8\,304\,000 \text{ pound-feet, there being no uniform load}$$

To illustrate the method when applied to points in the uniform load, assume the point to be 7 feet to the right of line 2. The position is illustrated in Fig. 87. The moment is:

$$M_{b\text{-}b} = 19\,304 + 304 \times 7 + \frac{7^2 \times 2}{2},$$
$$= 21\,481 = 21\,481\,000 \text{ pound-feet}$$

The use of the moment diagram is now apparent. Reactions due to any position of the engines may be determined by dividing the span into the value obtained for the moment at the right end of the span. Likewise, if the moment of the reaction about any panel point is determined and from it the moment of the wheel loads about that same panel point are subtracted, then the result, divided by the height of the truss, will give the chord stress. For example, if the right end of an 8-panel 196-foot span truss, height 25 feet, came 7 feet to the right of the vertical line 2, then the moment at this point (see Fig. 87) would be 21 481 000, and the reaction would be 21 481 000 ÷ 196 = 109 600. This position of the loads would cause the panel point L_6 to come 3 feet to the right of wheel 13. The moment of the reaction about L_6 is 109 600 × 6 × 24.5 = 16 111 200:

and the chord stress L_5L_6 for this position of the engine is:

$$\frac{16\,111\,200 - 8\,304\,000}{25} = -312\,000 \text{ pounds.}$$

In using the engine to determine the shear in any particular panel, it must be remembered that the shear is *not* the left reaction less all the loads to the left of the panel point on the right of the section, as the loads in the panel under consideration are carried on stringers, and these stringers transfer a portion of the loads to the

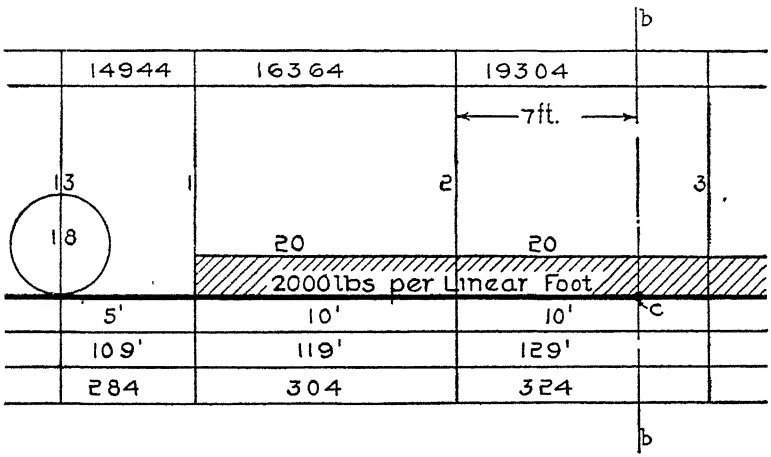


Fig. 87. Calculation of Moment at Point under Uniform Load

panel point on the left of the panel, and a portion to the panel point on the right of the panel. Only that portion of the loads in the panel which is transferred to the left panel point should be subtracted from the reaction, as should all of the loads to the left of the panel under consideration. If, in a 6-panel 120-foot span Pratt truss, the wheel 6 comes at L_2, the left reaction will be:

$$R_1 = \frac{1}{120}\left(16\,364 + 3 \times 284 + \frac{3^2 \times 2\,0}{2}\right) = 143.6;$$

and the loads in the first two panels will be in position as indicated by Fig. 88, the wheel 3 being 1 foot to the right of point L_1. Let it be required to determine the shear in the panel L_1L_2 when the loads are in this position. It will be the reaction 143.6 minus loads 1 and 2 and also that portion of the loads 3, 4, and 5 which will be transferred by the stringers to the point L_1. As the stringers are simple

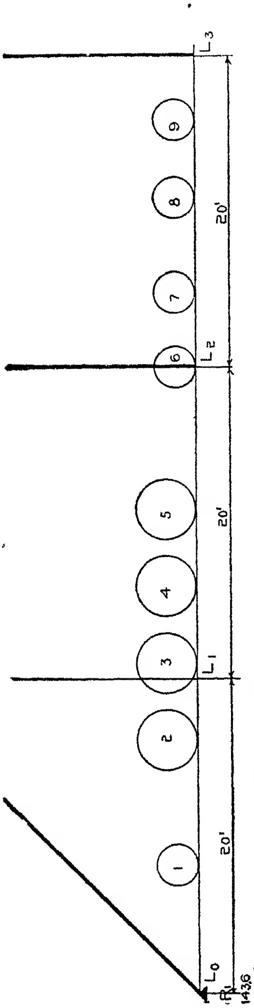


Fig. 88. Determination of Shear in Panel under Engine Load.

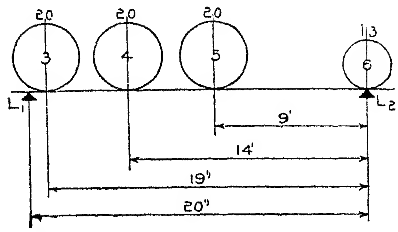


Fig. 89 Shear Diagram for Panel under Engine Load

beams, the amount transferred to L_1 will be the reaction of the stringer L_1L_2. Referring to Fig. 89, the reaction is:

$$R_{L_1} = (20 \times 9 + 20 \times 14 + 20 \times 19) \div 20$$
$$= 42.0$$

The shear in the second panel is now found to be:

$$V_2 = 143.6 - (10 + 20 + 42.0) = +71.6$$

In the majority of cases where it is necessary to determine the shear in a panel, none of the loads will be in the panel to the left of the one under consideration. In this case the operation is somewhat simplified, as the engine diagram can be used directly. If the engine be placed so that the third wheel is at L_2, wheel 16 will be just over the right support, and the left reaction will be:

$$R_1 = 12\,041 \div 120 = 100.3$$

As there are no wheel loads in the first panel, the amount to be subtracted from the reaction will be that proportion of the loads 1 and 2 which is transferred to L_1; and this (see Fig. 90) is $230 \div 20 = 11.5$. The shear in the second panel is then $100.3 - 11.5 = +88.8$.

From inspection of the resulting shear in the second panel when wheel 6 is at L_2 and when wheel 3 is at L_2, it is seen that different wheels at L_2 will give different shears in the panel to the left. Evidently there is some wheel which will give the greatest shear possible. The same is true of the relation between wheels and moments. The

next two articles are devoted to subject-matter which will enable one to tell which of several wheels is the correct wheel at the point, without the necessity of solving for the shear each time every wheel is at the point.

45. **Position of Wheel Loads for Maximum Shear.** By methods of differential calculus, it can be proved that, for any system, either of wheel loads or wheel loads followed by a uniform load (see Fig. 91), the correct wheel that should be at the panel point b in order to

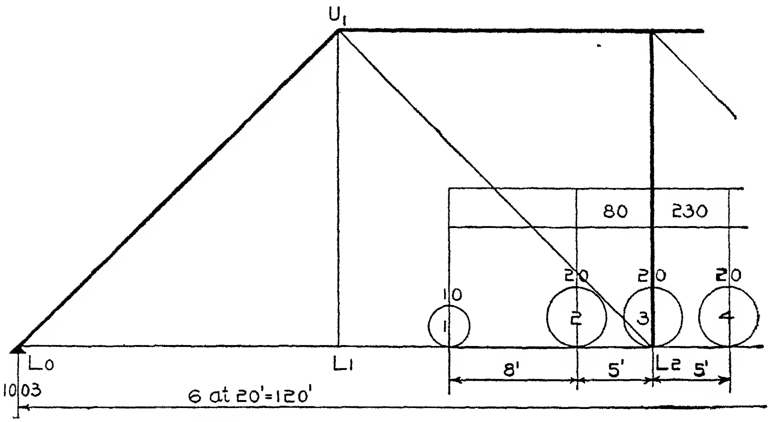


Fig. 90 Determination of Shear in Panel under Engine Load.

give a very great or maximum shear in the panel $a - b$, is such a wheel that the quantity $Q = \frac{W}{m} - G$ is positive when $q = \frac{W}{m} - (G + P)$ is negative In these equations,

W = Total load on the truss,
m = Number of panels in the truss;
G = Load in panel under consideration, and
P = Load at panel point on right of panel.

If a load is directly over the panel point a, it is not to be included in the weight G; neither is P included in the weight G. If a wheel load should come directly over the right end of the truss, it should not be considered in the quantity W.

The only way to determine which wheel is the correct one, is to try wheel 1, then wheel 2, and so on, until the wheel or wheels are reached that will give the Q and q signs of an opposite character.

The process should not be stopped there, but the next succeeding wheels should be tried until Q and q again have the same sign.

As an example, let it be required to determine the position of the wheel loads to produce the maximum positive shears in a 6-panel 120-foot Pratt truss. This work should be arranged in tabular form, and Table V is found to be convenient.

TABLE V

Determination of Position of Wheel Loads for Maximum Shear

($m = 6$)

Panel Point	Wheel at Point	G	$\frac{W}{m}$	P	$G+P$	Q	q	Remarks
L_1	2	10	$\frac{284}{6}$ = 47 33	20	30	+	+	
L_1	3	30	$\frac{292}{6}$ = 48 67	20	50	+	−	gives a maximum
L_1	4	50	$\frac{302}{6}$ = 50 30	20	70	+	−	gives a maximum
L_1	5	60	$\frac{302}{6}$ = 50 30	20	80	−	−	
L_2	2	10	$\frac{232}{6}$ = 38 67	20	30	+	+	
L_2	3	30	$\frac{245}{6}$ = 40 83	20	50	+	−	gives a maximum
L_2	4	50	$\frac{258}{6}$ = 43 00	20	70	−	−	
L_3	1	0	$\frac{152}{6}$ = 25 33	10	10	+	+	
L_3	2	10	$\frac{172}{6}$ = 28 67	20	30	+	−	gives a maximum
L_3	3	30	$\frac{192}{6}$ = 32.0	20	50	+	−	gives a maximum
L_3	4	50	$\frac{212}{6}$ = 35 3	20	70	−	−	
L_4	1	0	$\frac{116}{6}$ = 19 33	10	10	+	+	
L_4	2	10	$\frac{129}{6}$ = 21 50	20	30	+	−	gives a maximum
L_4	3	30	$\frac{142}{6}$ = 23.67	20	50	−	−	
L_5	1	0	$\frac{70}{6}$ = 11 67	10	10	+	+	
L_5	2	10	$\frac{90}{6}$ = 15 00	20	30	+	−	gives a maximum
L_5	3	30	$\frac{103}{6}$ = 17 17	20	50	−	−	

A study of Table V shows the fact that wheel 1 can never produce a maximum. It also shows that there are in some cases two positions which will give large values of the shear. In these cases the shears for each position of the engines must be determined in order to tell which wheel at the panel point in reality gives the greatest. In practical work it is customary to use the first position found, as the difference in the shears resulting from the use of the two positions is not large enough to affect the final design.

Fig. 92 shows the engine diagram on the truss in the correct position to give the maximum shear in the second panel. The weight of wheel 16 is not included in the weight W, as it is directly over the right support.

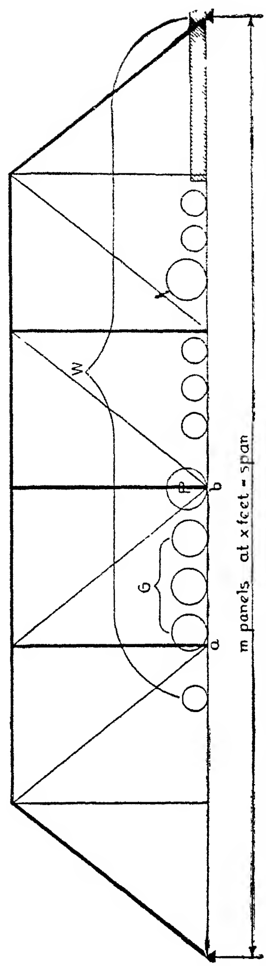


Fig. 91. Engine Diagram for Determining Position of Wheel Loads to Give Maximum Shear in Panel

46. **Position of Wheel Loads for Maximum Moments.** In this case the methods of differential calculus are employed to determine which wheels will, if placed at a point, give a maximum moment at that point. For any system, either of wheel loads or of wheel loads followed by a uniform load, that wheel which will cause $K = \frac{Wn}{m} - L$ to be positive, and $k = \frac{Wn}{m} - (L + P)$ to be negative, is the wheel. Here n is the number of the panel under consideration, and is to be reckoned from the left end; L = the load to the left of the point under consideration; and the remainder of the letters signify the same as they do in Article 45. In some cases there will be more than one position of the loads which will satisfy the above condition. It is then necessary to work out the actual moments created by the loads in each position, in order to find out which is the largest. The

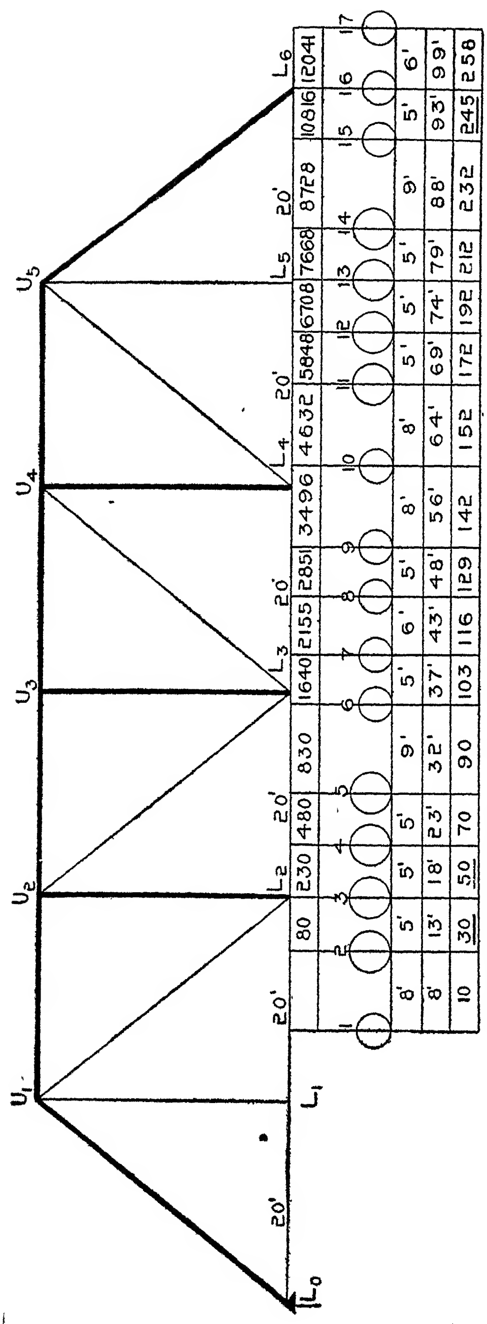


Fig 92 Diagram of 6-Panel Pratt Truss Loaded with Engine in Position to Give Maximum Shear in Second Panel

position of the loads for the greatest moments should be determined for all panel points except the one on the extreme right, as the greatest moment possible may be caused by wheels of the rear engine being on the point on the right-hand side of the truss, instead of the wheels of the front engine being at the corresponding point on the left-hand side.

In general, it may be said that there will be a number of wheels which, if placed at the panel point in the center of the span, will satisfy the given conditions. In this particular case, it is not necessary to determine all of the moments. The greatest moment possible will occur when that one of the heaviest wheels of the second locomotive which gives the heaviest load upon the truss is at the point. In case several of the heavy wheels give the

same maximum load W, use the first wheel which gives this maximum W.

Let it be required to determine the position of the wheel loads for maximum moments at the lower panel points of the 6-panel 120-foot Pratt truss of Article 45. The necessary work can be conveniently arranged in the form of a table, as is done in Table VI.

TABLE VI

Determination of Position of Wheel Loads for Maximum Moments

Panel Point	Wheel at Point	L	$\frac{Wn}{m}$	P	$L+P$	K	k	Remarks
L_1	2	10	284 − 6 = 47 3	20	30	+	+	
L_1	3	30	292 − 6 = 48 7	20	50	+	−	gives a maximum
L_1	4	50	302 − 6 = 50 3	20	70	+	−	gives a maximum
L_1	5	60	302 − 6 = 50 3	20	80	−	−	wheel 1 is off bridge
L_2	5	70	(271 − 6) × 2 = 90 3	20	90	+	+	
L_2	6	90	(290 − 6) × 2 = 96 7	13	103	+	−	gives a maximum
L_2	7	103	(300 − 6) × 2 = 100 0	13	116	−	−	
L_3	8	116	(271 − 6) × 3 = 135 5	13	129	+	+	
L_3	9	129	(284 − 6) × 3 = 142 0	13	142	+	0	gives a minimum
L_3	10	142	(298 − 6) × 3 = 149 0	10	152	+	−	gives a maximum
L_3	11	142	(304 − 6) × 3 = 152 0	20	162	+	−	gives a maximum
L_3	12	142	(294 − 6) × 3 = 147 0	20	162	+	−	gives a maximum
L_3	13	142	(284 − 6) × 3 = 142 0	20	162	0	−	gives a minimum
L_3	14	142	(274 − 6) × 3 = 137 0	20	162	−	−	
L_4	11	152	(271 − 6) × 4 = 180 6	20	172	+	+	Note wheel 18 not included
L_4	12	172	(284 − 6) × 4 = 189.4	20	192	+	−	gives a maximum
L_4	13	192	(294 − 6) × 4 = 196 0	20	212	+	−	gives a maximum
L_4	14	212	(304 − 6) × 4 = 202.6	20	232	−	−	

One should carefully note that in certain positions, as when wheels 11, 12, 13, and 14 are at L_3, some wheels are to the left of the left support; that is, they are not upon the bridge. In all such cases they are counted neither in the quantity L nor in W.

In the case of L_3, wheel 11, being the first large driver of the second engine, will give the greatest moment, as it is the first driver to come at the point when the maximum load of 304 000 pounds is on the truss. Fig. 93 shows the engine diagram on the truss in correct position to give the maximum moment at point L_2.

47. **Pratt Truss under Engine Loads.** In order to exemplify the use of the engine-load diagram, let it be required to determine the stresses in the Pratt truss of Article 45 due to E 40 loading, the

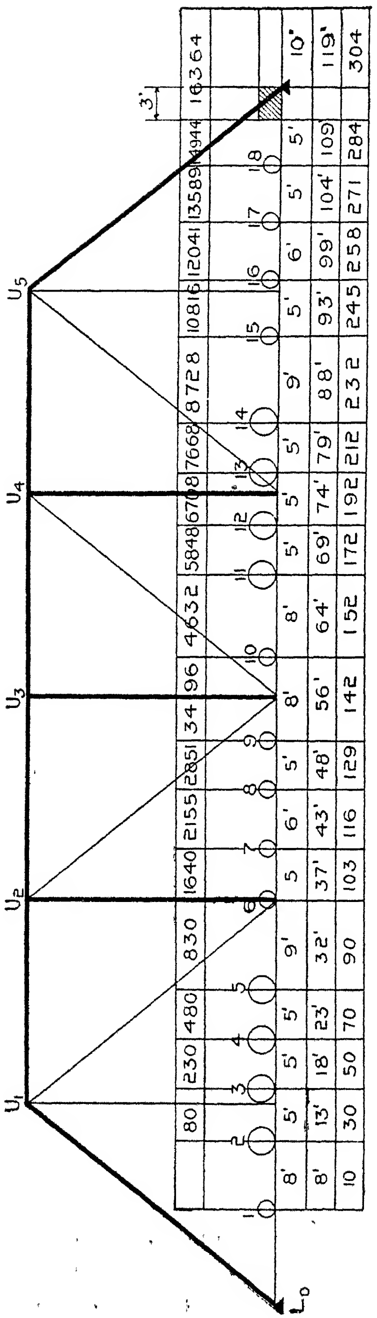


Fig 93. Diagram of 6-Panel Pratt Truss Loaded with Engine in Position to Give Maximum Moment at Panel Point

height being 25 feet. The secant is $(25^2 + 20^2)^{\frac{1}{2}} \div 25 = 1.28$.

The maximum positive shears in the various panels should first be computed These, written in reverse order, will be the maximum negative or minimum shears. Table V should now be referred to, and an outline diagram drawn to the same scale as the engine used, on which to place the engine diagram in the correct position. The various values can then be read off the diagram at the right-hand end of the truss. It will be found convenient to lay off to scale the first ten feet of the lower chord of the truss from the right support, making the divisions one foot apart. This will enable one to ascertain the distance of the last wheel load from the right support, or the amount of uniform load upon the bridge, without scaling or further computation. In case it is desired to have the wheel loads appear on the lower chord, as in Fig. 93, the outline of the truss should be on tracing cloth or transparent paper. This is not to be advised, however, as errors are likely to occur because of failure to distinguish clearly

the various numerical values. It is far better to place the diagram as in Fig. 92, in which case the outline of both the truss and the diagram can be drawn on good stiff paper.

For wheel 3 at point L_1 (see Articles 44 and 45), the left reaction is as follows, there being four feet of uniform load upon the truss:

$$R_1 = \left(16\,364 + 284 \times 4 + \frac{4^2 \times 2\,0}{2}\right) - 120 = 146\,0,$$

and the proportion of loads in the panel which is transferred to the point L_0 by the stringers is $230 \div 20 = 11.5$. The shear is therefore $V_1 = +146.0 - 11.5 = +134.5$. The computation for the shear when wheel 4 is at the point, will not be made; for, as has been noted before, the result will not be much different from the above.

For wheel 3 at L_2, wheel 16 comes over the right support. The left reaction is:

$$R_1 = 12\,041 - 120 = 100\,3,$$

the proportional part of the loads which is transferred to L_1 is 11.5; and the shear is:

$$V_2 = (+100\,3 - 11\,5) = +88\,8$$

For wheel 2 at L_3, wheel 11 comes four feet from the right support. The left reaction is:

$$R_1 = (5\,848 + 172 \times 4) - 120 = 54\,5$$

That part of wheel 1 which is transferred to L_2 is $80 \div 20 = 4.0$, and the shear is therefore:

$$V_3 = (+54\,5 - 4) = +50.5.$$

For wheel 2 at L_4, wheel 9 comes over the right support. The left reaction is:

$$R_1 = 3\,496 - 120 = 29\,1$$

That part of wheel 1 which is transferred to L_3 is 4.0, and the shear is therefore:

$$V_4 = (+29\,1 - 4\,0) = +25.1.$$

For wheel 2 at L_5, wheel 5 is five feet from the right support, and the left reaction is:

$$R_1 = (830 + 90 \times 5) - 120 = 10.7$$

and the shear is:

$$V_5 = (+10.7 - 4\,0) = +6\,7.$$

If the dead panel load is 20 000 pounds, all the shears may now be written as follows:

Dead-Load V	+ Live-Load V	− Live-Load V
$V_1 = +50.0$	$+134.5$	$\pm\ 0.0$
$V_2 = +30.0$	$+\ 88.8$	$-\ 6.7$
$V_3 = +10.0$	$+\ 50.5$	-25.1
$V_4 = -10.0$	$+\ 25.1$	-50.5
$V_5 = -30.0$	$+\ 6.7$	-88.8
$V_6 = -50.0$	$\pm\ 0.0$	-134.5

A comparison of the shears in the third and fourth panels shows that counters are required. The stress in these counters is:

$$U_3L_2 = U_3L_4 = +1.28 \times (25.1 - 10.0) = +19.2$$

As it is known that positive shears cause a compressive stress in L_0U_1 and tensile stresses in the diagonals, and that negative shears produce compressive stresses in the intermediate posts, the left half of the bridge being considered, the web stresses for dead and live load can be determined without in all cases writing the stress equations in order to determine the sign. It should be remembered that one-third of the dead panel load, or 6 700 pounds, is applied at the panel points of the top chord.

Dead-Load Stresses in the Diagonals—

$$L_0U_1 = -1.28 \times 50 = -64.0$$
$$U_1L_2 = +1.28 \times 30 = +38.4$$
$$U_2L_3 = +1.28 \times 10 = +12.8$$

Dead-Load Stresses in the Verticals. For U_2L_2, the section passed will cut U_1U_2, U_2L_2, and L_2L_3, and the shear on this section will be $50 - 2 \times 13.3 - 6.7 = +16.7$. The stress equation is $+16.7 + U_2L_2 = 0$, from which $U_2L_2 = -16.7$.

The dead-load stress in U_3L_3 is found by passing a circular section around U_3. Then $-U_3L_3 - 6.7 = 0$, from which $U_3L_3 = -6.7$. In a similar manner, by passing a section around L_1, the stress is found to be $+13.3$.

Live-Load Stresses in the Diagonals—

	Maximum	Minimum
L_0U_1	$= -1.28 \times 134.5 = -172.2$	0
U_1L_2	$= +1.28 \times 88.8 = +113.6$	$-1.28 \times 6.7 = -8.6$
U_2L_3	$= +1.28 \times 50.5 = +\ 64.7$	0

Live-Load Stresses in the Verticals. The maximum stress in U_1L_1 occurs when one of the large drivers is at L_1, and the loads in

the first panel are as near as possible one-half the sum of the loads in panels 1 and 2 and the load at L_1. This can be established as a fact by use of the differential calculus. In the present case, this condition is satisfied when wheel 4 is at L_1. Then the weight of the wheels in panel 1 is 50 000 pounds, and the sum total is 116 000 pounds. If wheel 13 be placed at L_1, the result will be the same, and then the engine diagram can be used. Fig. 94 represents the engine diagram in place, ready to use. According to Article 44, the value 480 is the moment of wheels 10 to 12 about L_1. Therefore $480 \div 20$ (20 is the panel length) $= 24.00$, is that amount of wheels 10 to 12 which is transferred to L_0. In like manner, $529 \div 20 = 26.45$ is the amount of wheels 14 to 16 transferred to L_2. As the total weight of the loads in the two panels is 116 000 pounds, the amount transferred to L_1 must be $116.0 - (24.00 + 26.45) = 65.55$, and the stress in U_1L_1 is therefore $+65.55$.

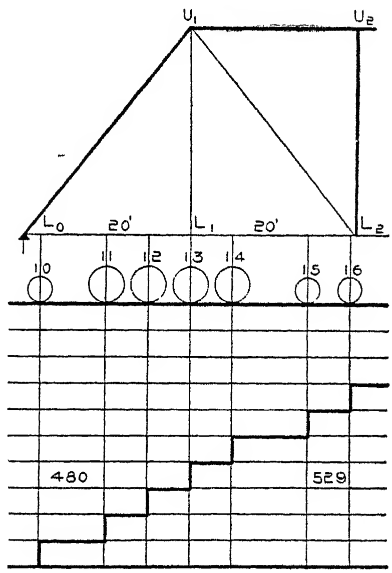


Fig 94 Engine Diagram for Determination of Live-Load Stress in Vertical of Pratt Truss

The maximum live-load stress in L_2U_2 occurs when the loading is in a position to give the maximum shear in the third panel, as the shear at a section cutting U_1U_2, U_2L_2, and L_2L_3 is the same as that at a vertical section in the panel. The stress equation is $+U_2L_2 + 50.5 = 0$, from which $U_2L_2 = -50.5$. In a similar manner, the stress equation for the maximum live-load stress in U_3L_3 is $+U_3L_3 + 25.1 = 0$, as U_3L_4 is working, and therefore $U_3L_3 = -25.1$. As in the case of the analysis of the Pratt truss under uniform load (see Article 39), the dead-load stress of -6.7 cannot be added to this stress of -25.1 to obtain the maximum; but the dead-load stress in U_3L_3 must be obtained when diagonals U_2L_3 and U_3L_4 are in action. In the manner explained in Article 39, this is found to be $+3.30$.

It will be found that as the engines come on the bridge from the

left, the counters come into action in the case of U_2L_2; and in the case of U_3L_3, both U_2L_3 and L_3U_4 act, thus causing the live-load stress in these verticals to be zero; and when this is the case, the dead-load stress is $-$ 6.7, which is the minimum.

Dead-Load Chord Stresses. The dead-load chord stresses can be found by any of the methods previously given; but they will be found by the tangent method as indicated below, the tangent being $20 \div 25 = 0.8$:

$$L_0L_1 = +0.8 \times 50 = +40.0 = L_1L_2$$
$$U_1U_2 = -(50 + 30) \times 0.8 = -64.0$$
$$U_2U_3 = -(50 + 30 + 10) \times 0.8 = -72.0$$
$$L_2L_3 = -U_1U_2 = -(-64.0) = +64.0$$

Live-Load Chord Stresses. On account of the wheel loading, no ratio can be established between these stresses and the dead-load chord stresses. The maximum moments at each point must be determined, and these divided by the height of the truss will give the chord stresses. For all points to the left of the center of the bridge, the main diagonal will act. For points to the right of the center, an uncertainty exists. The shear in the panels on either side of the point under consideration should be determined when the loading is in position to give the maximum moment at that point. This will indicate which diagonals act, which fact will indicate for what chord member that point is the center of moments.

When wheel 3 is at L_1, four feet of uniform load are on the truss, and the left reaction is:

$$R_1 = (16\,364 + 284 \times 4 + \frac{4^2 \times 2}{2}) \div 120 = 146.0.$$

The moment of this reaction about L_1, less the moment of wheels 1 and 2 about L_1, will be the moment at L_1 due to this loading. The moment of wheels 1 and 2 about L_1 is taken from the diagram, where it occurs in the first line of values just to the right of the vertical line through wheel 3, and therefore:

$$M_1 = 146.0 \times 20 - 230 = 2\,690\,000 \text{ pound-feet.}$$

When wheel 4 is at L_1, there are nine feet of uniform load on the truss, and the left reaction is:

$$R_1 = (16\,364 + 284 \times 9 + \frac{9^2 \times 2}{2}) \div 120 = 158.0;$$

and in this case,

$$M_1 = 158.0 \times 20 - 480 = 2\,680\,000 \text{ pound-feet.}$$

As this is less than when wheel 3 is at the point, wheel 3 gives the greatest moment.

When considering the point L_2 with wheel 6, the left reaction is:

$$R_1 = (16\ 364 + 284 \times 3 + \frac{3^2 \times 2}{2}) - 120 = 143\ 5$$
$$M_2 = 143\ 5 \times 2 \times 20 - 1\ 640$$
$$= 4\ 100\ 000 \text{ pound-feet}$$

The conditions at L_3 indicate that there are several wheels which give large moments; but according to Article 46, wheel 11 gives the maximum moment. When this wheel is at L_3, wheel 1 is off the truss, and 15 feet of uniform load are on the truss. The moment of *all* the loads about the *right support* is $19\ 304 + 304 \times 5 + \frac{5^2 \times 2}{2} = 20\ 849$, from which should be subtracted the moment of wheel 1 about the right support. This moment of wheel 1 is $10 \times 124 = 1\ 240$, and the moment about L_6 of all loads on the truss is $20\ 849 - 1\ 240 = 19\ 609$. The left reaction is:

$$R_1 = 19\ 609 - 120 = 163\ 4$$
$$M_3 = 163\ 4 \times 3 \times 20 - (5\ 848 - 10 \times 64)$$
$$= 4\ 596\ 000 \text{ pound-feet.}$$

In the case of L_4, the reactions and the moments for the two positions are:

For wheel 12, $R_1 = 16\ 364 - 120 = 136.4$
$$M_4 = 136.4 \times 4 \times 20 - 6\ 708$$
$$= 4\ 204\ 000 \text{ pound-feet.}$$

For wheel 13, $R_1 = (16\ 364 + 5 \times 284 + \frac{5^2 \times 2}{2}) - 120 = 148.4$
$$M_4 = 148\ 4 \times 4 \times 20 - 7\ 668$$
$$= 4\ 204\ 000 \text{ pound-feet,}$$

which shows that each wheel gives the same moment, and also that the moment is greater than that at L_2, the corresponding point on the left-hand side of the center of the truss. As L_2 is the center of moments of U_1U_2, then, if the center of moments for U_4U_5 falls at L_4 (that is, if L_4U_5 acts), the stress in U_4U_5 will be greater than the stress in U_1U_2 when wheel 6 is at L_2. Of course, if the engine came on the truss from the left, U_1U_2 would receive the same stress that U_4U_5 now receives. According to the shears, L_4U_5 always acts, and therefore the center of moments for U_4U_5 does fall at L_4.

TABLE VII

Stresses in a Pratt Truss

Kind of Stress	End-Post	Verticals			Diagonals			Upper Chord		Lower Chord		
	L_0U_1	U_1L_1	U_2L_2	U_3L_3	U_1L_2	U_2L_3	L_2U_3	U_1U_2	U_2U_3	L_0L_1	L_1L_2	L_2L_3
Dead-Load.	− 64.0	+ 13.3	− 16.7	− 6.7	+ 38.4	+ 12.8	− 12.8	− 64.0	− 72.0	+ 40.0	+ 40.0	+ 64.0
Live-Load..	− 172.2	+ 65.6	− 50.5	+ 3.3 / − 25.1	+ 113.6	+ 64.7	0.0	− 168.2	− 183.8	+ 107.5	+ 107.5	+ 168.2
	0.0	0.0	0.0	0.0	− 8.6	0.0	+ 32.0					
Maximum..	− 236.2	+ 78.9	− 67.2	− 21.8	+ 157.0	+ 77.5	+ 19.2	− 232.2	− 255.8	+ 147.5	+ 147.5	+ 232.2
Minimum.	− 64.0	+ 13.3	− 16.7	− 6.7	+ 29.8	0.0	0.0	− 64.0	− 72.0	+ 40.0	+ 40.0	+ 64.0

The various moments are written in order, as such action will facilitate the remainder of the computations.

$$M_1 = 2\,690$$
$$M_2 = 4\,100$$
$$M_3 = 4\,596$$
$$M_4 = 4\,204$$

The chord stresses are now found to be:

$$L_0L_1 = L_1L_2 = +\frac{2\,690}{25} = +107.5$$
$$U_1U_2 = -\frac{4\,100}{25} = -164\,0$$
$$U_2U_3 = -\frac{4\,596}{25} = -183\,8$$
$$U_4U_5 = -\frac{4\,204}{25} = -168.2$$
$$L_2L_3 = -U_1U_2 = -(-164.0) = +164\,0$$
$$L_3L_4 = -U_4U_5 = -(-168\,2) = +168.2$$

When the load comes on from the left, the stresses in U_1U_2 and L_2L_3 will be -168.2 and $+168.2$ respectively, which are the maximum live-load stresses for these members.

Instead of placing the values of the stresses on a truss outline, they are sometimes put in tabular form, as in Table VII.

48. **Impact Stresses.** When an engine is at rest on a bridge, the stresses in the members are the same as those computed for that loading. When the loads move across the bridge at any speed, the vibrations and the shocks produced by the counterweights in the drivers and by other causes create stresses in the various members in excess of those computed by aid of the engine diagram. The excess stresses

are designated as *impact stresses.* This term, however, is misleading to a certain extent, as causes other than the impact or pounding of the engine wheels help to produce the stresses referred to.

It is a well-known fact capable of mathematical demonstration, that a load, if suddenly applied, will produce a stress equal to twice that which it will produce as a static load; also, that as the ratio of the weight of the load to the weight of the structure decreases, the vibrations produced by the impact will be less. These two facts are the basis of most of the empirical formulæ for impact stresses; and empirical formulæ are used to obtain these stresses, as the existing conditions and producing causes are not such as to make them susceptible of mathematical treatment. The result of experiments on actual bridges under the effect of passing engines and trains, have been the basis of many formulæ. One of these is:

$$I = S\left(\frac{300}{L + 300}\right),$$

where I = Impact stress in the member,
S = Live-load stress in the member caused by the engine load when at rest;
L = Length of that part of the bridge which is loaded when the stress S is produced, and
300 = A constant value derived from experiments.

This formula was proposed by C. C. Schneider in 1887, and is given in the "Transactions" of the American Society of Civil Engineers, Vol. 34, page 331. While it does not take into consideration the relative weights of the bridge and the live-load loads, this formula does make allowance for the time it takes to produce the stress, by introducing L, the distance over which the engine passes before causing the stress S. It is seen that the smaller the distance L, the greater will be the impact stress for any given value of S. When L becomes exceedingly small, the effect would be that of a suddenly applied load, and the impact stress would equal the stress S. Table VIII gives the values of $\frac{300}{L + 300}$, which is called the *impact coefficient*, for different values of L. Values not given may be interpolated.

For example, by consulting Fig. 92, which gives the position of the engines for the maximum live-load stress in U_1L_2, it is seen that 93 feet (the distance from wheel 1 to the right support) is the loaded

TABLE VIII

Values of the Impact Coefficient

L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$
5	0 984	31	0 906	57	0 840	83	0.783	145	0 674
6	0 980	32	0 904	58	0 838	84	0.781	150	0 667
7	0 977	33	0 901	59	0 836	85	0.779	155	0 659
8	0 974	34	0 898	60	0 833	86	0.777	160	0 652
9	0 971	35	0 896	61	0 831	87	0 775	165	0 645
10	0 968	36	0 893	62	0 829	88	0 773	170	0 638
11	0 965	37	0 890	63	0 826	89	0 771	175	0 632
12	0 962	38	0 888	64	0.824	90	0 769	180	0 625
13	0 958	39	0 885	65	0 822	91	0 767	185	0 619
14	0.955	40	0 882	66	0 820	92	0 765	190	0.612
15	0 952	41	0 880	67	0 817	93	0 763	195	0 606
16	0 949	42	0 877	68	0 815	94	0 761	200	0.600
17	0.946	43	0 875	69	0 813	95	0 759	210	0 588
18	0 943	44	0.872	70	0 811	96	0 758	220	0 577
19	0.940	45	0 870	71	0 809	97	0 756	230	0 566
20	0.937	46	0 867	72	0 806	98	0.754	240	0 556
21	0 935	47	0 865	73	0 804	99	0 752	250	0 546
22	0 932	48	0 862	74	0 802	100	0 750	260	0 536
23	0 929	49	0 860	75	0 800	105	0 741	270	0.526
24	0 926	50	0 857	76	0 798	110	0 732	280	0 517
25	0 923	51	0 855	77	0 796	115	0 725	290	0 508
26	0 920	52	0 852	78	0 794	120	0 714	300	0 500
27	0 917	53	0 850	79	0 792	125	0 706	400	0.429
28	0.915	54	0 847	80	0 789	130	0 698	500	0 375
29	0 912	55	0 845	81	0 787	135	0 690	600	0.333
30	0 909	56	0 843	82	0 785	140	0 682		

length. From Table VII, it is seen that the stress in U_1L_2 produced by this loading is +118.6; and from Table VIII, the impact coefficient for 93 feet is found to be 0.763. The impact stress is now computed:

$$I = 0\ 763 \times 118\ 6 = +90\ 6.$$

The maximum stress in U_1L_2 is now:

Dead-load	=	+ 38 4
Live-load	=	+118 6
Impact	=	+ 90.6
Maximum	=	+247.6

Table IX gives the necessary information for computing the impact stresses, and also gives the impact stresses corresponding to the maximum live-load stresses in the members of the truss of Article 47.

TABLE IX

Impact Stresses in a Pratt Truss

MEMBER	S	L	$\frac{300}{L+300}$	I	REMARKS
L_0U_1	−172.2	113	0.727	−125.2	4 ft. of uniform load on truss
U_1L_1	+ 65.6	37	0.890	+ 58.4	See succeeding text
U_2L_2	− 50.5	68	0.815	− 41.3	Same as for U_2L_3
U_3L_3	− 25.1	48	0.862	− 21.6	Same as for U_3L_4
U_1L_2	+118.6	93	0.763	+ 90.6	Wheel 16 at L_6
U_2L_3	+ 64.7	68	0.815	+ 52.7	Wheel 11 is 4 ft. from L_6
U_3L_4	+ 32.0	48	0.862	+ 27.6	Wheel 9 at L_6
U_1U_2	−168.2	114	0.724	−121.8	Wheel 13 at L_4 5 ft. of uniform load on bridge.
U_2U_3	−183.8	114	0.724	−133.2	15 ft. of uniform load on bridge. Wheel 1 off bridge
L_0L_2	+107.5	113	0.727	+ 78.2	Same loading as for L_0U_1
L_2L_3	+168.2	114	0.724	+121.8	Same loading as for U_2U_3

In the case of U_1L_1, it should be noted that only the wheels 10 to 16 inclusive cause the stress (see Fig. 94), and that the loaded length is the distance from wheel 10 to wheel 16.

Some specifications do not call for impact stresses. The unit-stresses in these specifications are made low, and the sections designed are large enough to withstand the additional stresses due to impact. In cases where the impact stresses are required, they must be considered in computing the maximum and minimum stresses.

49. **Snow-Load Stresses.** In some localities the snowfall is considerable, and its weight should be taken into account in computing stresses. This should be done by considering it as an additional dead load of 15 pounds per square foot of floor surface for every foot of snowfall. As it covers the entire floor surface, the stresses will be proportional to the dead-load stresses. Also it is evident that the snow load should not be taken into account in railroad bridges unless they have solid floors, as most of it falls through the open spaces between ties and stringers.

As an example, let it be required to determine the snow-load stresses in a member of a highway bridge, the dead-load stress in the member being +84.0, the dead panel load being 12 000 pounds, and

the snow being $1\frac{1}{2}$ feet deep on the roadway, which is 14 feet wide. The snow panel load is:

$$\frac{1}{2}(14 \times 15 \times 1\tfrac{1}{2} \times 20) = 3\,150 \text{ pounds}$$

In the above equation, 14 is the width of roadway; 15 is the weight in pounds of one square foot of snow one foot deep; and 20 is the length of one panel. One-half of the weight of snow must be taken, as half is carried by each truss. The snow-load stress is then:

$$\frac{3\,150}{12\,000} \times 84.0 = +\,22.1.$$

In like manner, all snow-load stresses can be computed.

Most of the standard specifications which have been published do not specify snow loads; and in fact it is not customary to include the snow load in any designs except those for bridges in extreme northern latitudes. It is hardly probable that the greatest load will come upon a country bridge when it is covered with snow. Also, in cities, the sidewalks are cleaned of snow; and so is the roadway if the city is of large size.

WIND-LOAD EFFECTS

50. Top Lateral System Through-Bridges. The unit-loads for this system are given in Article 26. Common practice is to take 150 pounds per linear foot of top chord, the end-post being considered part of the top chord in this computation.

In many of the longer-span modern bridges, the diagonals of this system are designed to take either tension or compression; but in the majority of the shorter spans, 200 feet and under, while generally consisting of angles or other stiff shapes, they are designed to take tension only. The verticals or top lateral struts take compression. This combination of tension diagonals and compression verticals makes the so-called *Pratt system of webbing;* and indeed the lateral systems, both top and bottom, are Pratt trusses in a horizontal position. Fig. 95 shows the side elevation of the truss of Article 47, and also the top and bottom laterals. The diagonals shown in full lines act when the wind is *right,* and those shown by dotted lines act when the wind is *left.* Wind *right* indicates that the wind is blowing from the right when a person stands facing the right-hand end of

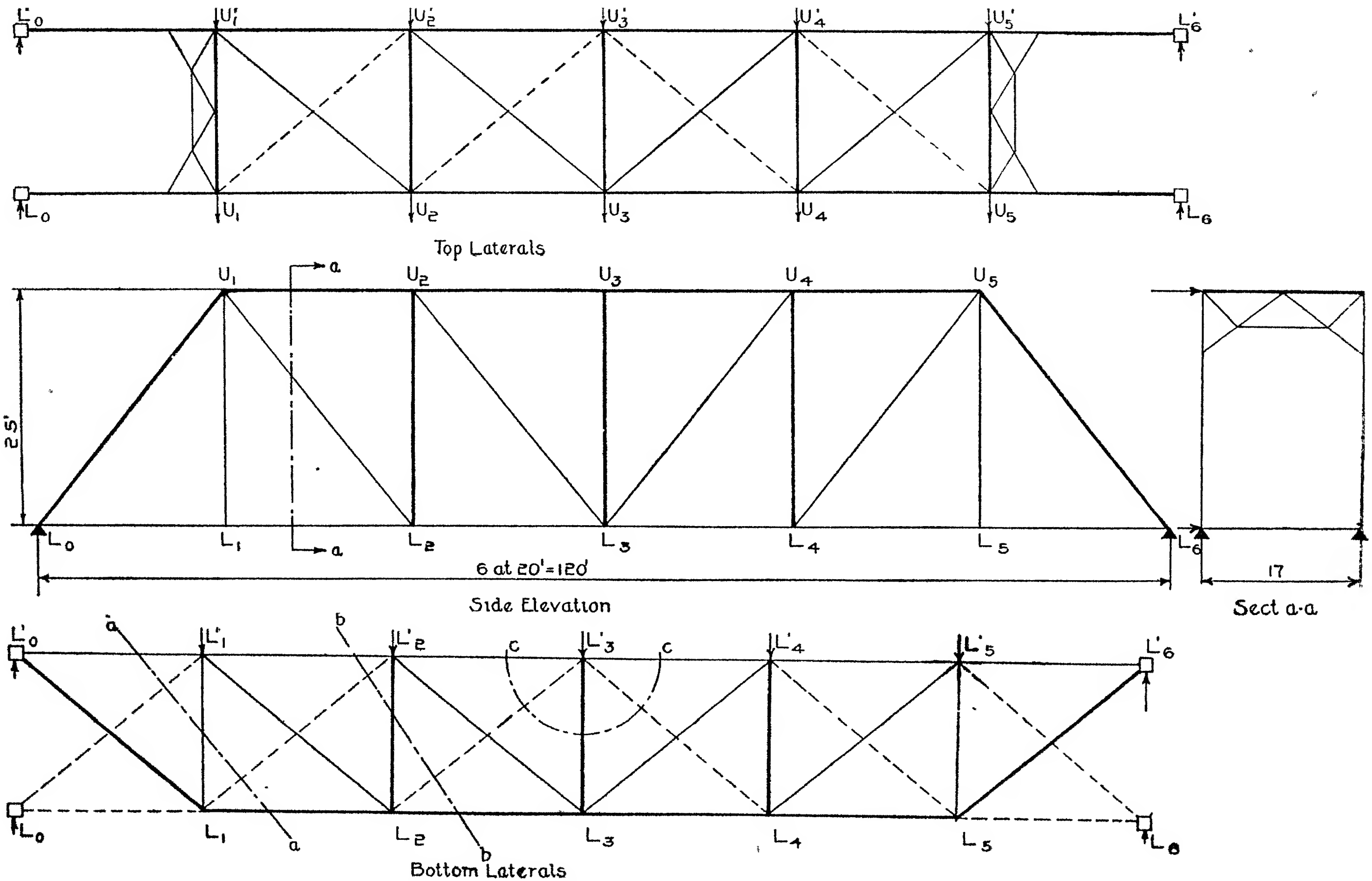


Fig. 95. Side and End Elevations and Top and Bottom Laterals of 6-Panel Pratt Truss.

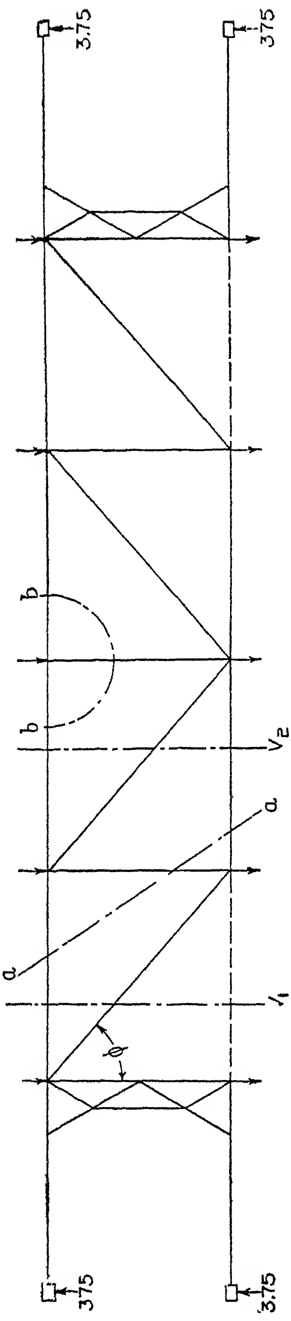


Fig 96. Diagram Showing Distribution of Loads and Reaction in Pratt Truss of Fig 95

the bridge. Wind *left* indicates that the wind blows from a person's left when standing as above described.

The wind load of 150 pounds is divided between the two trusses, this being exact enough for practical purposes; for, by actual experiment, the difference between the readings of wind-pressure gauges placed at points opposite each other in the top chords of a through-bridge was only from 8 to 10 per cent.

The problem, then, is one of a deck Pratt truss with a dead panel load of $150 \times 20 = 3.0$ divided between the two chords. Fig. 96 shows the distribution of loads and the reaction, it being considered that the portal bracings and the end-posts (see Fig. 95) are stiff enough to distribute the reaction equally between the bearing points L_0, L_0', L_6, L_6'. Each panel load is indicated by an arrow, and is equal to $3.0 \div 2 = 1.5$. The reaction at each of the points L_0, L_0', L_6, and L_6' is $10 \times 1.5 \div 4 = 3.75$. The truss being symmetrical, the stresses in like members on each side of the center will be the same. The shears in the top system are:

$$V_1 = +2 \times 3.75 - 2 \times 1.5 = +4.5$$
$$V_{a\text{-}a} = +2 \times 3.75 - 3 \times 1.5 = +3.0$$
$$V_2 = +2 \times 3.75 - 4 \times 1.5 = +1.5$$

and the secant ϕ_1 is $(17^2 + 20^2)^{\frac{1}{2}} \div 17 = 1.544$. The stresses in the diagonals are:

$$U_1'U_2 = +1.544 \times 4.5 = +6.95$$
$$U_2'U_3 = +1.544 \times 1.5 = +2.32.$$

The vertical $U_2'U_2 = -3.0$; and by passing a section $b - b$ around U_3', the stress in $U_3'U_3$ is found to be -1.5.

In obtaining the chord stresses in this system, the reactions of the top lateral truss are at U_1' and U_5' as the portal and end-posts are not in the same plane as the lateral system. The tangent method is the simplest to use in this case. The tangent is $20 \div 17 = 1.176$, and the stresses (see Fig. 95) are:

$$U_1'U_2' = -4.5 \times 1.176 = -5.29$$
$$U_2'U_3' = -(4.5 + 1.5) \times 1.176 = -7.06$$
$$U_1U_2 = 0$$
$$U_2U_3 = -U_1'U_2' = +5.29$$

Fig. 97 is a diagram with the stresses caused by wind right and

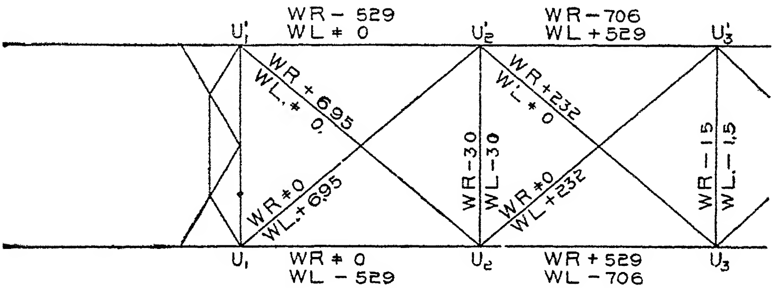


Fig. 97. Wind Stress Diagram of Pratt Truss of Fig 95

wind left indicated thereon. The stresses for wind left can easily be written by inspection.

51. **Bottom Lateral Bracing, Through-Bridges.** Fig. 95 shows the lower lateral system with the panel points loaded with the fixed or dead wind load. In this case it is all taken as acting on one side, it being assumed that the floor system protects the leeward truss. The problem then becomes that of determining the stresses in a deck Pratt truss of 6 panels of 20 feet each, the height being 17 feet. When wind is *right*, the members shown by broken lines in Fig. 95 do not act.

The fixed wind load (Article 26) is 150 pounds per linear foot of chord. The panel load will be the same as before, 3.0, but all will be on one chord. The shears are:

$$V_1 = 2\tfrac{1}{2} \times 3.0 = +7.5$$
$$V_{a\text{-}a} = +7.5$$
$$V_2 = +7.5 - 3.0 = +4.5$$
$$V_{b\text{-}b} = +4.5$$
$$V_3 = +7.5 - 2 \times 3.0 = +1.5$$

The secant being 1.544, as previously computed, the web stresses are:

$$L_0'L_1 = +7\ 5 \times 1\ 544 = +11\ 60 \qquad L_1'L_1 = -7\ 5$$
$$L_1'L_2 = +4\ 5 \times 1\ 544 = +\ 6\ 95 \qquad L_2'L_2 = -4.5$$
$$L_2'L_3 = +1\ 5 \times 1\ 544 = +\ 2\ 32 \qquad L_3'L_3 = -3\ 0$$

The stress in $L_3'L_3$ is determined by passing section $c - c$ and resolving the vertical forces at L_3' (see Fig. 95).

By using the tangent method, the chord stresses are computed as follows:

$$L_0'L_1' = -7\ 5 \times 1\ 176 = -8\ 82$$
$$L_1'L_2' = -(7\ 5 + 4\ 5) \times 1\ 176 = -14\ 12$$
$$L_2'L_3' = -(7\ 5 + 4\ 5 + 1\ 5) \times 1\ 176 = -15\ 88$$
$$L_1L_2 = -L_0'L_1' = -(-8\ 82) = +8\ 82$$
$$L_2L_3 = -L_1'L_2' = -(-14.12) = +14\ 12$$

The wind load acting on the train is 450 pounds per linear foot. It is evident that the train may cover the span either partially or entirely, and therefore its action on the lower lateral system is the same as if it were stressed by a live load of 450 pounds per linear foot of span.

The live panel load is $450 \times 20 = 9.0$. The maximum live-load reaction is $5 \times 9.0 \div 2 = 22.5$, and the positive live-load shears are:

$$V_1 = +22\ 5$$
$$V_2 = (1 + 2 + 3 + 4)\frac{9\ 0}{6} = +15.0$$
$$V_3 = (1 + 2 + 3)\frac{9\ 0}{6} = +9\ 0$$

It is unnecessary to go further than the center, as only the maximum stresses are required in the members. The web stresses are computed as given below:

$$L_0'L_1 = +22\ 5 \times 1\ 544 = +34.75 \qquad L_1'L_1 = -22\ 5$$
$$L_1'L_2 = +15.0 \times 1.544 = +23\ 15 \qquad L_2'L_2 = -15\ 0$$
$$L_2'L_3 = +\ 9\ 0 \times 1\ 544 = +13\ 91 \qquad L_3'L_3 = -\ 9\ 0$$

The maximum chord stresses due to this load of 450 pounds per linear foot of train, occur when the train covers the entire span; and they are directly proportional to the stresses produced by the fixed load, in the same ratio as the live panel load is to the fixed panel load. This ratio is $\frac{9.0}{3.0} = 3.0$. The chord stresses, therefore, are:

$L_0'L_1' = -\ 8.82 \times 3 = -26.46$
$L_1'L_2' = -14.12 \times 3 = -42.36$
$L_2'L_3' = -15.88 \times 3 = -47.64$
$L_1L_2 = +\ 8.82 \times 3 = +26.46$
$L_2L_3 = +14.12 \times 3 = +42.36$

Table X, Article 53, gives the stresses in the top and bottom lateral systems for wind right and wind left.

52. Overturning Effect of Wind on Truss. When the wind blows on the top chord, it tends to overturn the truss. As the truss is held down by its own weight, the action of the wind does not overturn it, but causes the dead-load reaction on the windward side to be less and that on the leeward side to increase by a like amount. The amount is $\pm V = \frac{\Sigma w}{2} \times \frac{h}{b}$, where Σw = the sum of all the wind panel loads, h = the height of the truss, and b = the distance center to center of trusses. The effect upon the leeward truss is the same as if two ver-

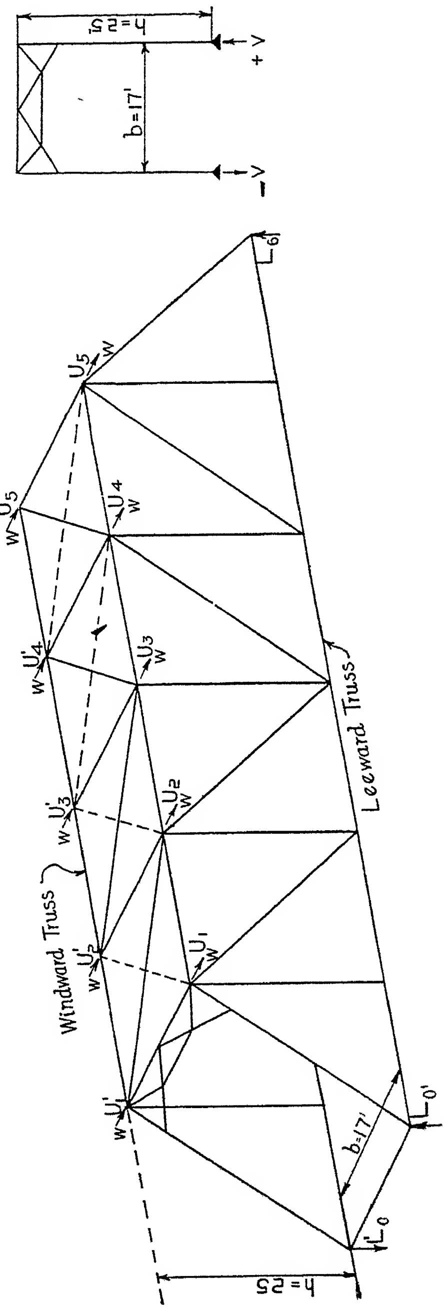


Fig. 98. Diagram Illustrating Overturning Effect of Wind on Truss.

tical loads, each equal to V and acting downward, were placed at the hips U_1 and U_5 (see Fig. 98). The effect on the windward truss is the same as if two vertical loads, each equal to V and acting upward, were placed at the hips U_1' and U_5'.

The stresses in the leeward truss will now be worked out. The stresses in the windward truss are the same, but with opposite signs. The truss is that of Article 47. Here $V = \frac{10 \times 1.5}{2} \times \frac{25}{17} = 11\ 000.$ Fig. 99 shows the truss with the loads in the correct position, the

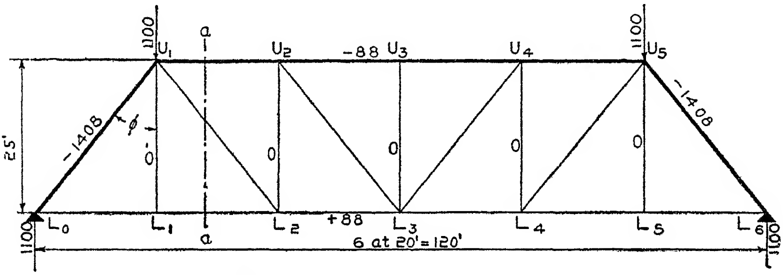


Fig 99. Truss under Wind Loads.

reactions each being 11.00. $V_1 = +11.00$, and $V_2 = +11.00 - 11.00 = 0$. The shears in the 2d, 3d, 4th, and 5th panels are also zero. As the shear in these panels is zero, the stress in the diagonals and vertical posts is zero $\times$ secant ϕ = zero. The stress in the hip verticals U_1L_1 and U_5L_5 is zero, as there are no loads at L_1 and L_5. The stress in the end-post is $-11.00 \times 1.28 = -14.08$. Taking the center of moments at U_1, the stress equation of $L_0L_1 = L_1L_2$ is: $-L_1L_2 \times 25 + 11.00 \times 20 = 0$; whence $L_1L_2 = +8.8$. The stress in all the lower chord members will be found to be $+ 8.8$. By summing the horizontal forces at the section $a - a$, noting that, as U_1L_2 is zero, its component is also zero, there results: $+L_1L_2 + U_1U_2 = 0$; whence $U_1U_2 = -L_1L_2 = -(+8.80) = -8.80$. This is also the stress in all members of the top chord.

It is now seen that the overturning effect of the wind on the truss causes stresses only in the end-posts and chords. The wind on the lower chord causes no overturning effect, as it is transferred directly to the abutments.

53. **Overturning Effect of Wind on Train.** The wind blowing upon the train tends to overturn it, and in so doing the pressure on

the leeward stringer is increased and that on the windward stringer decreased by the same amount. This difference in pressures is transferred to the floor-beam and then to the panel points (see Fig. 100), where its value is:

$$\pm L = \frac{W \times (8.5 + a)}{b},$$

where W = Panel load due to wind on train,
8.5 = A constant established by the Specifications (see Article 26, p. 15);
a = Distance from top of rail to center line of lower chord. It may be taken as 3 feet in most cases, as this is approximately the usual depth.
b = Distance center to center of trusses.

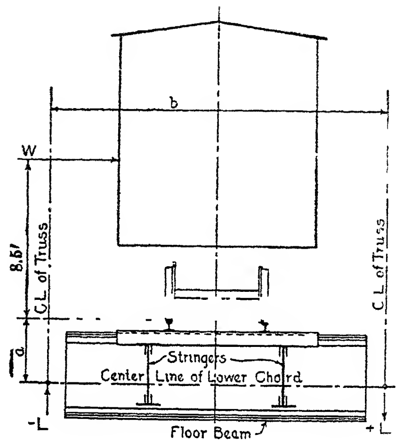


Fig. 100. Illustrating Overturning Effect of Wind on Train.

For the case in hand, $W = 20 \times 450 = 9\,000$. Therefore,

$$L = \pm \frac{9\,000 \times (8.5 + 3)}{17} = \pm 6\,090 \text{ pounds.}$$

The action of the wind in tending to overturn the train is the same as if the truss were under a live panel loading of L, the panel load L acting upward on the windward and downward on the leeward truss.

The chord stresses due to this will be proportional to the dead-load stresses in the same ratio as this panel load L is to the dead panel load. For the truss of Article 47, this ratio is $\frac{6\,090}{20\,000} = 0.303$, and the chord stresses caused by the overturning effect of the wind on the train (see Table VII) are:

$$U_1U_2 = -64.0 \times 0.303 = -19.39$$
$$U_2U_3 = -72.0 \times 0.303 = -21.82$$
$$L_0L_1 = L_1L_2 = +40.0 \times 0.303 = +12.12$$
$$L_2L_3 = +64.0 \times 0.303 = +19.39$$

The stress in U_1L_1 is $+6.09$, the panel load at L_1.

The maximum positive shears are:

$$V_1 = \frac{6.09}{6}(1 + 2 + 3 + 4 + 5) = +15.22$$

$$V_2 = \frac{6\ 09}{6}(1 + 2 + 3 + 4) = +10.15$$

$$V_3 = \frac{6\ 09}{6}(1 + 2 + 3) = +6\ 09$$

$$V_4 = \frac{6.09}{6}(1 + 2) = +3\ 05$$

and the maximum web stresses are found to be:

$$\begin{aligned} L_0U_1 &= -15\ 22 \times 1.28 = -19\ 50 \\ U_1L_2 &= +10\ 15 \times 1.28 = +13\ 00 \\ U_2L_3 &= +\ \ 6\ 09 \times 1.28 = +\ \ 7\ 80 \\ U_3L_4 &= +\ \ 3\ 05 \times 1.28 = +\ \ 3.91 \end{aligned}$$

$$\begin{aligned} U_2L_2 &= -6\ 09 \\ U_3L_3 &= -3\ 05 \end{aligned}$$

It is unnecessary to compute the shears further than one panel past the middle of the span, as only the maximum stresses are usually required.

The wind stresses from various causes are grouped together and given in Table X.

From Table X it is seen that large wind stresses occur in some of the members. Most specifications require that the stresses due to wind shall be neglected in the design unless they exceed 25 per cent of the sum of the dead-load and live-load stresses.

The subject of wind stresses does not ordinarily receive the consideration it should have; in fact, it appears to be common practice, in the case of spans up to 200 feet, to neglect the action of the wind in all members of the bridge except the top and bottom lateral diagonals, the top struts, the portal, and the bending effect in the end-post. For the last two effects mentioned, see the next succeeding article.

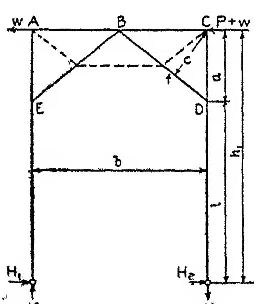


Fig. 101. Style of Portal Bracing in Common Use on Spans up to 250 Feet.

54. Portals and Sway Bracing. One-half of the wind on the top chord is transferred to the hips $U_1'U_1$ and $U_5'U_5$. From there it is carried to the abutments by means of the portal bracing and the end-posts. Various styles of portal bracing are in use, but few are so easily analyzed and constructed as that of Fig. 101. This form

TABLE X
Wind Stresses in Pratt Truss

WEB MEMBERS

Overturning	L_0U_1	U_1L_2	U_2L_3	U_3L_2	U_1L_1	U_2L_2	U_3L_3
Wind Right							
on Truss	−14 08	0	0	0	0	0	0
on Train	−19 50	+13 00	+7 80	+3 91	+6 09	−6 09	−3 05
Wind Left							
on Truss	+14 08	0	0	0	0	0	0
on Train	+19 50	−13 00	−7 80	−3 91	−6 09	+6 09	+3 05
Maximum +Stress	+33 58	+13 00	+7 80	+3 91	+6 09	+6 09	+3 05
Maximum −Stress	−33 58	−13 00	−7 80	−3 91	−6 09	−6 09	−3 05

CHORDS

Member		L_0L_1	L_1L_2	L_2L_3	U_1U_2	U_2U_3
Direct	Wind Right	0	+ 8 82	+14 12	0	+ 5 29
		0	+26 46	+42 36		
	Wind Left	− 8 82	−14 12	−15 88	− 5 29	− 7 06
		−26 46	−42 36	−47 64		
Overturning Truss	Wind Right	+ 8 80	+ 8 80	+ 8 80	− 8 80	− 8 80
	Wind Left	− 8 80	− 8 80	− 8 80	+ 8 80	+ 8 80
Overturning Train	Wind Right	+12 12	+12 12	+19 39	−19 39	−21.82
	Wind Left	−12 12	−12 12	−19 39	+19 39	+21 82
Maximum Tensile Stress		+20 92	+56 20	+84 67	+22.9	+23.56
Maximum Compressive Stress		−56 20	−77 40	−91 71	−28.19	−25 33

LATERAL SYSTEMS

Member	$U_1'U_2$	$U_2'U_3$	$U_2'U_2$	$U_3'U_3$	$L_0'L_1$	$L_1'L_2$	$L_2'L_3$
Wind Right							
on Truss	+6 95	+2 32	−3 0	−1 5	+11 60	+ 6 95	+ 2 32
on Train					+34 75	+23 15	+13 91
Wind Left							
on Truss	0	0	−3 0	−1 5	0	0	0
on Train					0	0	0
Maximum	+6 95	+2 32	−3.0	−1 5	+46 35	+30 10	+16.23

The stresses in $L_1'L_1$, $L_2'L_2$, and $L_3'L_3$ are not given in the above table These members are the floor-beams, and the small stress due to wind is neglected in their design

of portal is at present being used almost universally on all spans up to 250 feet.

Let it be required to analyze a portal of this form, all the distances being as indicated in Fig. 101; and let:

$$w = \text{Wind panel load of upper chord,}$$
$$m' = \text{Number of panels in upper chord;}$$

Then,

$$P = (m' - 1)\,w,$$

and,

$$V = \pm \left\{ (P + w) + w \right\} \frac{h_1}{b};$$

also,

$$H_1 = H_2 = \left\{ (P + w) + w \right\} \div 2$$

The stress in BC, the center of moments being at D, is:

$$S_{BC} = -\frac{(P + w)\,a + H_2 l}{a} = -\left\{ (P + w) + H_2 \frac{l}{a} \right\}.$$

The stress in AB, the center of moments being at E, is:

$$S_{AB} = +\frac{wa + H_1 l}{a} = +w + H_1 \frac{l}{a}$$

For the stress in BD, the center of moments is taken at C, and the perpendicular distance c to BD is determined. The stress in BD, then, is:

$$S_{BD} = +H_2 \frac{h_1}{c}.$$

The stress in BE is:

$$S_{BE} = -H_1 \frac{h_1}{c}.$$

It must be remembered that h_1 is not the height of the truss, but is the length of the end-post from L_0 to U_1.

For the truss of Article 47, $w = 1.5$; $m' = 4$; and $P = 4.5$. The value $h_1 = (20^2 + 25^2)^{\frac{1}{2}} = 32.0$ feet. The distance a must be so chosen that BD will not interfere with engines or other traffic which passes through the bridge. It will be assumed as 5 feet in this case. Then $V = (4.5 + 1.5 + 1.5)\,\frac{32}{17} = 14.08$; and $H_1 = H_2 = \frac{7.5}{2} = 3.75$; whence,

$$S_{BC} = -\left(6.0 + 3.75 \times \frac{27}{5}\right) = -26.25$$

$$S_{AB} = 1.5 + 3.75 \times \frac{27}{5} = +21.75$$

The distance $BD = \sqrt{\overline{BC}^2 + \overline{CD}^2} = \sqrt{\overline{8.5}^2 + \overline{5.0}^2} = 9.85$. Then, from similar triangles DCB and DFC, is obtained the proportion:

$$\frac{Cf}{CD} = \frac{BC}{BD},$$

$$Cf = \frac{5 \times 8.5}{9.85} = 4.3 \text{ feet, and}$$

$$S_{BD} = +3.75 \times \frac{32.0}{4.3} = +27.90$$

$$S_{BE} = -3.75 \times \frac{32.0}{4.3} = -27.90$$

When the wind blows from the other side, the stresses in the diagonals are reversed, and those in the top are transposed. The members shown by broken lines take no stress. When the wind blows, the end-posts tend to bend as shown in Fig. 102. This is with-

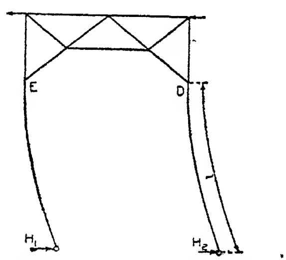


Fig. 102. Illustrating Tendency of End-Posts to Bend under Wind Load.

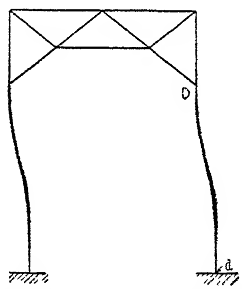


Fig. 103. Bending Tendency when End-Posts are Fixed at Lower End.

stood by the cross-section of the post at the points E and D. The bending moment caused at these points by the wind is $H_1 \times l$ and $H_2 \times l$. For the truss under consideration,

$$M_D = M_E = 3.75 \times 27 \times 12 = 1\,215\,000 \text{ lb.-ins.}$$

If the posts are fixed at the lower end, then they will tend to bend as shown in Fig. 103, the post resisting the bending at two points D and d. The section at each point withstands in this case only half of the moment just computed, or $1\,215\,000 \div 2 = 607\,500$ lb.-ins. A further discussion of this will be given in Part II, on "Bridge Design."

Various forms of sway bracing are used to connect the intermediate posts and thus stiffen the cross-section of the bridge at those points. The form of portal just given is often used, as is also the form shown in Fig. 104. Here h is the height of the truss. The braces BD are called *knee-braces*. Here w is the wind panel load of the top chord, and

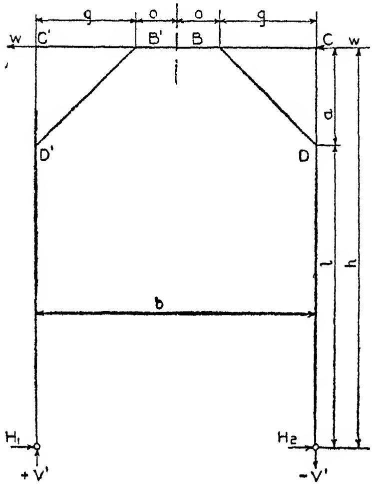


Fig 104 A Type of Bracing in Frequent Use.

$$V' = \pm\frac{2wh}{b}$$

$$H_1 = H_2 = \frac{2w}{2}$$

$$S_{BC} = -(wa + H_2 l) - a$$
$$= -(w + H_2\frac{l}{a})$$

$$S_{B'C'} = +(w + H_1\frac{l}{a})$$

$$S_{BD} = +H_2\frac{h}{c}$$

$$S_{B'D'} = -H_1\frac{h}{c}$$

The stress in $B'B$ is the direct compression due to wind right or left, and differs in accordance with the position of the top strut. There is also a bending moment at B' and B, which is:

$$M_{B'} = M_B = -V'g + H_2 h$$

The bending moment at D and D' is equal to $H_2 l$ or $H_2 l \div 2$, according to whether or not the lower ends of the posts are fixed.

The determination of the stresses for the truss of Article 47 is left to the student.

When the wind is from the other side of the truss, the signs of the stresses in the knee-braces and the members $C'B'$ and CB are reversed.

55. **Final Stresses.** The class of stresses which go to make up the maximum or minimum for which the member is designed, is determined by the specifications used. The dead-load and live-load stresses are always included, and then those due to impact and wind should be added if required. In computing the maximum stresses, the *algebraic* sum should always be used. In a large majority of cases, all stresses which go to make up the maximum have the

same sign, but some exceptions have been noted, as in the middle vertical of a Pratt or Howe truss The minimum stresses are, with rare exceptions, obtained by combining stresses with signs of opposite character.

GIRDER SPANS

56. **Moments and Shears in Floor-Beams.** In any bridge the floor-beam acts as a support for either the joists or stringers, and the moments and shears occurring in it are due to the loads which come on the joists or stringers In a highway bridge the joists are spaced so closely that the load which they transmit to the floor-beams may be considered as uniformly distributed, providing the live load is a uniform load, in which case,

$$M = \frac{(2P_L + P_D) \times \text{length of floor beam in inches}}{8}$$

$$V = (2P_L + P_D) \div 2,$$

where M = Maximum moment in pound-inches;
V = Maximum shear,
P_L = Live panel load,
P_D = Weight of stringers and floor material in one panel

It will be seen that these formulæ are those for the maximum moment and shear in a uniformly loaded beam, the total load being $2P_L + P_D$.

As an example, let it be required to determine the maximum moment and shear in the floor-beam of a highway bridge whose panels are 20 feet long, and trusses 16 feet center to center, the live load being 100 pounds per square foot of floor surface, the flooring weighing 10 pounds per square foot, and there being 5 lines of joists weighing 15 pounds per linear foot, and 2 lines of joists weighing 8 pounds per linear foot.

$$P_L = \frac{16}{2} \times 20 \times 100 = 16\,000 \text{ pounds.}$$

$$P_D = 5 \times 20 \times 15 + 2 \times 20 \times 8 + 16 \times 20 \times 10 = 5\,020 \text{ pounds.}$$

Therefore,

$$M = \frac{(2 \times 16\,000 + 5\,020)\,16 \times 12}{8}$$

$$= 888\,480 \text{ pound-inches at center of floor-beam.}$$

$$V = (2 \times 16\,000 + 5\,020) \div 2$$

$$= 18\,510 \text{ pounds at ends of floor-beam.}$$

In the case of a single-track railroad bridge, there are only two stringers upon which the weight of the track, the engine, and the train is supported. These join the floor-beam at points equally distant from the center of same. The weight of the ties, rails, and fastenings is usually taken at 400 pounds per linear foot of *one track*. As regards the live load, the proposition reduces itself to placing the wheel loads so that the sum of the reactions of stringers in the adjacent panels will be a maximum on the floor-beam under consideration. This is discussed in Article 47, page 87 (see Fig. 94).

In determining the values of the maximum moment and shear in the floor-beam, the case is that of a beam symmetrically loaded with two equal concentrated loads. Each load is equal to the dead weight of one stringer, one-half the track weight in one panel, and the maximum sum total of the reactions due to the wheel loads on the stringers in adjacent panels which meet at that point. This latter quantity is called the *floor-beam reaction*. For a general arrangement of the loads, see Fig. 105. The distance a has become standard for single-track spans, and is 6 feet 6 inches.

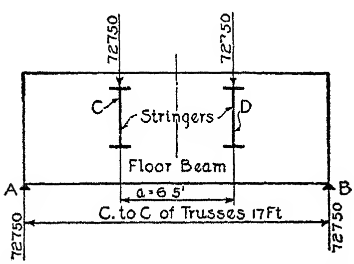


Fig 105. Arrangement of Loads for Calculating Moments and Shears in Floor-Beams

Let it be required to determine the maximum shears and moments in the floor-beam of the truss of Article 47.

The weight of the stringer may be obtained by the formula of Table II, and is:

$$\text{Stringer} = 20\,(123.5 + 10 \times 20) - 2 = 3\,200 \text{ pounds.}$$

The weight of one-half of the ties, rails, etc., in one panel is:

$$\tfrac{1}{2}\text{ Track} = (400 \times 20) - 2 = 4\,000 \text{ pounds}$$

The weight that comes from the engine wheels is given in Article 47, page 87 (see Fig. 94), and is 65.55. Each load is therefore the sum of all the above weights, as follows:

$$3\,200 + 4\,000 + 65\,550 = 72\,750.$$

The maximum shear (see Fig. 105) is seen to be 72 750 pounds; and the maximum moment occurs at C and D, and is:

$$M = 72\,750 \times \left(\frac{17}{2} - \frac{6.5}{2}\right) \times 12 = 4\,583\,250 \text{ pound-inches}$$

For any particular engine the floor-beam reactions for different length panels are easily tabulated for future reference. Table XI gives the floor-beam reactions for panel lengths from 10 to 24 feet inclusive.

TABLE XI
Floor-Beam Reactions
E 40 Loading

Panel Length	Maximum Floor-Bm Reaction	Panel Length	Maximum Floor-Bm Reaction	Panel Length	Maximum Floor-Bm Reaction
10	41 000	15	55 000	20	66 55
11	43 800	16	57 000	21	67 95
12	44 400	17	59 000	22	70 14
13	46 600	18	61 000	23	72 13
14	52 200	19	63 000	24	73 96

In many cases it is desirable to keep the dead-load shears and moments separate from those of the live load; and this can easily be done.

In neither of the above cases has the weight of the beam itself been taken into account. This should be done in the final design. The method of procedure is to compute the moment and shears as above; then make a provisional design of the beam. Next, compute the weight of the beam thus designed, and add the moments and shears caused by this weight to the other dead-load moments and shears; then re-design the beam and compute its weight. If this last weight varies 10 per cent from the previous weight, another re-design should be made. The above proceeding belongs to Bridge Design, Part II, and will there be treated.

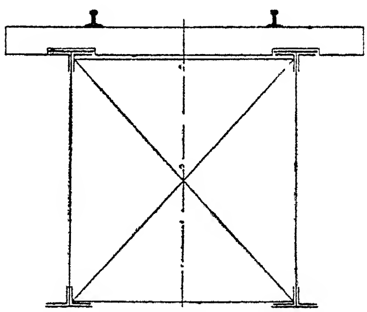


Fig. 106 Cross-Section of Deck Plate-Girder Railway Bridge

57. **Moments in Plate-Girders.** Plate girders are of two classes—namely (1) those which have the ties or floor laid directly upon the upper flanges of the girders; these are called *deck plate-girder bridges*,

and (2) those in which the webs of the girders are connected with each other at intervals by floor-beams which in turn carry stringers or joists in exactly the same manner as in the floor system of a railroad or highway truss-bridge; this latter type is called a *through plate-girder bridge.* Figs. 106 and 107 show cross-sections of deck and through plate-girder bridges respectively, for railway service. Fig. 108 is a side view of a deck plate-girder bridge. Fig. 109 is a longitudinal section of a through plate-girder railroad bridge. The section is taken down the middle of the track. The bridge shown has 5 panels. An odd number of panels should be chosen, as this does not bring a floor-beam at the center of the span, and hence the great moment which would then be caused is avoided.

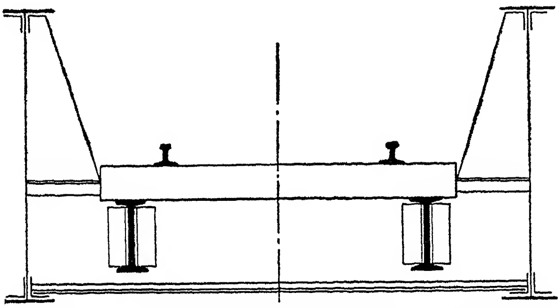


Fig. 107. Cross-Section of Through Plate-Girder Railway Bridge.

The analysis of the shears and moments of a through plate-girder is precisely the same as that for a truss bridge. The shear is constant between any two panel points as 0-1 or 1-2, etc., and the moments are computed for the points 1, 2, 3, and 4.

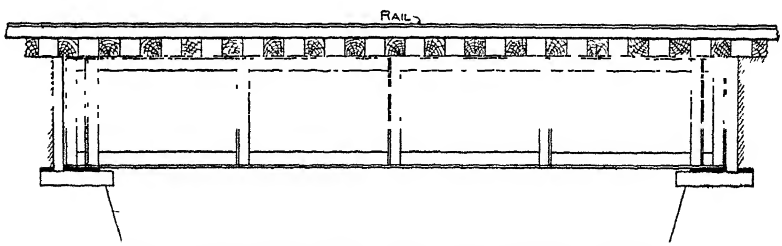


Fig 108. Side View of Deck Plate-Girder Railway Bridge.

If wheel loads are used for moments, the relation that $K = \frac{Wn}{m} - L$ must be $+$, and that $k = \frac{Wn}{m} - (L + P)$ must be $-$, holds true when the loads are in correct position for maximum moments. Here $m =$ the number of panels, and $n =$ the panel under considera-

tion and is to be reckoned from the left end; in fact, all terms have the same value as mentioned in Article 46 A careful review of Articles 44 and 46 should enable the student to follow the example which will now be given.

EXAMPLE. It is required to determine the moments at the points of floor-beam support for a 5-panel through plate-girder of 75-foot span. The live loading is Cooper's E 40.

Dead-Load Moments. Through plate-girders, on account of the heavy floor system and the fact that the floor system transfers its own weight and that of the live load to the girders as concentrated loads, are about 40 per cent heavier than deck plate-girder bridges of the same span. The weight of the entire span, therefore, is;

$$1\ 4 \times 75\ (123\ 5 + 10 \times 75) = 91\ 700 \text{ pounds.}$$

Part of this 91 700 pounds (the weight of the girders themselves) acts as a uniform load; the remainder (the weight of the floor-beams and stringers) acts as concentrated loads at the points where the floor-beams join the web. Experience has shown that the weight of the floor for a single-track railroad system is about 400 pounds

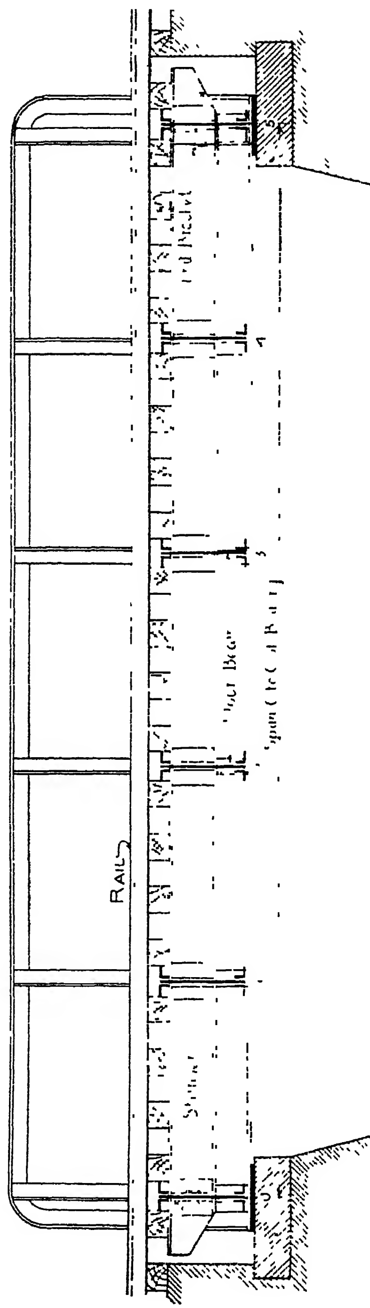


Fig. 109. Longitudinal Section of a Through Plate-Girder Railway Bridge

TABLE XII

Wheel Position, Moments in a Through Plate-Girder

Panel Point	Wheel at Point	L	$\frac{Wn}{m}$	P	$L+P$	K	k	Remarks
1	2	10	$\frac{172}{5} = 34\ 4$	20	30	+	+	
1	3	30	$\frac{192}{5} = 38\ 4$	20	50	+	−	Maximum
1	4	40	$\frac{202}{5} = 40\ 4$	20	60	+	−	Maximum
1	5	40	$\frac{202}{5} = 40\ 4$	20	60	+	−	Maximum
1	6	40	$\frac{195}{5} = 39\ 0$	13	53	−	−	
2	4	50	$\frac{152 \times 2}{5} = 60\ 8$	20	70	+	−	Maximum
2	5	70	$\frac{172 \times 2}{5} = 68\ 8$	20	90	−	−	
3	6	90	$\frac{152 \times 3}{5} = 91\ 2$	13	103	+	−	Maximum
3	7	103	$\frac{172 \times 3}{5} = 103\ 2$	13	116	+	−	Maximum
3	8	116	$\frac{192 \times 3}{5} = 115\ 2$	13	129	−	−	
4	6	90	$\frac{129 \times 4}{5} = 103\ 2$	13	103	+	+	
4	7	103	$\frac{142 \times 4}{5} = 113\ 6$	13	116	+	−	Maximum
4	8	116	$\frac{152 \times 4}{5} = 121\ 6$	13	129	+	−	Maximum
4	9	129	$\frac{152 \times 4}{5} = 121\ 6$	13	142	−	−	

per linear foot. The weight of the stringers and floor-beams for this bridge is therefore $75 \times 400 = 30\,000$ pounds, and $91\ 700 - 30\,000 = 61\ 700$ pounds acts as a uniform load. This 61 700 pounds is distributed over two girders, and so gives $61\ 700 \div (2 \times 75) =$ say, 412 pounds per linear foot of one girder.

The dead load which is concentrated at each panel point is that due to the weight of the steel floor and the weight of ties, rails, and fastenings. It is, for one girder,

$$15 \times (400 + 400) \div 2 = 6\,000 \text{ pounds.}$$

The dead-load moments are now computed by the methods of Strength of Materials, and are found to be:

$$M_0 = M_5 = 0,$$
$$M_1 = M_4 = +4\,390\,000 \text{ pound-inches},$$
$$M_2 = M_3 = +6\,580\,000 \text{ pound-inches}.$$

Live-Load Moments. The positions of the wheels for maximum moments are now determined (see Table XII).

The computations for the reactions are best arranged in tabular form. Table XIII gives the values.

TABLE XIII

Reactions for a Through Plate-Girder

Point	Wheel at Point	Equation for Reaction	Reaction
1	3	$R = (6\,708 + 192 \times 4) - 75$	99 7
1	4	$R = (7\,668 + 212 \times 4 - 10 \times 78) - 75$	103 2
1	5	$R = (8\,728 + 232 \times 4 - 10 \times 83 - 20 \times 75) - 75$	97 8
2	4	$R = (4\,632 + 152 \times 7) \div 75$	76 0
3	6	$R = (4\,632 + 152 \times 6) - 75$	73 9
3	7	$R = (5\,848 + 172 \times 3) - 75$	84 9
4	6	$R = (2\,851 + 129 \times 4) - 75$	44 9
4	7	$R = (3\,496 + 142 \times 4) \div 75$	54 2
4	8	$R = (4\,632 + 152 \times 2) - 75$	65 4

The live-load moments are computed as follows:

Point 1:
- Wheel 3, $M = 99\ 7 \times 15 - 230 = 1\,265\,000$ pound-feet.
- Wheel 4, $M = 103\ 2 \times 15 - 20 \times 5 - 20 \times 10 = 1\,247\,000$ pound-feet
- Wheel 5, $M = 97\ 8 \times 15 - 20 \times 5 - 20 \times 10 = 1\,167\,000$ pound-feet.

Point 2 Wheel 4, $M = 76\ 0 \times 30 - 480 = 1\,800\,000$ pound-feet.

Point 3:
- Wheel 6, $M = 73\ 9 \times 45 - 1\,640 = 1\,785\,000$ pound-feet.
- Wheel 7, $M = 84.9 \times 45 - 2\,155 = 1\,665\,000$ pound-feet.

Point 4:
- Wheel 6, $M = 44\ 9 \times 60 - 1\,640 = 1\,054\,000$ pound-feet.
- Wheel 7, $M = 54\ 2 \times 60 - 2\,155 = 1\,097\,000$ pound-feet.
- Wheel 8, $M = 65\ 4 \times 60 - 2\,851 = 1\,073\,000$ pound-feet.

The above values show that the greatest live-load moments are:

M at Point 1, by wheel 3 = 1 265 000 pound-feet
M at " 2, " " 4 = 1 800 000 " "
M at " 3, " " 4 = 1 800 000 " "
M at " 4, " " 3 = 1 265 000 " "

The last two values are obtained when the load comes on the bridge from the left. Inspection of the results obtained at points 3 and 4 when the load comes on from the right, shows that they are considerably smaller than the results obtained at their symmetrical points 1 and 2, and therefore it was not necessary to determine the moments for any points to the right of the center. This is practically true of all girder spans, deck or through.

The method of procedure when the girder is a deck plate-girder is the same as that just illustrated, except that in the computation of the dead-load moments there is no concentration of certain portions of the dead load, the weight of the girders themselves being a uniform load, as is also the weight of the ties and rails or, if it be a highway bridge, the floor-joists which run transversely. Highway spans are seldom built of deck plate-girders, it being preferable to use the through girders, as then the girders themselves serve as a railing and keep the traffic confined to the roadway. The girder span is usually divided into ten equal divisions, the points of division being called the *tenth-points*. The shears and moments are computed for the center point and those points which lie to the left of the center. After the values are computed, they are laid off as ordinates, with the corresponding tenth-points as abscissæ. A curve is then drawn through their upper ends, and the *curve of maximum shears or moments* is the result. To get the maximum shear or moment at any point other than a tenth-point, the ordinate is scaled at the desired point.

EXAMPLE. Let it be required to determine the maximum moments at the tenth-points of a 100-foot-span deck plate-girder.

Dead-Load Moments. The weight of steel in the span is 100 $(123.5 + 10 \times 100) = 112\,350$ pounds, and the weight of the track is $400 \times 100 = 40\,000$ pounds, making a total of 152 350 pounds, or $152\,350 \div (2 \times 100) =$ say, 762 pounds per linear foot per girder. The dead-load moments are now determined according to the methods of Strength of Materials, and are:

$$M_1 = 342\ 900 \text{ pound-feet}$$
$$M_2 = 609\ 600 \text{ pound-feet}$$
$$M_3 = 800\ 100 \text{ pound-feet}$$
$$M_4 = 914\ 400 \text{ pound-feet}$$
$$M_5 = 952\ 500 \text{ pound-feet}$$

Live-Load Moments. The determination of the wheel load

positions is made by the use of the formulæ $K = (\frac{Wn}{m} - L)$ and $k = \frac{Wn}{m} - (L + P)$; only, in this case, n is the number of divisions from the left support to the section, and m is the number of divisions into which the girder is divided.

The determination of the wheel positions is given in Table XIV.

TABLE XIV

Wheel Positions, Moments in Deck Plate-Girder

Point	Wheel at Point	L	$\frac{Wn}{m}$	P	$L+P$	K	k	Remarks
1	2	10	258 × 0.1 = 25.8	20	30	+	−	Maximum
1	3	20	261 × 0.1 = 26.1	20	40	+	−	"
1	4	20	254 × 0.1 = 25.4	20	40	+	−	"
1	5	20	242 × 0.1 = 24.2	20	40	+	−	"
1	6	20	240 × 0.1 = 24.0	13	33	+	−	"
1	7	13	230 × 0.1 = 23.0	13	26	+	−	"
2	2	10	232 × 0.2 = 46.4	20	30	+	+	
2	3	30	245 × 0.2 = 49.0	20	50	+	−	Maximum
2	4	50	258 × 0.2 = 51.6	20	70	+	−	"
2	5	60	261 × 0.2 = 52.2	20	80	−	−	
3	3	30	232 × 0.3 = 69.6	20	50	+	+	
3	4	50	232 × 0.3 = 69.6	20	70	+	−	Maximum
3	5	70	245 × 0.3 = 73.5	20	90	+	−	"
3	6	80	261 × 0.3 = 78.3	13	93	−	−	
4	4	50	212 × 0.4 = 84.8	20	70	+	+	
4	5	70	232 × 0.4 = 92.8	20	90	+	+	
4	6	90	245 × 0.4 = 98.0	13	103	+	−	Maximum
4	7	103	258 × 0.4 = 103.2	13	116	+	−	
4	8	106	261 × 0.4 = 104.4	13	119	−	−	
5	6	90	232 × 0.5 = 116.0	13	103	+	+	
5	7	103	232 × 0.5 = 116.0	13	116	+	0	Maximum
5	8	116	245 × 0.5 = 122.5	13	129	+	−	"
5	9	129	258 × 0.5 = 129.0	13	142	0	−	"
5	10	132	274 × 0.5 = 137.0	10	142	+	−	"
5	11	102	244 × 0.5 = 122.0	20	122	+	0	"
5	12	102	234 × 0.5 = 117.0	20	122	+	−	"
5	13	102	224 × 0.5 = 112.0	20	122	+	−	"
5	14	122	234 × 0.5 = 117.0	20	142	−	−	

While many wheels on point 1 satisfy the condition, the greatest moment will occur when one of the large drivers is at the point, and it is therefore unnecessary to examine the point for other wheels. The same is true at the center point, 5, the maximum occurring under one of the heavy driver wheels. The reactions and the computations for the same are given in Table XV.

TABLE XV

Reactions for a Deck Plate-Girder

Point	Wheel at Point	Equation for Reaction	Reaction
1	2	$R = (12\,041 + 5 \times 258) \div 100$	133 31
1	3	$R = (12\,599 + 4 \times 261) - 100$	136 43
1	4	$R = (11\,984 + 4 \times 254) \div 100$	130 00
1	5	$R = (11\,334 + 4 \times 234 + 2 \times 4^2 - 2) \div 100$	122.86
2	3	$R = 12\,041 - 100$	120 41
2	4	$R = (12\,041 + 5 \times 258) \div 100$	133 31
3	4	$R = 10\,816 \div 100$	108 16
3	5	$R = 12\,041 - 100$	120 41
4	5	$R = (8\,728 + 4 \times 232) \div 100$	96 56
4	6	$R = (10\,816 + 4 \times 245) \div 100$	117 96
4	7	$R = (12\,041 + 4 \times 258) \div 100$	130 73
5	7	$R = (8\,728 + 8 \times 232) - 100$	105 84
5	8	$R = 12\,041 - 100$	120 41
5	9	$R = (12\,041 + 5 \times 258) - 100$	133 31
5	10	$R = (13\,904 + 2 \times 274) - 100$	144 52
5	11	$R = (11\,334 + 5 \times 234 + 2 \times 5^2 - 2) - 100$	125 29
5	12	$R = (9\,514 + 10 \times 214 + 2 \times 10^2 \div 2) - 100$	117 54
5	13	$R = (7\,794 + 15 \times 194 + 2 \times 15^2 - 2) \div 100$	109 29

TABLE XVI

Maximum Moments in a Deck Plate-Girder

Point	Wheel at Point	Equation for Moment	Moment in Pound-Feet
1	2	$M = 133\ 31 \times 10 - 80$	1 253 100
1	3	$M = 136\ 43 \times 10 - 5 \times 20$	1 264 000
1	4	$M = 130\ 00 \times 10 - 5 \times 20$	1 200 000
1	5	$M = 122\ 86 \times 10 - 5 \times 20$	1 128 000
2	3	$M = 120\ 41 \times 20 - 230$	2 178 200
2	4	$M = 133\ 31 \times 20 - 480$	2 186 200
3	4	$M = 108.16 \times 30 - 480$	2 764 800
3	5	$M = 120\ 41 \times 30 - 830$	2 782 300
4	5	$M = 96\ 56 \times 40 - 830$	3 032 400
4	6	$M = 117.96 \times 40 - 1\,640$	3 078 400
4	7	$M = 130.73 \times 40 - 2\,155$	3 074 200
5	7	$M = 105\ 84 \times 50 - 2\,155$	3 137 000
5	8	$M = 120\ 41 \times 50 - 2\,851$	3 169 500
5	9	$M = 133\ 31 \times 50 - 3\,496$	3 169 500
5	10	$M = 144\ 52 \times 50 - 4\,072$	3 154 000
5	11	$M = 125\ 29 \times 50 - 3\,068$	3 196 500
5	12	$M = 117\ 54 \times 50 - 2\,658$	3 219 000
5	13	$M = 109.29 \times 50 - 2\,248$	3 216 500

Table XVI gives the computations of the live-load moments at the tenth-points, the final results being in pound-feet.

Whenever any loads were off the left end of the bridge, the lines 7 to 16 of the engine diagram were used (Fig. 85). For example, with wheel 10 at 5, wheel 1 would be off the left end. By looking in the second space of line 8, there is found the quantity 13 904, which is the moment of wheels 2 to 18 inclusive about a point directly under wheel 18. Just to the right of the vertical line through wheel 18, is the value 284, which is the weight of wheels 1 to 18 inclusive; but this must be decreased by 10, the weight of wheel 1, as that wheel is off the span. As wheel 18 is 2 feet from the right end of the girder, the moment about the point is $13\,904 + 274 \times 2$. By looking in the second space of line 16, the value 4 072 is found. This is the value of the moment of loads 2 to 9 inclusive about a point directly under wheel 10, and must be subtracted from the moment of the reaction in order to get the moment at 5 for this loading. See Articles 21 and 47 for further information regarding the use of the values in lines 7 to 16 of the engine diagram.

By the help of differential calculus it can be proved that *the greatest possible moment does not occur at the middle of a beam loaded either with concentrated loads or with concentrated loads followed by a uniform load, but it occurs under the load nearest the middle of the beam when the loads are so placed that the middle of the beam is half way between the center of gravity of all the loads and the nearest load.*

The wheel which produces this greatest moment is the same one which produces the maximum moment at the middle of the beam. The exact solution of this problem involves the use of quadratic equations, but for all practical purposes the following rule will suffice:

> Place the loading so that the wheel which produces the maximum moment at the middle of the beam is at that point Find the distance of the center of gravity of all the loads from the right end Move the loads so that the middle of the beam is half way between the center of gravity as found above and the load which produced the maximum moment at the middle of the beam Find the moment under that load, with the loads in the position just mentioned.

For the case in hand, wheel 12 at 5 gives the maximum moment. The moment at the right end of the span, wheel 12 being at 5, is:

$$9\,514 + 10 \times 214 + 2 \times \overline{10}^2 \div 2 = 11\,754\,000 \text{ pound-feet.}$$

The center of gravity is $\frac{11\ 754\ 000}{234} = 50.2$ feet from the right support, or 0.2 foot to the left of the center of the girder. Now place wheel 12 one-tenth of a foot to the right of the center, and determine the moment under it. The reaction will be:

$$R = (9\ 514 + 9.9 \times 214 + 2 \times \overline{9.9}^2 \div 2) \div 100 = 117\ 306,$$

and,

$$M = 117.306 \times 50.1 - 2\ 658 = 3\ 219\ 030 \text{ pound-feet.}$$

In this particular case the difference between the greatest moment possible and the greatest moment at the middle is not sufficient to warrant the extra labor involved in computing it. In general it may be said that *if the greatest moment possible occurs within six inches of the middle of the beam, it is not necessary to compute it, the moment at point* 5 *being taken.*

58. **Shears in Plate-Girders.** In the case of through plate-girders, the maximum live-load shears are determined by placing the wheels in such a position that $Q = \frac{W}{m} - G$ is $+$, and $q = \frac{W}{m} - (G + P)$ is $-$.

In these equations, m is the number of panels into which the span is divided; and the other quantities are the same as given in Article 45, which should now be reviewed.

For example, let it be required to determine the dead and live load shears in the through plate-girder of Article 57, p. 111.

The weight of	one girder	$= 61\ 700 \div 2$	$=$	30 850 lbs.
" "	" $\frac{1}{2}$ the floor	$= 5 \times 6\ 000$	$=$	30 000 lbs.
	Total weight on one girder		$=$	60 850 lbs.

The dead-load shears are then computed by the methods given in Strength of Materials, and are given as follows, it being remembered that the concentrated load which comes at the end is one-half a panel load, or 3 000 pounds:

$$V_0 = 60\ 850 \div 2 = 30\ 425 \text{ lbs} = \text{end shear;}$$

$$V_1 = 30\ 425 - 3\ 000 - \frac{30\ 850}{5} = 21\ 255 \text{ lbs ;}$$

$$V_2 = 30\ 425 - 3\ 000 - 6\ 000 - 2 \times \frac{30\ 850}{5} = 9\ 085 \text{ lbs.}$$

$$V_3 = 30\ 425 - 3\ 000 - 2 \times 6\ 000 - 2 \times \frac{30\ 850}{5} = 3\ 085 \text{ lbs.}$$

$$V_c = 0,$$

where V_0 = the shear at the end; V_1 = the shear just to the left of point 1; V_2 = the shear just to the left of point 2; V_3 = the shear just to the right of point 2; and V_c = the shear at the middle of the girder.

The determination of the wheel load position for maximum live-load shears is given in Table XVII. By comparing the formulæ Q and K, it will be seen that for the first panel $Q = K$, and $q = k$, as $n = 1$. The position of wheel loads for maximum moments at point 1 is the same as for maximum shear in the first panel. According to Table XII, wheels 3, 4, and 5 at point 1 all give maximum shears in the first panel. In this case, as in previous ones, only the shear for the first position of the loading found for any particular point will be determined, as the difference between this and the other cases is too small to warrant the additional labor necessary in computing them. It is evidently unnecessary to go past panel 3, as only the maximum shears are required.

TABLE XVII

Wheel Positions, Shears in a Through Plate-Girder

($m = 5$)

Point	Wheel at Point	G	$\frac{W}{m}$	P	$P+G$	Q	q	Remarks
1								See Table XII, and text above this table
2	2	10	142 − 5 = 28 4	20	30	+	−	Maximum
2	3	30	152 − 5 = 30 4	20	50	+	−	Maximum
2	4	40	152 − 5 = 30 4	20	60	−	−	
3	2	10	116 − 5 = 23 2	20	30	+	−	Maximum
3	3	30	116 ÷ 5 = 23 2	20	50	−	−	

For wheel 3 at point 1, the left reaction (see Table XIII) is 99 7. That portion of wheels 1 and 2 which is transferred to point 0 is $230 \div 15 = 15\ 33$; and the shear in the first panel, therefore, is:

$$V_1 = 99\ 70 - 15\ 33 = +84.37.$$

For wheel 2 at point 2, the left reaction is:

$$R = (3\ 496 + 142 \times 5) \div 75 = 56.10;$$

therefore,

$$V_2 = 56\ 10 - \frac{80}{15} = +\ 50\ 67.$$

A computation with wheel 3 at point 2 will give a shear only 560 pounds greater, which difference would not influence the design to any appreciable extent.

For wheel 2 at point 3, the left reaction is:

$$R = (2\,155 + 116 \times 1) - 75 = 30.30;$$

therefore,

$$V_3 = 30\ 3 - \frac{80}{15} = +24\ 97.$$

When the girder is a deck one, the computation of the dead shears is very much simplified, as all of the load is uniform.

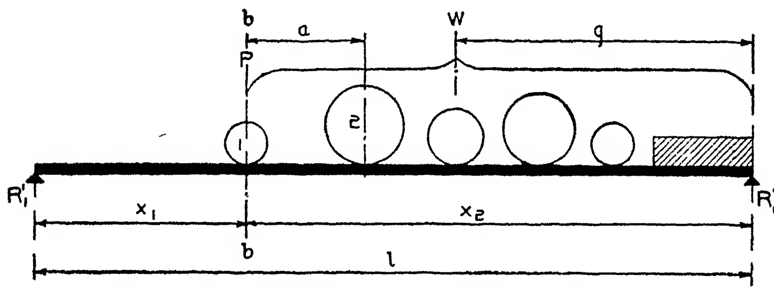


Fig 110. Beam under Wheel Loads Followed by Uniform Load.

Let it be required to determine the dead and live load shears at the tenth-points of the deck plate-girder of Article 57.

The total weight of one girder and track is 76 175 lbs.

$$V_0 = 76\ 175 \div 2 = +38\ 088 \text{ pounds}$$

$$V_1 = 38\ 088 - \frac{76\ 175}{10} = +30\ 470 \text{ pounds.}$$

$$V_2 = 38\ 088 - \frac{2}{10} \times 76\ 175 = +22\ 850 \text{ pounds.}$$

$$V_3 = 38\ 088 - \frac{3}{10} \times 76\ 175 = +15\ 230 \text{ pounds.}$$

$$V_4 = 38\ 088 - \frac{4}{10} \times 76\ 175 = +\ \ 7\ 618 \text{ pounds.}$$

$$V_5 = 0.$$

The position of the wheel loads to produce the maximum shear cannot be determined by the same relation as that used in structures which have a system of floor-beams and stringers, for here not a portion, but all of the load to the left of the section, must be subtracted from the left reaction in order to give the shear.

The correct relation for the wheel load position will now be deduced.

Let Fig. 110 represent a beam of span l loaded with a series of wheel loads followed by a uniform load. Let P equal the weight of the first wheel, W equal the weight of all the loads, and g the distance from the center of gravity of all of the loads to the right abutment. The distance between the first and second wheel centers is a, and the first wheel is at the section $b-b$ at a distance x_1 from the left support. Then,

$$R_1' = \frac{Wg}{l},$$

and

$$V'_{b-b} = R_1' - \text{(loads to left of section)} = \frac{Wg}{l}$$

Now, assume that the loads move forward the distance a. The wheel 2 will be at section $b-b$, and Fig. 111 will represent the position of the loads. Then,

$$R_1'' = \frac{W(g+a)}{l};$$

and

$$\begin{aligned} V''_{b-b} &= R_1'' - P \\ &= \frac{W(g+a)}{l} - P \\ &= \frac{Wg}{l} + \frac{Wa}{l} - P. \end{aligned}$$

It is now evident that in order to get the greatest shear at section $b-b$, wheel 2 must be placed at the section whenever V''_{b-b} is greater than V'_{b-b}. Then,

$$V''_{b-b} > V'_{b-b};$$

$$\frac{Wg}{l} + \frac{Wa}{l} - P > \frac{Wg}{l}.$$

Now, canceling out the term $\frac{Wg}{l}$, which appears on both sides of the equation, there results:

$$\frac{Wa}{l} > P.$$

For the engine under consideration, $a = 8$ feet, and $P = 10\,000$ pounds, and the equation reduces to:

$$W > \frac{10.0}{8}l;$$

$$W > 1\frac{1}{4}l,$$

which is to say that when the load on the girder is greater than $1\frac{1}{4}$ times the span, then wheel 2 should be placed at the section in order to give the maximum shear.

For loading E 40, the following is true:

For all sections up to and including the center of all spans, place wheel 2 at the section to give the maximum shear.

In Fig. 111 it is immaterial whether or not any additional loads come on the span at the right end when the loads move forward the

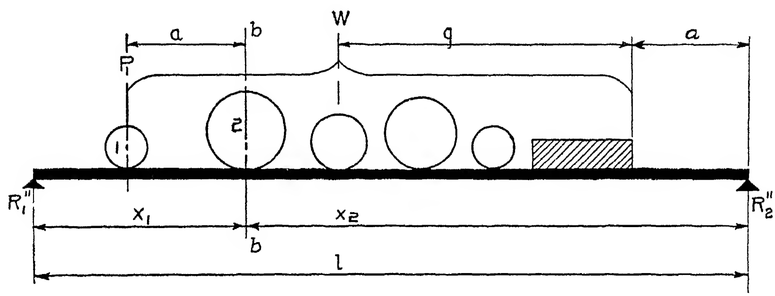


Fig 111 Beam of Fig 110 with Loads Moved Forward.

distance a, as they would only tend to increase the left reaction and therefore the shear V''_{b-b}. If the relation deduced is true for the case when no extra loads come on at the right end, it will be true when they do.

The live-load shears at the left end and at the tenth-points, wheel 2 being at the section in all cases, are computed from the general formula, which is:

$$V = R - \Sigma P,$$

in which,

R = Left reaction;

ΣP = All loads to left of section, and is equal to 10 000 pounds for all sections except the end of the girder.

The computations and results can be conveniently placed in tabular form, and are given in Table XVIII.

In order to illustrate the use of the relation $W > 1\frac{1}{4}l$, let point 3 in the above span be taken. Place wheel 2 at point 3; then, as wheels 1 to 13 are on the girder, the total weight W is 212. As l = 100, $1\frac{1}{4}l$ = 125. Therefore, as 212 is greater than 125, wheel 2 is the correct wheel.

TABLE XVIII
Maximum Shears in a Deck Plate-Girder

Point	Reaction Equation	R	ΣP	V	Remarks
0	$(13\,904+4\times274)-100$	150 00	0	150 00	Wheel 18, 4 ft from rt. end
1	$(12\,041+5\times258)-100$	133 31	10	123 31	Wheel 16, 5 ft. from rt end
2	$10\,816-100$	108 16	10	98 16	Wheel 15 at right end
3	$(\ 7\,668+4\times212)-100$	85.16	10	75 16	Wheel 13, 4 ft. from rt. end
4	$(\ 5\,848+4\times172)-100$	65 36	10	55 36	Wheel 11, 4 ft. from rt. end
5	$(\ 4\,632+2\times152)-100$	49 36	10	39 36	Wheel 10, 2 ft from rt. end

The curves of maximum live-load moments and shears are shown in Fig. 112. They should always be drawn. From them the shear or moment at any desired section can be determined. For example, let it be desired to determine the maximum live-load shear and moment at a point 24 feet from the left end of the girder. By drawing the ordinate, shown by a broken line in Fig. 112, and scaling, the following values are found:

$V_{24} = 88\,000$ pounds;
$M_{24} = 2\,440\,000$ pound-feet.

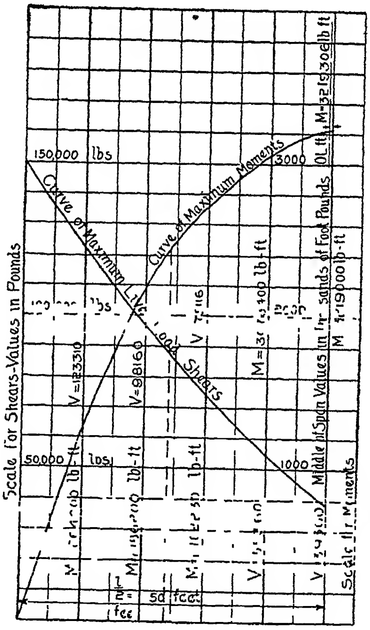


Fig. 112. Curves of Maximum Live-Load Moments and Shears

A similar set of curves for the dead-load shears and moments should be made. The set for the deck plate-girder in hand is shown in Fig. 113. These are easily constructed by laying off the maximum values of the shear at the end, and the maximum value of the moment at the center. The shear curve is a straight line from the end to the center, while the moment curve is a parabola from the center to the end.

The stresses in the lateral systems of plate-girders are computed in a manner the same as that employed for the lateral systems of trusses, the unit-load being taken according to the specifications used.

59. Stresses in Plate-Girders. The stresses in plate-girders are treated in the Instruction Paper on Steel Construction, Part II, pages 135 to 158, and the student is referred to this treatise for information regarding this subject.

The stress in the flange is seen to depend upon the distance from center of gravity to center of gravity. This distance, in turn, depends

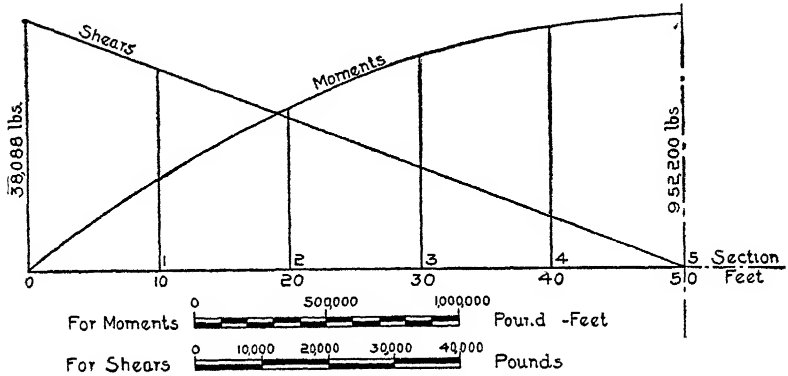


Fig 113. Curves of Dead-Load Shears and Moments in a 100-Foot Span Deck Plate-Girder.

upon the depth of the girder. Certain approximate rules have been proposed in order to determine this, but the following formula will give the width of the web plate in accordance with best modern practice:

$$d = \frac{l}{0.005\,l + 0.543},$$

in which

d = Width of the web plate, in the even inch;
l = Span, in feet.

For example, let it be required to determine the width of the web plate of a plate-girder of 80-foot span center to center of end bearings.

$$d = \frac{80}{0.005 \times 80 + 0.543} = \frac{80}{0.94} = 85.2 \text{ (say 86) inches.}$$

If the resultant value had been 85 inches, the width would have been taken as either 84 or 86. The reason for this is that the wide plates kept in stock at the mills are usually the even inch in width and can therefore be procured more quickly than if odd-inch widths were ordered, in which case the purchaser would be forced to wait until

they were rolled—often a period of several months The distance back to back of flange angles, the so-called *depth* of girder, is one-half inch more than the width of the web. This is due to the fact that each pair of flange angles extend one-fourth inch beyond the edge of the web plate, so as to keep any small irregularities caused on the edge of the web plate by the rolling, from extending beyond the backs of the angles.

EXERCISES AND PROBLEMS

1. Determine the maximum positive shears in the first six panels of a 9-panel 114-foot Pratt truss, the live panel load being 8.0. Use the exact and also the conventional method.

ANSWER:

	x	Exact Shears	Conventional Shears
V_1	16	+32 00	+32 00
V_2	14	+24 54	+24 90
V_3	12	+18 05	+18 70
V_4	10	+12 48	+13 35
V_5	8	+ 7 85	+ 8 90
V_6	6	+ 4 50	+ 5 34

2. Find the maximum and minimum stresses in L_1U_2 and U_3L_3 of an 8-panel 160-foot through Warren truss Height 20 ft., dead panel load 10 00, all on lower chord, live panel load 12 00

ANSWER: In L_1U_2: d. l., -28.00; l. l., -35.30 and $+1.68$; max., -63.30; min., -26.32. In U_3L_3: d. l., $+16.80$; l.l., $+25.20$ and -5.04; max., $+42.00$; min., $+11.76$.

3. In the truss of Problem 2, determine the maximum stress in L_2L_3 by the method of moments, and also by the tangent method.

ANSWER: d. l. = $+67.50$; l. l. = $+81\ 00$; max. = $+148.50$.

4 Determine the dead-load stresses in the members U_2L_2 and L_4U_5 of a 9-panel 180-foot through Warren truss. Height is 24 feet, dead panel load is 10 0, one-third being at each panel point of the upper chord, and two-thirds being at each panel point of the lower chord.

ANSWER: $U_2L_2 = +30.60$; $L_4U_5 = -1.80$.

5. Determine the stress in the counter of a through Howe truss of 8 panels and 160-foot span. Height is 30 ft.; dead panel load, 9.6; live panel load, 11.5.

ANSWER: -4.59.

6. In the truss of Problem 5, determine the maximum and minimum stress in U_2L_2, L_2U_3, and L_3U_4.

ANSWER:

	U_2L_2	L_2U_3	L_3U_4
d. l l. l. l. l.	+20 80 +30 30 −1.44	−17 30 −25 90 + 5 18	− 5 76 −17.30 0 00
Max. Min	+51.10 +19 36	−43 20 −21 20	−23 06 ± 0 00

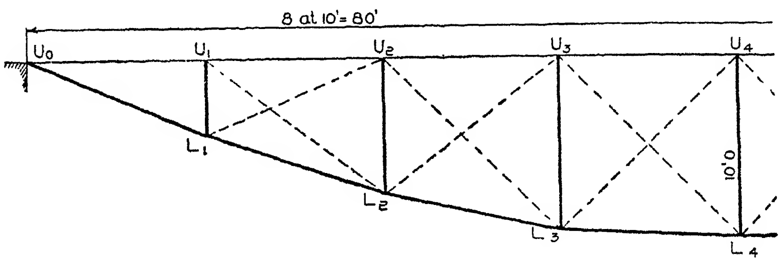


Fig 114 Deck Parabolic Bowstring Truss

7. In the deck parabolic bowstring truss of Fig 114, determine the maximum stress in L_1L_2, L_1U_2, and U_3L_3. The dead panel load is 4.0, all on upper chord, and the live panel load, 20 0.

ANSWER: $L_1L_2 = +201\ 9$; $L_1U_2 = +21.8$; $U_3L_3 = -33.6$.

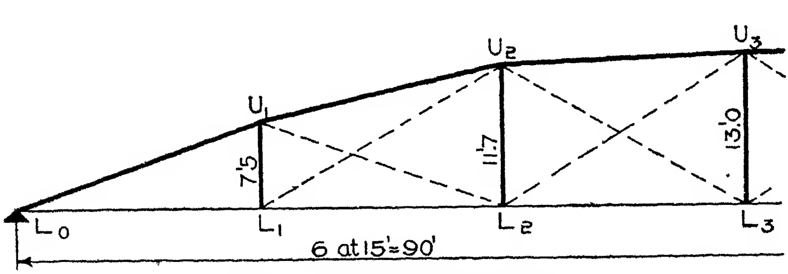


Fig. 115. Through Bowstring Truss.

8. In the through bowstring truss of Fig 115, determine the maximum stress in U_1L_2 and L_1U_2, the dead panel load being 5.0, and the live panel load 15.0

ANSWER: $U_1L_2 = +33.50$; $L_1U_2 = +38.0$.

9 Determine the maximum and minimum stresses in the members U_1L_1, U_1L_2, U_2L_2, and U_2L_3 of a 7-panel 175-foot through Pratt truss 30 feet high Dead panel load is 10.0, all on lower chord, live panel load is 15 0

ANSWER:

	U_1L_1	U_1L_2	U_2L_2	U_2L_3
d l	+10 0	+26 00	−10 00	0 00
l l	+15 0	+41 70	−21 40	−12 85
l l	0 0	− 2 78	+ 6 42	0 00
Max	+25 0	+67 70	−31 40	−12 85
Min.	+10 0	+23 22	− 3 58	0 00

10. Determine the maximum and minimum stresses in the members U_1m_2, m_3L_3, U_2L_2, and m_2U_2 of the deck Baltimore truss shown in Fig 116. Dead panel load, 30 000 lbs.; live panel load, 50 000 lbs. One-third of dead panel load is applied at the lower ends of all the verticals.

ANSWER:

	U_1m_2	m_3L_3	U_2L_2	m_2U_2
d l	+190 8	+ 84 8	−110 0	+21 2
l l	+333 5	+191 5	−211 0	+35 4
l l.	− 15 1	− 50 5	+ 10 7	0 0
Max.	+524 3	+276 3	−321 0	+56 6
Min	+175 7	+ 34 3	− 99 3	+21 2

11 In the truss of Problem 10, determine the maximum stress in M_2U_2 and L_3L_4.

ANSWER: $M_2U_2 = -840\ 0$; $L_3L_4 = +960\ 0$.

12 Determine the position of the wheel loads of Cooper's E 40 loading to produce the maximum positive live-load shears in the panels of a 7-panel 175-foot Pratt truss.

ANSWER: L_1, wheel 4; L_2, wheels 3 and 4; L_3, wheel 3; L_4, wheel 3; L_5, wheel 2; L_6, wheel 2.

13 Determine the maximum positive live-load shears for the truss of Problem 12.

ANSWER: $V_1 = 192\ 8$; $V_2 = 137.8$; $V_3 = 90.8$; $V_4 = 52.6$; $V_5 = 25.0$; $V_6 = 6.8$.

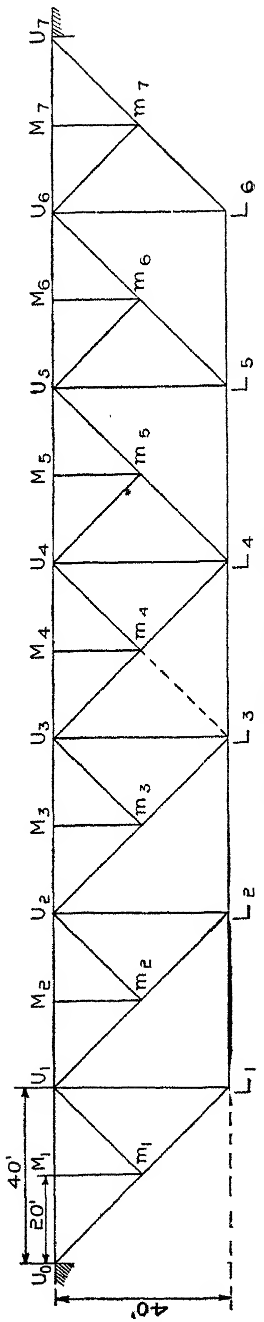


Fig. 116 Deck Baltimore Truss.

14 Determine the position of the wheel loads of Cooper's E 40 loading to produce maximum moments at the panel points of the truss of Problem 12

ANSWER: L_1, wheel 4; L_2, wheel 7; L_3, wheels 11 and 12; L_4, wheels 13 and 14.

15 Determine the maximum moments at the panel points of the truss of Problem 12 Loading, Cooper's E 40.

ANSWER: $M_1 = 4\,820\,000$; $M_2 = 7\,745\,000$; $M_3 = 9\,192\,000$; $M_4 = 9\,082\,000$, all in pound-feet.

16 Compute the maximum live-load web stresses in the truss of Problem 12, the height being 32 feet. Loading, E. 40.

17. Compute the maximum live-load chord stresses in the truss of Problem 12, the height being 32 feet. Loading, E 40.

18 Compute the impact stresses for all members of the truss of Problem 12.

19. Determine the maximum live-load shears at the tenth-points of a 65-foot span deck plate-girder. Loading, E 40.

ANSWER: $V_0 = 103.0$; $V_1 = 86$; $V_2 = 69\ 7$; $V_3 = 54.5$; $V_4 = 40.8$; $V_5 = 28.4$.

20. Compute the shear due to impact in the girder of Problem 19.

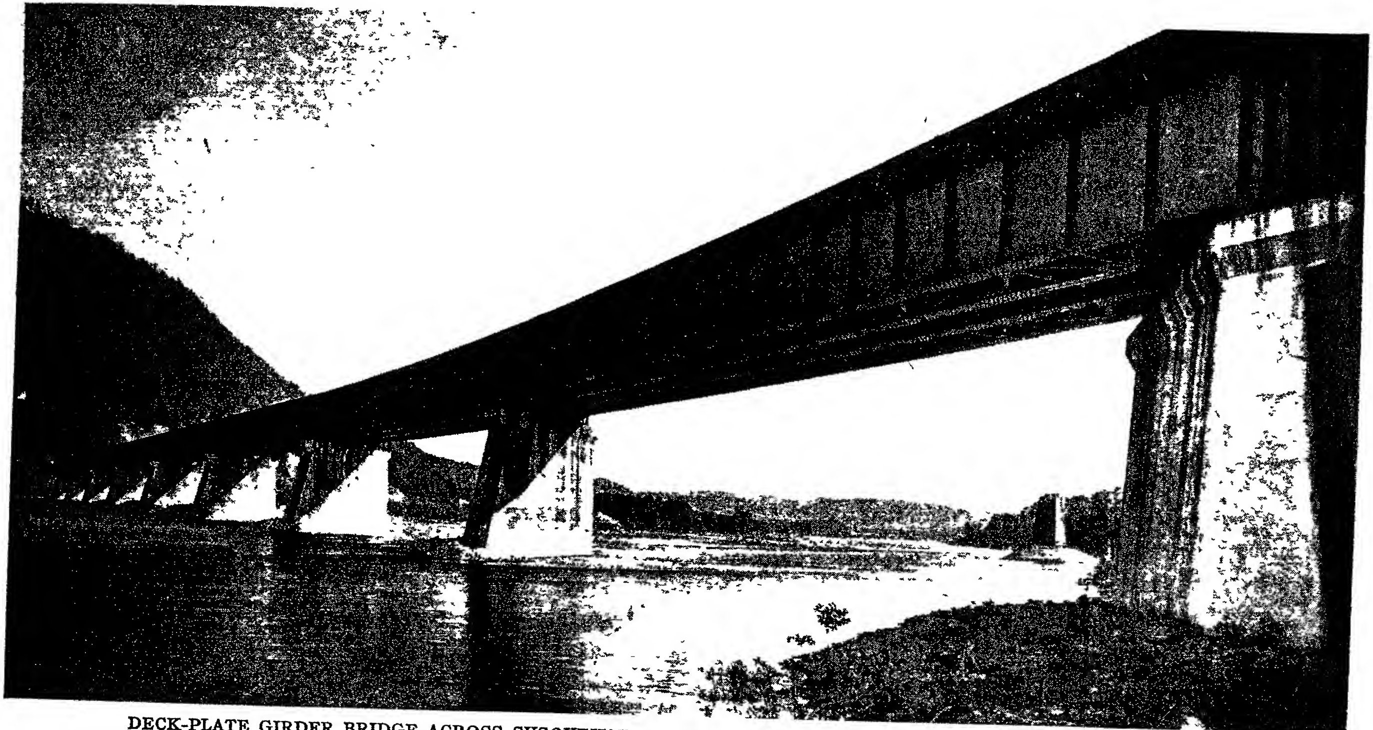
ANSWER: $V_0 = 84.7$; $V_1 = 71.5$; $V_2 = 58.8$; $V_3 = 47.0$; $V_4 = 35.6$; $V_5 = 25.4$.

21. Compute the maximum live-load moments at the tenth-points of the girder of Problem 19. Loading, Cooper's E 40.

ANSWER:

POINT	WHEEL	MOMENT	IMPACT MOMENT
1	2	6 540	5 520
2	2	11 320	9 520
3	3	14 860	12 500
4	4	16 850	14 050
5	4	16 860	14 530
1 45′ from center	4	16 920	14 650

All moments are in thousands of pound inches.



DECK-PLATE GIRDER BRIDGE ACROSS SUSQUEHANNA RIVER AT TOWANDA, PENNSYLVANIA, ON THE LEHIGH VALLEY RAILROAD

This bridge consists of thirteen 130-foot double-track spans and one 120-foot double-track span

Courtesy of Phoenix Bridge Company, Phoenixville, Pennsylvania

BRIDGE ENGINEERING

PART II

BRIDGE DESIGN

60. **General Economic Considerations.** The prime consideration which influences the decision to build is *cost*. After the decision to build has been made, the problem is one of a purely engineering character, whereas in the first case it was one of either a political or an engineering character, or both. The engineering problem is an economic one, in which maximum benefits must be obtained at a minimum cost.

A map of the proposed bridge site and the approaches, as well as of the country for a considerable distance up and down stream, should be made. This map should show the contours, the soundings, the borings, the high and low water-mark elevations, and the exceptional flood line. On this map the bridge should be plotted in its proposed location and also in various others. In the case of each of these locations, various schemes taking into account different numbers of piers and spans should be considered.

Several authors have attempted to present formulæ having a more or less theoretical derivation and purporting to indicate the correct number of piers and spans for a minimum cost. The use of these formulæ should not be encouraged, since they do not in any case give results close enough to serve for anything but a rough guide.

The cost of abutments will vary somewhat with the location and the character of the approach. This variation is usually small, and ordinarily an approximate location of the abutments can be quickly made. As the number of abutments is in all cases constant, their effect upon the problem of the location of the bridge is small, the main proposition being that of the cost and the number of piers and spans.

The cost of the piers will usually not be constant, those closer to the middle of the stream costing more on account of the depth of the water and the more difficult character of the foundation. Piers

should not be placed on a skew; neither should they be placed directly in the maximum line of action of the current. If a skew is unavoidable, it should be as small as possible. The cost of piers should be ascertained by the most careful estimates. In the case of small bridges where there are only one or two piers, the matter is very simple, but with a considerable number of piers the problem becomes very complicated and requires weeks and sometimes months or years for its solution.

The determination of the cost of the superstructure is a comparatively simple matter. In certain instances the class of bridge is limited to some extent by the specifications. Cooper, in Article 2 of his "Specifications for Steel Railroad Bridges and Viaducts" (edition of 1906), gives the following:

Types of Bridges for Various Spans

Spans	Kind of Bridge
Up to 20 feet	Rolled beams
20 to 75 "	Riveted plate-girders
75 to 120 "	Riveted plate- or lattice-girders
120 to 150 "	Lattice or pin-connected trusses
Over 150 "	Pin-connected trusses

One railroad expresses a preference for plate-girders for all spans from 20 to 115 feet; and for spans from there to 150 feet, riveted trusses.

The question as to whether the bridge will be deck or through is one which is decided by the controlling influences of *water-way, false work, time of erection,* and *extra cost of masonry.* If the clear height required for the water-way is sufficiently small, the deck bridge should be chosen, as in this class the cost of false work is less, the time of erection is less, and the cost of masonry is less by an amount equal to the cross-section of the piers times the depth of the truss. Deck bridges also cost less than through bridges of equal span.

The conditions permitting, girders should be used in preference to trusses. While for equal spans girders are heavier and therefore cost more, the steel work alone being considered, little or no false work is required, and the time of erection is much less than in the

case of trusses. This makes the total cost of girder bridges less than those in which trusses are used. Another item in favor of girders is their great stiffness.

While pin-connected bridges cost less and are easier to erect, their stiffness is not so great as that of riveted bridges, which cost more. The time required for the erection of riveted bridges is also greater than that for pin-connected bridges. This is on account of the great amount of time required to make the riveted connections. For long spans, say over 200 feet, it is necessary to use pin-connected bridges, as the extreme size of the connection plates prohibits the use of the riveted type. Also, it is unnecessary to use riveted long-span trusses to obtain stiffness, as the weight of the pin-connected bridges is so great when compared with the live load that sufficient stiffness is obtained.

The cost of spans of different lengths and character may be obtained directly from the bridge companies; or their weights may be computed from the formulæ given in Article 20, p. 9 (Part I, "Bridge Analysis"), and multiplied by the unit price which your experience indicates is correct, thus giving the total cost.

Evidently the solution of problems of this nature cannot be made within the limits of this text, but the following example will tend to indicate somewhat the manner of procedure in a problem of this kind. For example, if the length between abutments is 1 400 ft., the cost of each abutment is $12 000, and the cost of each pier is $15 000, then, if we have fourteen 100-foot plate-girder spans, each costing $4 300, and thirteen piers, the total cost will be $279 200. On the other hand, if nine piers and ten 140-foot truss spans, each costing $9 200, are used, the cost will be $251 000, showing a balance of $28 200 in favor of the truss scheme. The live loading is E 50.

61. **Economic Proportions.** The depth of girders is given in Article 59, Part I.

In the case of trusses, the effect of an increase in the height is to increase the stresses in the web members and to decrease the stresses in the chord members. This variation does not affect the weights to any considerable extent; in fact, a variation of 20 per cent in the height will not affect the weight more than 2 or 3 per cent.

The height of the bridge is usually fixed by some considerations which are in turn determined by the specifications. The height must

be sufficient to clear whatever traffic will pass through. It should also be sufficient to prevent overturning on account of the wind pressure on the truss or on the traffic. In addition, the height of the bridge is influenced by the depth of the portal bracing. A deep portal bracing is desirable, in that it stiffens the trusses under the action of the wind and the vibration due to the passing traffic; but a deep portal bracing increases the height of the truss and therefore the bending in the end-posts due to the wind. Judgment on the part of the engineer should be used in order to determine the limiting height for securing a maximum amount of benefit as regards stiffness and a minimum amount of bad effect due to the bending in the end-posts. Fig. 117, which gives the height for any given length of span, may be said to represent the best modern practice (1908). Variations of a foot or more from those given do not affect the weight to any appreciable extent.

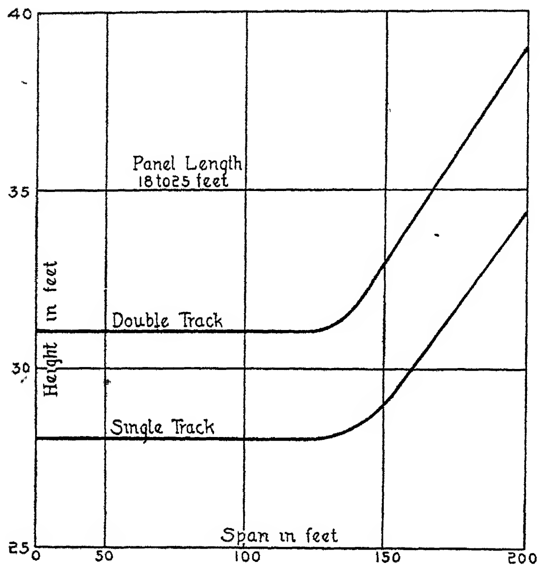


Fig. 117. Curves Showing Relation between Height of Trusses and Length of Span in Double- and Single-Track Railway Bridges.

The distance from center to center of trusses for highway bridges depends upon the width of the street or, if in the country, the width of the roadway. Streets, of course, vary in width in different localities, but country highway bridges usually have a roadway of from 14 to 16 feet in the clear.

In the case of railroad bridges, the distance from center to center of trusses depends upon whether the track is straight or on a curve, and also upon whether the bridge is a deck or a through bridge.

The actual amount varies in most cases, and is fixed by specification. Some specifications require that when the track is straight, the distance from center to center of trusses shall be 17 feet; or that, in case one-twentieth of the span exceeds the 17 feet, then one-twentieth of the span shall be used.

For deck plate-girders the common practice appears to be to space them as given below:

Width of Plate-Girder Bridges for Various Spans

Spans	Distance Center to Center of Plate-Girders
Up to 65 feet	6 feet 6 inches
65 to 80 "	7 feet 0 inches
80 to 115 "	7 feet 6 inches

For through plate-girders the spacing should be such that no part of the clearance diagram will touch any part of the girder. In case of double-track plate-girders with one center girder, great care should be exercised in order that the center girder shall not be so deep nor have so wide a flange as to interfere with the clearance diagram (see Fig. 126).

On account of the wind on a train which runs on track placed at the elevation of the top chord of deck bridges, the overturning effect is exceedingly great, and special care should be taken that the height and width are such as to prevent overturning.

In through bridges the clearance must be such as to allow the clearance diagram to pass. Special attention should be paid to the knee-braces and also to the portal braces. When the bridge is on a tangent, the spacing of the trusses is a comparatively simple matter, being just sufficient for the clearance diagram; but on curves, allowance must be made for the tilt of the diagram due to the superelevation of the outer rail, and also allowance must be made for the fact that the length of the cars between trucks forms a chord to the curve, and as such the middle ordinance must be taken into account. It is also necessary to allow for that part of the car which projects over the trucks, as this will extend beyond the outer rail by an amount greater than one-half the width of the clearance diagram. (See Figs. 119 and 120.)

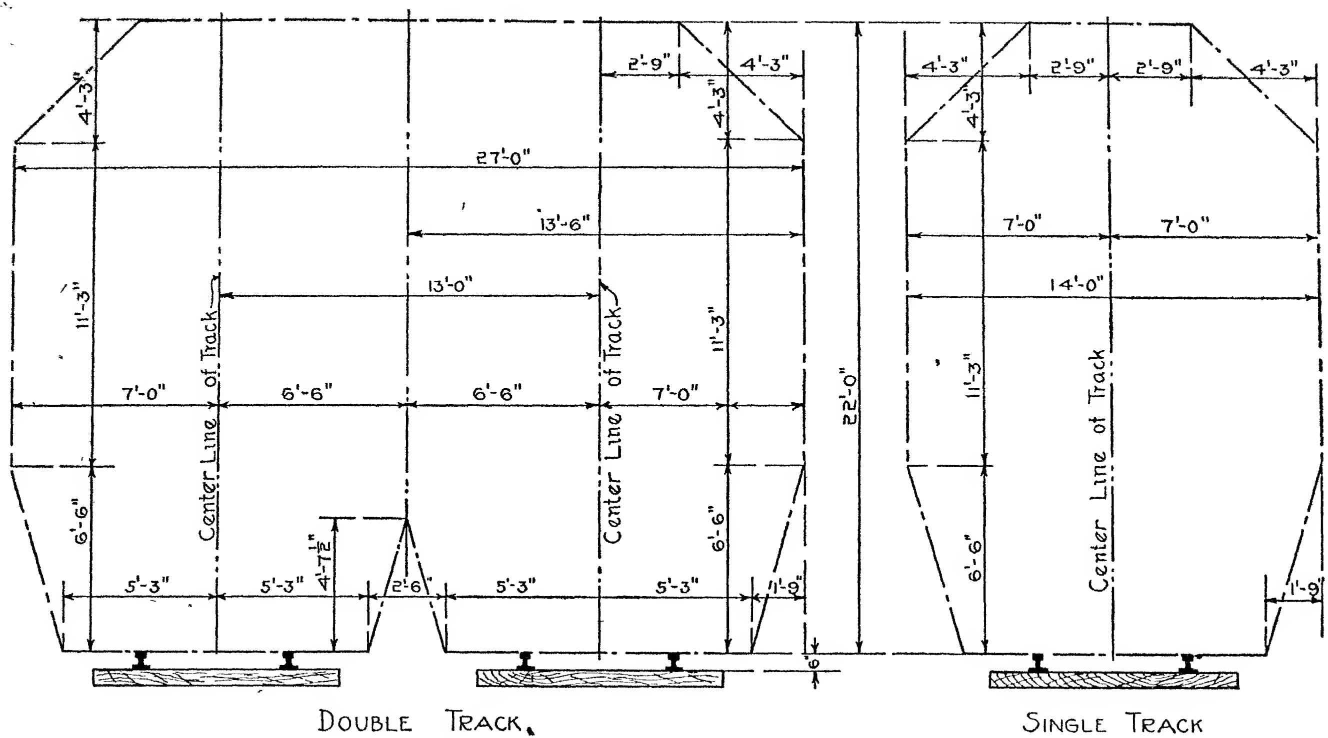


Fig 118 Standard Clearance Diagram (Straight Track) Used by Lehigh Valley Railroad Courtesy of F E Schall, Bridge Engineer

Dimensions given show standard clearance for straight tracks. When tracks are on a curve, equivalent clearance is to be provided, assuming length of cars 75 feet over all and 54 feet center to center of trucks Also, on account of tipping of cars, for structures extending over 12 feet 6 inches above top of rail, an additional clearance of 2½ inches is to be provided on the inside of curves for each inch of elevation of outer rail For structures of lesser height, proportionate allowance is to be made.

62. The Clearance Diagram. The clearance diagram is not supposed to represent the outline of the largest engine or car which may run over the line, but represents the maximum amount of space which may be taken up by objects which are to be shipped over the line. For instance, the lower part of the clearance diagram may allow for snow-plow or ballast distributors, and the upper part may take into account the passage of such material as carloads of lumber, piles, or telegraph poles. The standard clearance diagram of the Lehigh Valley Railroad is given in Fig. 118. This diagram is for the clearance on straight track only. On curves, the diagram tilts as shown in Fig. 119, and to allow for this tilting the Lehigh Valley Railroad requires 2½ inches additional clearance on the inside of curves for each inch of elevation of the outer rail. In addition to this tilting effect, the clearance should also be increased on account of the length of the cars and their projection over the outer and inner rails. Fig. 120 shows a standard car according to the specifications of the American Railway Engineering & Maintenance of Way Association, in such a position on a single-track span as to show the effect of the curve upon the widening of the spacing, center to center of trusses.

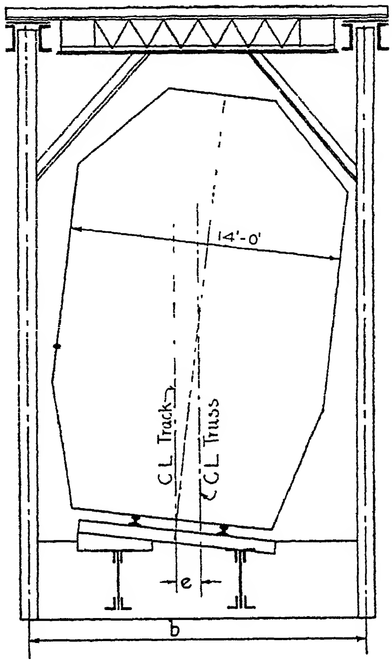


Fig. 119. Clearance Diagram on Curves, Showing Tilting

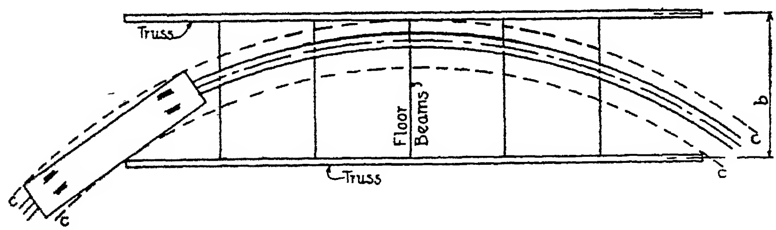


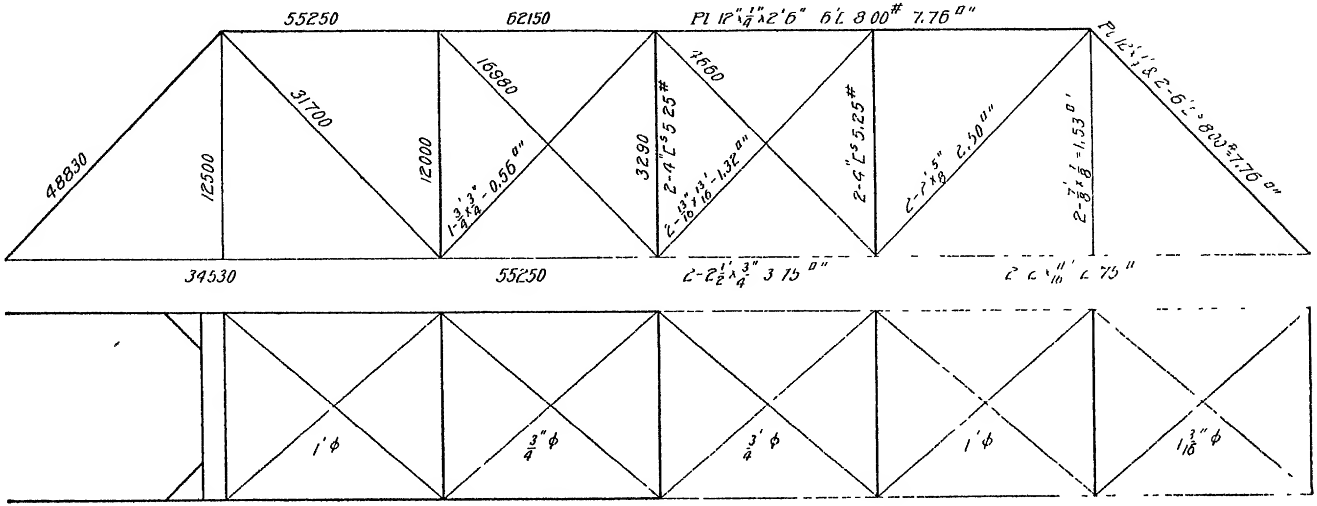
Fig. 120. Standard Car on Curve Showing Necessity for Wider Spacing of Trusses

This car is 80 feet long, 60 feet between centers of trucks, and is as wide as the clearance diagram, 14 feet for single track. It is evident that the trusses cannot be spaced so as to interfere with the clearance line of the body of the car and its projecting ends. These clearance lines are represented as broken lines in Fig. 120, and are marked *c–c*. Note that the center of the track is seldom in the center of the floor-beam. Also, it is evident that the sharper the curve, the greater the required distance between trusses, and accordingly the greater the floor-beams in length. This varies the moment in the different floor-beams and therefore makes them more costly. The stringers, also, are more costly, on account of the fact that their ends are skewed. On account of the eccentricity of the track, one truss takes more of the load than the other, and therefore the trusses are not the same—a fact which further increases the cost.

From the above it is seen that almost all conditions incident to the building of a bridge on a curve tend to increase the cost; and hence a fundamental principle of bridge engineering: *Avoid building bridges on curves.*

63. **Weights and Loadings.** For the weight of steel in any particular span, and for the loading required for any particular class of bridge, see Articles 20 to 23, Part I. The weight of the ties and the rails and their fastenings is usually set by the specifications at 400 pounds per linear foot of track. For highway bridges the weight of the wooden floor is usually taken at $4\frac{1}{2}$ pounds per square foot of roadway for every inch in thickness of floor.

Highway bridges are divided into different classes according to their loadings (see Cooper's Specifications). The decision as to the class to be employed depends somewhat upon the distance to the nearest bridge across the same stream. In case the nearest bridge is only a few miles away and is of heavy construction, it is not actually necessary to construct a heavy bridge at the proposed site, the heavier traffic being required to pass over the other bridge. In case a heavy bridge is not in the neighborhood, then one should be constructed at the proposed site. If the proposed site is on a road connecting adjacent towns of large size, then a heavy bridge should be constructed and provision made for future interurban traffic, even if none is at that time in view, since it will be more economical to do this than to erect a new bridge in the future.



Length of Span C. to C - 102'-0"
Clear Width of Roadway - 12'-0"
6 Panels - 17'-0" Each
Height of Truss C to C of Pins - 17'-0"
Portal Bracing 4Ls - $2\frac{1}{2}$" x 2" x 3.7#
Top Lateral Struts - 4Ls 2" x 2" x 3.2#
Sway Struts - Sway Rods -
Shoe Struts - 4" I - 7.5# Knee Braces 3" x 3" x $\frac{1}{4}$" L

Bottom Lateral Ties - $1\frac{3}{16}$" ϕ 1" ϕ $\frac{3}{4}$' ϕ
Top Lateral Ties - 1" ϕ $\frac{3}{4}$' ϕ
Floor Beam 18" x $\frac{1}{4}$' & 4Ls - 3x3" x 4.5#
Sidewalk Brackets —
Joists - 6 Lines 3x12'
Flooring - 2' Plank Joist Bearers 4x4"
Wheel Guards $2\frac{1}{2}$" x 8' Spiking Piece —
Hub Guards - 2 Lines $1\frac{1}{4}$' Gas pipe

Live Load 100 Lbs per Sq Ft of Floor

Plate I Stress Sheet of a Highway Bridge

In the case of railroad bridges, new ones are nearly always constructed to carry the heaviest main line engines. These are usually of a class corresponding to Cooper's E 40 or E 50. In some localities branch-line bridges are built for the same live loadings; but in the majority of cases the branch-line bridges consist of the old bridges from the main line.

64. **Specifications.** For any particular bridge the specifications are either written by the engineer in charge, or some of the very excellent general specifications which are on the market in printed form are used. Some railroads use these general specifications with the addition of certain clauses which are desired by the chief or bridge engineer. The principal differences in these general specifications are in regard to the allowance for impact.

Whenever highway design is mentioned in this text, it is to be in accordance with Cooper's Highway Specifications (edition of 1909). Wherever plate-girder design is given, it is in accordance with Cooper's Railway Specifications (edition of 1906); and wherever truss design is given, it is in accordance with the general specifications of the American Railway Engineering & Maintenance of Way Association (fourth edition, 1910).

65. **Stress Sheet.** Before the sections are designed, the computer makes a skeleton outline of the truss, and on this places the dead-load and live-load stresses, and, in case the wind should be considered, the wind-load stresses. This is sent to the designer. The designer determines the various sections, and also the moments and shears in the stringers and floor-beams. These are placed on a sheet usually 17 by 23 inches. This is called a *stress sheet.* This sheet is now given to the draftsman, who makes a shop drawing. The stress sheets for railroad bridges are usually more elaborate than those for highway bridges. Plate I is the stress sheet of a highway bridge; and Plate II (Article 78) and Plate III (Article 93) are examples of the best modern practice in the making of plate-girder and truss-bridge stress sheets.

66. **Floor System.** Perhaps no part of bridge design is better standardized than the construction of the open steel floors for railroad bridges. The stringers are usually placed 6 feet 6 inches apart, and consist of small plate-girders, or, if the panel length is short, of one or more I-beams. I-beams are economical in regard to

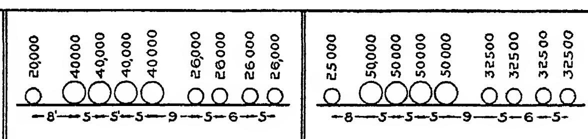
TABLE XIX

Safe Spans for I-Beams

(Based on unit-stress of 10 000 lbs per square inch in extreme fibre)

Size of I-Beam	Weight per Foot (Lbs.)	Moment of Inertia 1 I-Beam	Engine Class E 40 — Safe span C to C of Bearings: 1 Beam per rail	Engine Class E 40: 2 Beams per rail	Engine Class E 40: 3 Beams per rail	Engine Class E 50 — Safe span C to C of Bearings: 1 Beam per rail	Engine Class E 50: 2 Beams per rail	Engine Class E 50: 3 Beams per rail
9 in	25	92		5 ft 3 in	7 ft 9 in		4 ft 9 in	7 ft 0 in
9	30	102		5 9	8 9		5 0	7 0
9	35	112		6 6	9 6		5 3	7 9
10	30	135	3 ft 6 in	6 9	10 0	3 ft 0 in	5 9	8 6
10	35	163	4 3	8 6	11 0	3 6	7 0	10 0
10	40	175	4 6	9 0	12 0	3 9	7 6	11 0
12	31½	218	4 9	9 3	12 9	4 0	7 9	11 3
12	40	274	5 9	11 6	13 9	4 9	9 6	12 3
12	50	332	7 3	12 6	15 3	6 0	11 3	13 6
12	65	403	8 6	13 9	16 0	7 0	12 3	15 0
15	42	443	7 6	12 9	15 6	6 3	11 6	14 0
15	50	515	8 6	13 9	16 3	7 3	12 6	15 3
15	60	619	10 6	15 3	19 0	8 9	13 6	16 9
15	70	718	11 6	16 6	20 9	10 3	14 9	18 6
15	80	774	12 0	17 3	21 6	10 9	15 3	19 3
18	55	809	11 0	16 0	20 0	9 6	14 0	17 9
18	65	890	11 9	16 9	20 9	10 6	15 0	18 9
18	75	1 023	12 6	18 0	22 6	11 6	16 0	20 0
18	80	1 063	12 9	18 9	23 0	11 9	16 6	20 6
18	90	1 188	13 6	19 6	24 6	12 3	17 6	21 9
20	65	1 180	12 9	18 9	23 0	11 6	16 6	20 9
20	75	1 277	13 3	19 6	24 0	12 0	17 3	21 3
20	85	1 453	14 3	21 0	26 0	12 9	18 6	23 0
20	90	1 502	14 6	21 3	26 6	13 0	18 9	23 3
20	100	1 650	15 3	22 3	28 0	13 6	19 9	24 6
24	80	2 112	15 9	23 0	29 3	14 0	20 6	25 9
24	85	2 182	16 0	23 9	29 9	14 6	21 0	26 3
24	90	2 356	17 0	24 6	30 9	14 9	21 6	27 3
24	95	2 427	17 3	25 0	31 6	15 0	22 0	27 9
24	100	2 497	17 6	25 3	32 0	15 3	22 6	28 0

DIAGRAM OF ENGINES



first cost, but are disadvantageous on account of the eccentric connections which necessitate heavy brackets to resist part of their reaction. They are also somewhat undesirable on account of the fact that, the ties deflecting, most of the load is carried by the inner I-beam. However, I-beams for stringers and for short-span bridges (see Fig. 121) are much used in present practice, and give good results. Figs. 121 to 127 show the standard open floor sections of the Lehigh Valley Railroad. Table XIX gives the required number of I-beams, together with their weight, which are to be used for short-span bridges or as stringers in panels of given length.

Solid floors consist of angles and plates, channels and plates, or other shapes. They extend transversely across the bridge from truss

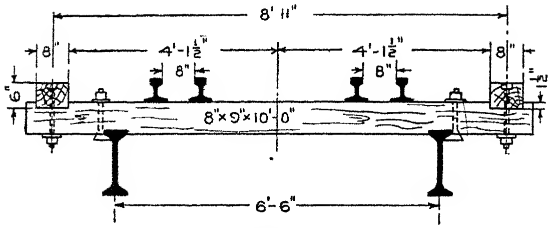


Fig. 121. Sectional View Showing Open-Floor Construction of Railroad Bridge of Short Span, Single Track. I-Beams used for Stringers. Lehigh Valley Standard

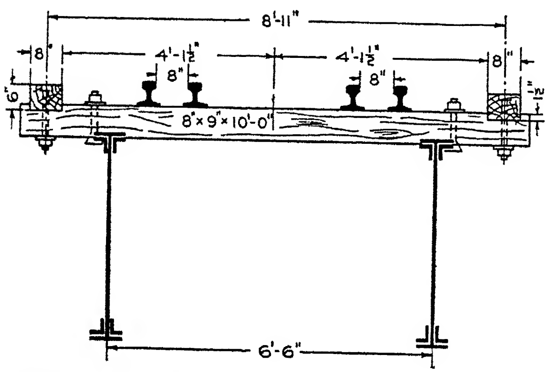


Fig. 122. Section of Open-Floor Construction of Deck-Girder and Truss Bridge, Single Track. Lehigh Valley Standard Plate-Girders used for Stringers

to truss, the lower chords, in case of truss bridges, being made heavy enough to act as girders as well as tension members. Figs. 128 to 130 show sections of solid floors. The ballast is laid directly upon these solid floors, which are first covered with a good damp-proof paint. The floors should also be supplied with good drainage facilities.

Concrete is sometimes laid directly upon the steel floor, and the ballast put upon this concrete,

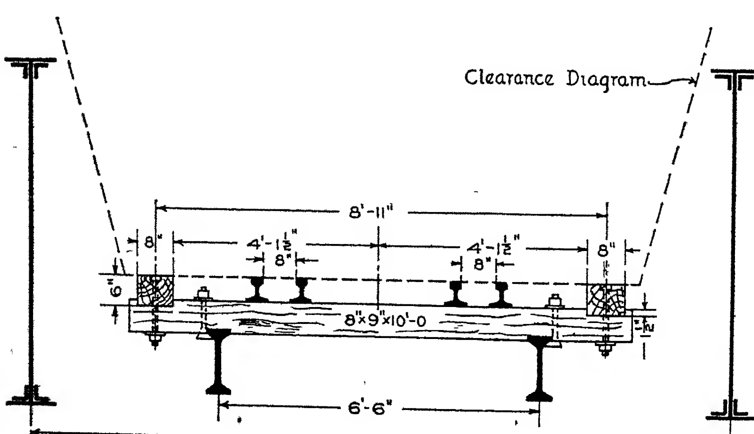


Fig. 123. Floor Construction of a Through-Girder Bridge, Single Track Lehigh Valley Standard.

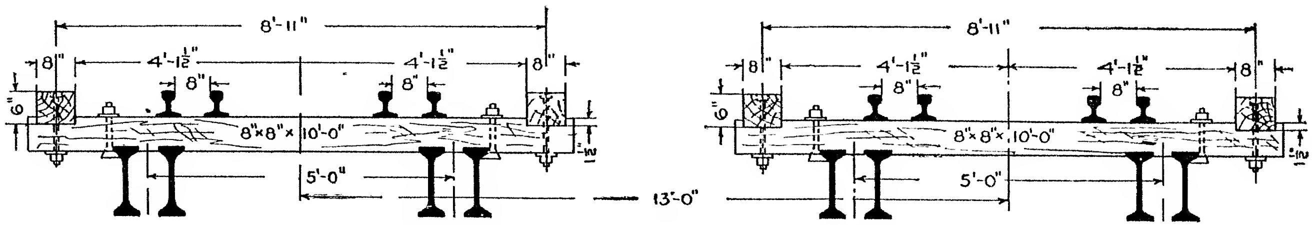


Fig. 124. Floor Construction of Double-Track I-Beam Bridge, Lehigh Valley Standard

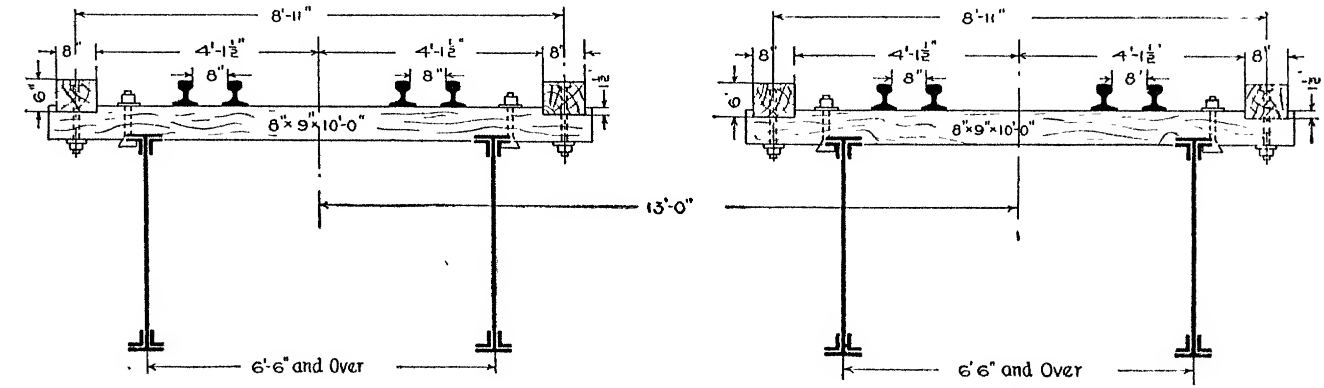


Fig 125. Floor Construction of Double-Track Deck-Girder and Truss Bridge, Lehigh Valley Standard

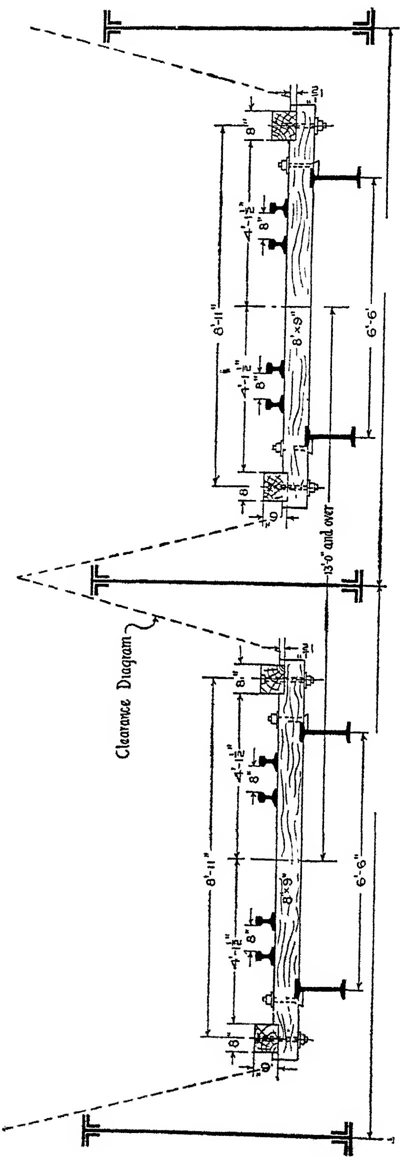


Fig. 126 Floor Construction of Double-Track Through-Girder Bridge, Lehigh Valley Standard

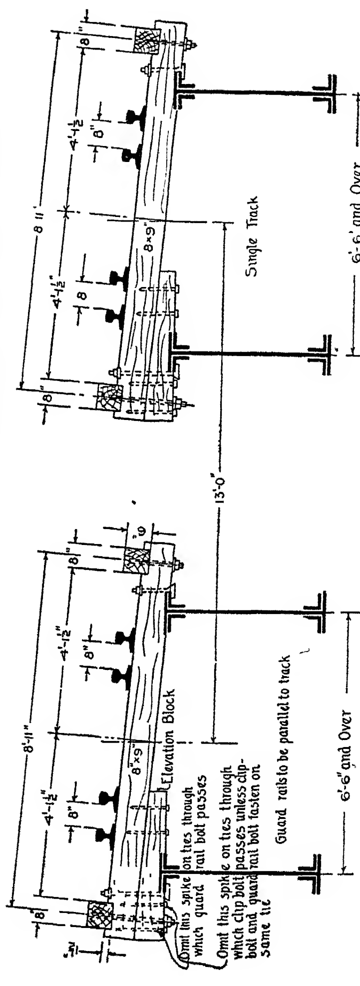


Fig 127 Floor Construction of Double-Track Deck-Girder and Truss Bridge on Curve, Lehigh Valley Standard.

which has previously had a layer of some good waterproofing applied on its upper surface.

67. Practical Considerations. The possibilities of the rolling mill and the various shops of a bridge company, such as the drafting room, forge, foundry, templet shop, assembling shop, and riveting and finishing shop, and also the shipping and erecting facilities, should be well known in order to make the most economical use of them. This requisite knowledge comes only from experience. The best way to obtain this experience without being actually employed in the shops, is to go into the shops every chance that presents itself, keep your eyes and ears open, and ask all the questions you can. The use to be made of handbooks of the various steel manufacturers is given in Part I of "Steel Construction," and should be thoroughly studied before going further. Some one of these handbooks is indispensable to persons designing steel structures. That of the Carnegie Steel Company (edition 1903) is one of the best, and will be frequently referred to in the present text.

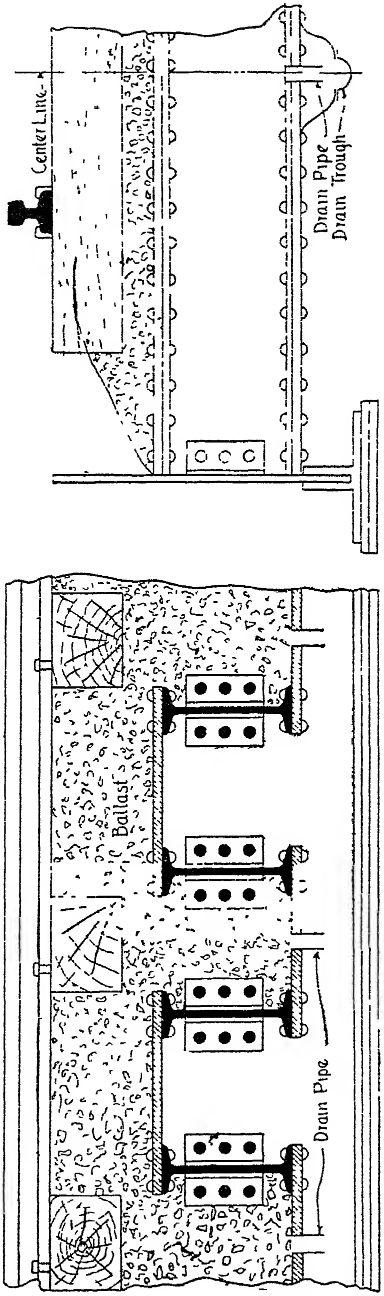


Fig. 128 Longitudinal and Transverse Section of Solid-Floor Construction

Copies may be procured from the Carnegie Steel Company, Frick Building, Pittsburg, Pa. The usual price to students is 50 cents, to others $2.00.

DESIGN OF A PLATE-GIRDER RAILWAY-SPAN

68. **The Masonry Plan.** In some cases the general dimensions of the masonry are limited; such a case, for example, would occur in the crossing of a street or narrow waterway. Here the length of the span and the distance above the street or the surface of the water,

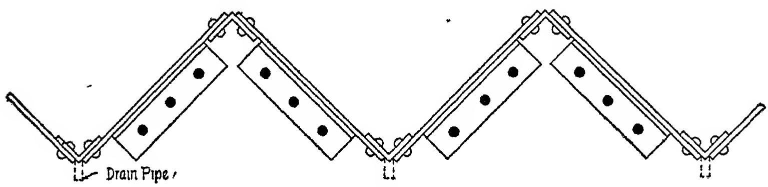


Fig. 129. Solid-Floor Construction of Plates and Angles.

are the limited dimensions. The span and under-clearance may be unlimited, as in the case of a country stream crossed by a roadway which is a considerable distance above the surface of the water. The term *unlimited* is not here used in its exact meaning, as the span in this case is really limited by the cost, which rapidly increases with the length of the span.

In some cases, as when the engineer is in a bridge office, the masonry plans are sent in by the railroad. In such cases many of the limited dimensions are fixed. The most usual dimensions to be fixed are the elevation of base of rail, the elevation and size of the bridge seat, and the length of the span under coping. These limit the depth of the girder, or the depth of the floor if it be a through girder, and also limit the length of the bearing plates at the end. Fig. 131, the masonry plan of a road crossing, shows in general what can be expected. All the dimensions usually fixed are given, and those marked x and y may or may not be, but x should never be less than 3 feet.

69. **Determination of the Class.** As before mentioned, the deck plate-girder should be used if possible, since its cost is less. There are some cases, however—such as track elevation in cities—where the additional cost required to elevate the track so as to use a

deck plate-girder will more than balance the saving in its favor. In such cases the through plate-girder is used.

The case whose design is under consideration will be taken similar to that of Fig. 131, and the span will therefore be a deck one.

70. **Determination of the Span, Center to Center.** Fig. 132 shows the various spans—namely, *under coping, center to center of end bearings,* and *over all.* The span under coping is that span from under coping to under coping lines of the abutments, and is so chosen as to give the required distance between the abutments at their base. The span center to center is equal to the span under coping *plus* the length of one bearing plate. The span over all is the extreme length of the girder. The length of the bearing plate is influenced by the width of the bridge seat, and also by the maximum reaction of the girder. The length should seldom be greater than 18 inches and never greater than 2 feet, as the deflection of the girder will cause a great amount of the weight to come on the inner edge of the bearing plate and also on the masonry, in which case the masonry is liable to fail at that point and the bearing plates are over-stressed.

Cast-steel bearings are now almost universally used. They decrease the height of the masonry, and distribute the pressure more evenly and for a greater distance over the bridge seat. When these castings are used, the bearing area between them and the girder may be made quite small, thus doing away to a great extent with the

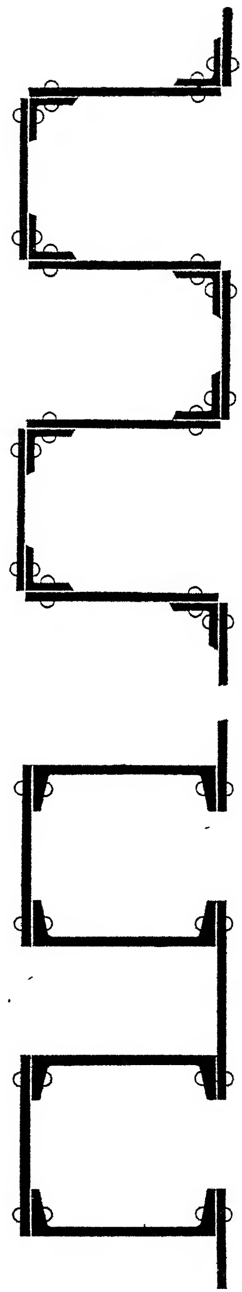


Fig 130 Methods of Solid-Floor Construction *A*—Plates and Channels, *B*—Plates and Angles

deteriorating effect due to the deflection of the girder as mentioned above. Fig. 133 shows the end of a girder equipped with a cast-steel pedestal. Table XX gives the length of the bearing on the

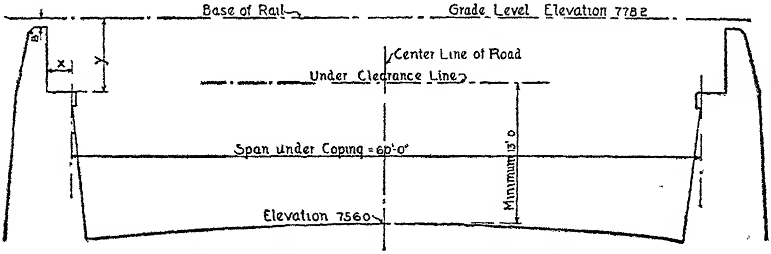


Fig. 131. Masonry Plan of a Road Crossing

masonry for various spans, Cooper's E 40 loading being used and cast-steel pedestals being employed.

TABLE XX

Length of Masonry Bearings

SPAN	LENGTH OF BEARING
15 to 24 feet	12 inches
25 to 44 "	16 "
45 to 69 "	21 "
70 to 79 "	23 "
80 to 89 "	29 "
90 to 115 "	36 "

As an example, let it be required to determine the span center to center of a deck plate-girder of 60-foot span under coping, the

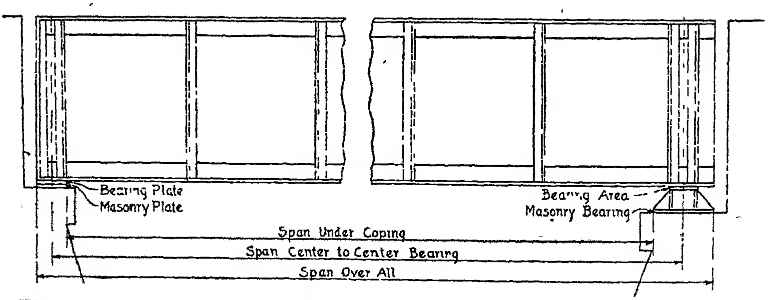


Fig. 132. Diagram Showing Various Spans Considered in Bridge Construction

loading being E 40. From Table XX it is seen that the length of the masonry bearing will be 21 inches, and therefore the span center

to center of bearings will be $60 + 2 \times (\frac{1}{2} \times 1 \text{ ft. } 9 \text{ in.}) = 61$ feet 9 inches.

In Articles 71 to 77 the above girder will be designed; and also such information as is of importance regarding the subject-matter of each article will be treated. The dead- and live-load shears are computed by the methods of Part I, and are given in Fig. 134.

71. **Ties and Guard-Rails.** The length of ties for single-track bridges is 10 feet. For double-track bridges the length is in most cases the same. In some double-track bridges, however, either each tie or every third tie extends entirely across the bridge. In other cases every third tie on one track extends to the opposite track, thus acting as a support for the foot-walk which is laid upon them. It is the best practice to limit the length of the ties on double-track bridges to 10 feet, since, if they extend into the opposite track in any way whatsoever, unnecessary expense is incurred whenever repairs or renewals are made, because both tracks must necessarily be disturbed to some extent.

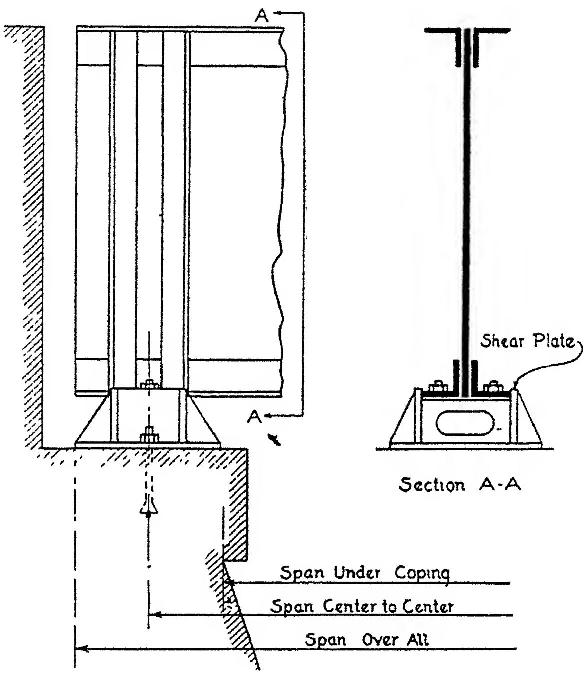


Fig. 133 End of a Girder Equipped with a Cast-Steel Pedestal.

The size of the ties varies with the weight of the engines and the spacing of the stringers or girders on which they rest. They are usually sawed to size instead of hewn, and the following sizes may be purchased on the open market—namely, 6 by 8, 7 by 9, 8 by 9, 9 by 10,

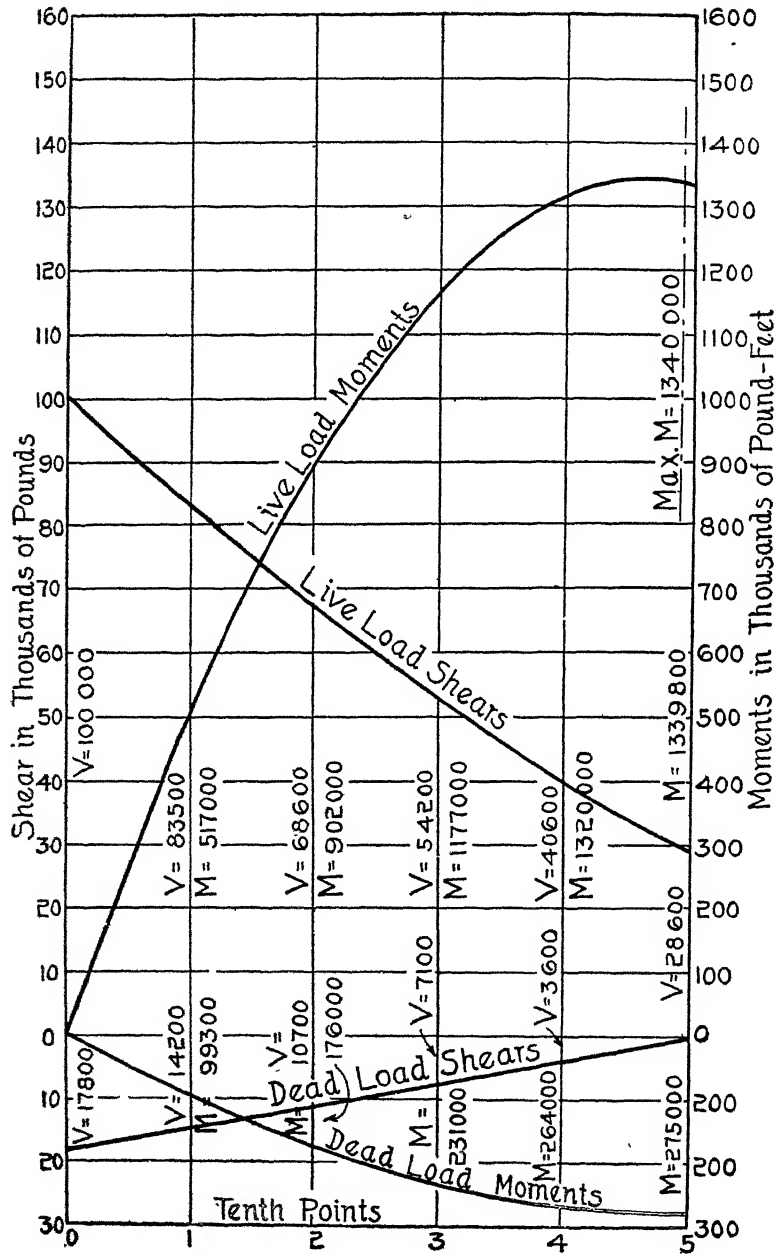


Fig 134 Shear and Moment Diagram.

and 10 by 12 inches. Larger sizes may be obtained on special order

The elevation blocks (see Fig. 127) should be of length to suit the width of the cover-plates and the spacing of the supports. They are usually made of the best quality of white oak, since the cost of renewal is great enough to demand that they be made of material as permanent as possible.

The guard-rails should be placed in accordance with the specifications (see Articles 13 and 14). Some railroads specify that the guard-rails shall be in 24-foot lengths unless the bridge is shorter than 24 feet, in which case one length of timber should be used. For method of connection and other details, consult Figs. 121 to 127. The guard-rails and the ties are usually made of Georgia long-leaf yellow pine, prime inspection. Other wood, such as chestnut, cedar, and oak, may be used.

In addition to the wooden guard-rail, a steel guard-rail usually consisting of railroad rails is placed within about 8 inches of the track rail.

In designing ties, the problem is that of a simple beam symmetrically loaded with two equal concentrated loads, the weight of the rail and tie itself usually being neglected. For the case in hand, which is that of a deck plate-girder, loading E 50, the concentrated load for which the tie must be designed is, according to Specifications (Article 23, 3d part), 8 333 pounds. According to Article 23, 100 000 pounds is on four wheels. This gives 25 000 pounds on one wheel, and according to Article 15, one-third of this, or 8 333 pounds, will come on one tie. Fig. 135 shows the condition of the loading, the space center to center of rail being taken as 4 feet 10 inches. Some designers take this distance as 5 feet; but as the standard rail head is about 2 inches, and the standard gauge 4 feet 8½ inches, the distance here taken seems to be the more logical one.

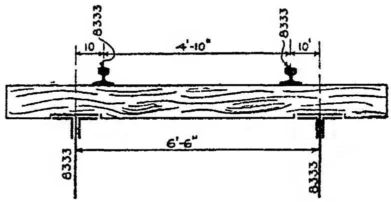


Fig 135 Distribution of Loading on Ties of Deck Plate-Girder Bridge

The formula to be used in the design of this beam is that given in "Strength of Materials," and is $M = \frac{SI}{c}$. In this case $M = 10 \times$

8 333 = 83 330 pound-inches. In the above formula, $I = bd^3 \div 12$, and $c = d \div 2$, and therefore $\frac{I}{c} = \frac{bd^2}{6}$. Substituting the value of the moment in the above formula, and solving for S, there results

$$S = \frac{499\ 980}{bd^2}.$$

For a 6 by 8-inch tie, the unit-stress would then be:

$$S = \frac{499\ 980}{6 \times 8^2} = 1\ 310 \text{ pounds}$$

If a 7 by 9-inch tie is used, the unit-stress is found to be 880 pounds. Since according to Article 15 of the Specifications, the unit-stress cannot be greater than 1 000 per square inch, it is necessary to use a 7 by 9-inch tie. If the engine loading had been E 50, the moment would have been 100 000 pound-inches, and then the stress in a 7 by 9-inch tie would be 1 060 pounds per square inch, and the stress in an 8 by 9-inch tie would be 930 pounds, which would necessitate the use of the latter.

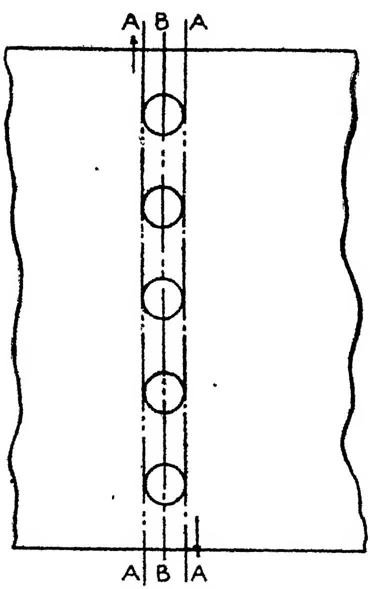


Fig 136 End Rivets Transferring Shear to Web.

The guard-rails on this bridge will be placed according to the Lehigh Valley standard, and hence their inner face will be 4 feet $1\frac{1}{2}$ inches from the center of the track.

Elevation blocks will not be required, as the bridge is on a tangent.

72. The Web. The economic depth of the web, according to Article 59, Part I, will be:

$$\text{Depth} = \frac{61 \text{ ft. } 9 \text{ in.}}{0.005 \times 61 \text{ ft. } 9 \text{ in} + 0\ 543} = 72\ 5.$$

The depth might be taken as 72 inches, but 74 inches will be decided upon, as this will decrease the area of the flange and also will not affect the total weight to any great extent. The unit-stress for shear

is 9 000 pounds per square inch (see Specifications, Articles 40 and 41).

The maximum shear in the girder occurs at the end, where it is 117 800 pounds. The area required for the web is then 117 800 ÷ 9 000 = 13.09 square inches, and the required thickness is 13.09 ÷ 74 = 0.177 inch. This latter value cannot be used, since, on account of Article 82 of the Specifications, no material less than $\frac{3}{8}$ inch can be used. The web plate will therefore be taken as 74 in. by $\frac{3}{8}$ in. in size.

Some engineers insist that the net section of the web should be considered. Consider Fig. 136, the shear being transferred to the web by the end rivets. The web will not tend to shear along the section *B–B*, in which case the rivet-holes should be subtracted; but it will shear along section *A–A*, a section which is unaffected by the rivet-holes. The web splice should come at one of the stiffeners, and will therefore be considered in Article 76.

73. **The Flanges.** This portion of the girder is usually built either of two angles or of two angles and one or more plates. In heavy girders where the flange areas are large, additional area is obtained by using side plates or side plates and four angles. Sometimes two channels are used in the place of side plates and angles. Fig. 137 shows the different methods of constructing the top flanges of girders. The lower flanges are usually of the same construction. Fig. 137 *b* has the web extending beyond the upper sur-

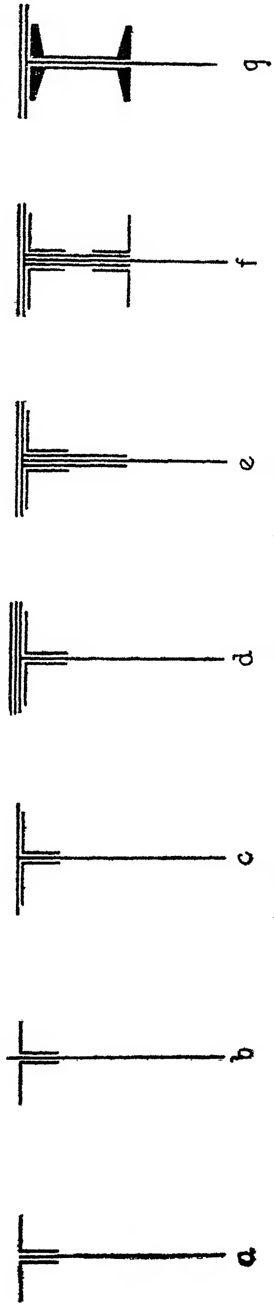


Fig. 137 Methods of Constructing Top Flanges of Girders.

faces of the upper flange angles. This is done in order that the ties may be dapped over it, and thus prevent the labor usually required for cutting holes in the lower face of the tie in order to allow for the projecting rivet-heads. Fig. 137 *g* is usually uneconomical, since the thinness of the channel web requires a great many rivets to sufficiently transmit the shear from the web to the flange, and also since the cover-plates must be very narrow.

Specifications usually state that the flanges shall have at least one-half of the total flange area in the angles, or that the angles shall be the largest that are manufactured. The largest angles are not usually employed, since their thickness is greater than three-quarters of an inch and therefore the rivet-holes must be bored, not punched. The reason for this is that the depth of the rivet-hole is too great in proportion to its diameter, and on this account the dies used for punching frequently break. Also, the punching of such thick material injures the adjacent metal, which makes it undesirable. In reality the flange area of only the short-span girders is small enough to allow the flange area to be taken up by the angles.

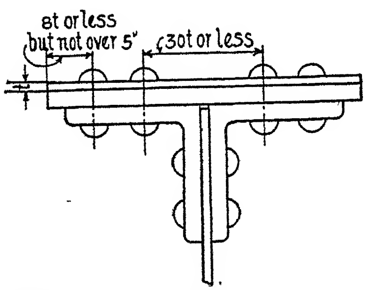


Fig. 138. Diagram Showing Relation between Thickness of Cover-Plates and Position of Rivets

In choosing the thickness of the cover-plates, care should be taken so that the outer row of rivets will not come further from the outer edge of the plate than eight times the thickness of the thinnest plate. In case eight times the thickness of the plate is greater than 5 inches, then 5 inches should be the limit. Also, the distance between the inner rows of rivets should not exceed thirty times the thickness of the thinnest plate. These limitations are placed by Article 77 of the Specifications, and Fig. 138 indicates their significance.

The determination of the required flange area depends upon the distance between the centers of gravity of the flanges; and in order to determine this exactly, the area and composition of the flanges should be known. The above condition requires an approximate design to be made, the supposition being that the flanges consist of two angles and one or more plates as shown in Fig. 138.

The distance back to back of angles will be taken as $74 + 2 \times \frac{1}{8} = 74\frac{1}{4}$ inches. Article 74 of the Specifications requires $\frac{1}{16}$ inch; but $\frac{1}{8}$ inch is better practice, since the edges of the web plate are very liable to overrun more than $\frac{1}{16}$ inch. Some specifications require $\frac{1}{4}$ inch.

In the computation of the approximate flange area, the center of gravity of each flange will be assumed as one inch from the back of the angles. The approximate effective depth is then $74\frac{1}{4}$ less 2×1 inch, which equals $72\frac{1}{4}$ inches. The approximate stresses in the flange areas are:

$$\text{For dead load, } \frac{275\,000 \times 12}{72.25} = 45\,600 \text{ pounds.}$$

$$\text{For live load, } \frac{1\,340\,000 \times 12}{72.25} = 222\,000 \text{ pounds.}$$

The approximate flange areas are now obtained by dividing these amounts by the allowable unit-stresses for dead and live load, which are (see Specifications, p. 8, Article 31): 20 000 and 10 000 pounds per square inch respectively; and the resulting areas are:

$$\text{For dead load, } \frac{45\,600}{20\,000} = 2.28 \text{ square inches}$$

$$\text{For live load, } \frac{222\,000}{10\,000} = 22.20 \text{ square inches}$$

These amounts give a total of 24.48 square inches as the approximate net flange area required.

It will be assumed that one-half the total area, or $24.48 \div 2 = 12.24$ square inches, is to be taken up by angles. If 12.24 sq. in. is distributed over two angles, then $12.24 \div 2 = 6.12$ square inches is the net area for one angle. Of course it is not to be assumed that the area of the angle chosen must be exactly 6.12, but that this 6.12 square inches is the approximate area of the angle to be chosen, and the net area of the angle (see Specifications, Article 149) must not be $2\frac{1}{2}$ per cent less than this, although it may be greater.

From Steel Construction, Part II, Table III, or Carnegie Pocket Companion, p. 146, a 6 by 6 by $\frac{3}{4}$-inch angle gives a gross area of 8.44 square inches and a net area of $8.44 - 2 \times (\frac{7}{8} + \frac{1}{8}) \times \frac{3}{4} = 6.94$ square inches, $\frac{7}{8}$-inch rivets being used and so spaced that two rivets are taken out of each angle (see Specifications, Article 63, and Fig. 139). A 6 by 4 by $\frac{13}{16}$-inch angle, giving a gross area of 7.47 and a net area of 6.66 square inches, one rivet-hole being out,

could have been used, but $\frac{11}{16}$ inch is too thick to punch, and therefore the above angle is chosen.

The required net area of the cover-plate is now found to be $24.48 - 2 \times 6.94 = 10.60$ square inches. Since the legs of the angles are 6 inches and the thickness of the web is $\frac{3}{8}$ inch, the outer edges of the angles are $12\frac{3}{8}$ inches apart; and since the cover-plate must extend somewhat over the edges of the angle, and the width of the cover-plate should be in the even inch, the width of the cover-plates must be at least 14 inches, as shown in Fig. 139.

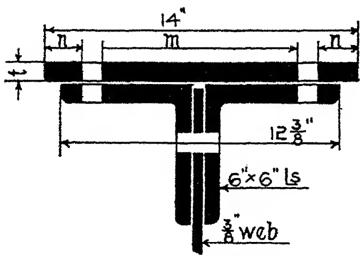


Fig 139 Calculation of Size of Angles and Cover Plate

On account of the 1-inch rivet-holes to be deducted, the real or net width of the cover-plate is: $2 \times n + m = 14 - 2 \times 1 = 12$ inches. The thickness of all the cover-plates at the center is now:

$$t = \frac{10.60}{12} = 0.885 \text{ inch—say, } \tfrac{7}{8} \text{ inch}$$

A thickness of $\frac{7}{8}$ of an inch is decided upon, for the reason that plates are rolled only to the nearest sixteenth of an inch.

The approximate section at the center has now been determined, and is:

2 Angles 6 by 6 by $\frac{3}{4}$-inch	=	13.88 sq. in. net.
Cover-plates $\frac{7}{8}$ inch thick	=	10.50 sq. in. net.
Total	=	24.38 sq. in. net.

This approximate section must now be examined, and, if it shows too great an excess or a deficiency, must be revised.

In order to determine the effective depth the distance between the centers of gravity of the flanges must first be computed, the gross areas being used. Theoretically, perhaps, the net areas should be used; but this is an unnecessary refinement, since the effect on the final result is of no practical importance.

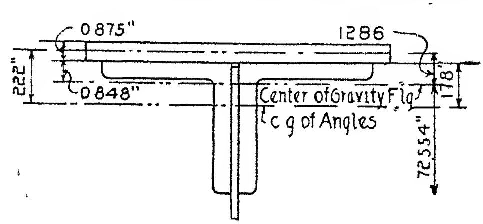


Fig. 140. Determination of Center of Gravity.

In computing the center of gravity (see Fig. 140), the axis is taken at the center of the cover-plates, as this reduces the moment of the cover-plates to zero. The distance of the center of gravity of the angles from their back (Carnegie Pocket Companion, p. 146, column 8) is 1.78 inches. The distance of this center of gravity from the center of the cover-plate, is $1.78+0.875\div2=2.22$ inches.

Gross area of the angles	$= 2 \times 8.44$	$= 16.88$ sq in
" " " cover-plates	$= \frac{7}{8} \times 14$	$= 12.25$ sq in
	Total	$= 29.13$ sq in

The center of gravity is now found to be $16.88 \times 2.22 \div 29.13 = 1.286$ inches from the center of the cover-plate, and $1.286-0.875\div2=0.848$ inch from the back of the angle. The effective depth h_e is $74.25 - 2 \times 0.848 = 72.554$ inches, and the required flange areas are:

$$\frac{275\,000 \times 12}{72.554 \times 20\,000} = 2.272 \text{ sq in for dead load.}$$

$$\frac{1\,340\,000 \times 12}{72.554 \times 10\,000} = 22.200 \text{ sq in for live load.}$$

$$\text{Total} = 24.472 \text{ sq in}$$

The values of the moments, as taken from the curves, must be multiplied by 12 in order to reduce them to pound-inches.

A total of 24.38 square inches is given by the section approximately designed, and the difference between that and the section as above determined is: $(24.472 - 24.38) \div 24.472 = 0.38$ per cent, and as this is less than $2\frac{1}{2}$ per cent (see Specifications, Article 149), it may be used without any further change. If there should have been a deficiency or an excess greater than $2\frac{1}{2}$ per cent, then it would have been necessary to revise. In case a revision of section is necessary, the size and thickness of the angles generally remain the same as those taken in the approximate design, the thickness of the cover-plates being decreased or increased as the case may be.

The total thickness of the cover-plates, $\frac{7}{8}$ inch, is too thick to be punched. In such cases as this, the section is made up of two or more plates whose total thickness is equal to that required. If plates of more than one thickness are decided upon, then their thickness should decrease from the flange angles outward. For the case in hand, one plate $\frac{3}{8}$ inch thick and one plate $\frac{1}{2}$ inch thick will be decided upon. The flange section at the center as finally designed is:

Shape	Net Section	Gross Section
2 Angles 6 by 6 by $\frac{3}{4}$ in.	13 88 sq in.	16 88 sq in
1 Cover-plate 14 by $\frac{3}{8}$ in	4 50 "	5 25 "
1 Cover-plate 14 by $\frac{1}{2}$ in.	6 00 "	7 00 "
Total	24 38 "	29 13 "

The above is the section required at the center of the girder; for any other point it will be less, decreasing toward the end, where it will be zero. Evidently, then, the cover-plates will not be required to extend the entire length. The following analysis will determine where they should be stopped. If the load were uniform, the moment

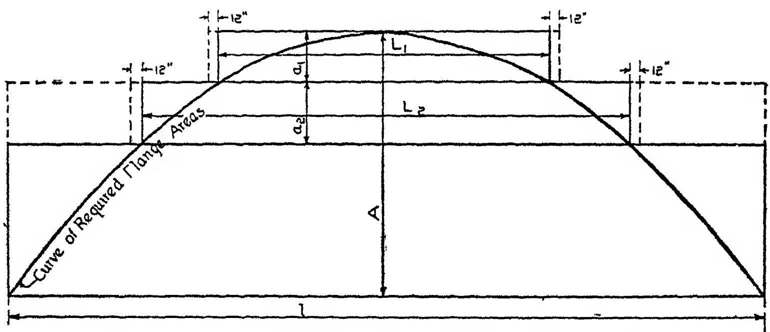


Fig 141. Diagram Showing Curve of Required Flange Areas.

curve would be a parabola. Although under wheel loading the curve of moments is not a parabola, yet it is sufficient for practical purposes to consider it as such. The curve of flange areas, like that of moments, is to be considered a parabola (see Fig. 141).

Let a_1 = Net area, in square inches, of the outer cover-plate;
a_2 = Net area, in square inches, of the next cover-plate;
a_3, etc. = Net areas of the other cover-plates,
A = Net area of all the cover-plates and the flange angle.

Then, from the properties of the parabola,

$$L = l\sqrt{\frac{\Sigma a}{A}},$$

where L = Length of cover-plate in question;
l = Length of span, center to center;
a = Net area of that cover-plate and all above it, and
A = Total net area of the flange, $\frac{1}{8}$ of the gross area of the web not being considered in this quantity.

The lengths of the cover-plates for the section above designed (see Fig 141) are:

$$L_1 = 61.75 \sqrt{\frac{4.50}{24.38}} = 26.45 \text{ feet}$$

$$L_2 = 61.75 \sqrt{\frac{4.50 + 6.00}{24.38}} = 40.00 \text{ feet.}$$

One foot is usually added on each end of the cover-plate as theoretically determined above The results are also usually rounded off to the nearest half-foot This is done in order to allow a safe margin because of the fact that the curve of flange areas is not a true parabola The final measurements of the cover-plates are

14 in by $\frac{3}{8}$ in by 28 ft 6 in. long
14 in. by $\frac{1}{2}$ in by 42 ft. 6 in. long.

In most cases the cover-plate next to the angle on the top flange only is made to extend the entire length of the girder Although this is not required for flange area, it is done in order to provide additional stiffness to the flange angles toward the ends of the span, and to prevent the action of the elements from deteriorating the angles and the web by attacking the joint at the top (see broken lines, Fig. 141, for length of first cover-plate extended).

EXAMPLES FOR PRACTICE

1. The dead-load moment equals 469 000 pound-inches, and the live-load moment, 4 522 000 pound-inches Design a flange section entirely of angles, if the distance back to back of angles is $45\frac{1}{4}$ inches.

2. The dead-load moment is 3 340 000 pound-inches, and the live-load moment, 21 235 000 pound-inches Design a flange section using 6 by 6-inch angles and three 14-inch cover-plates, the distance back to back of flange angles being $78\frac{1}{4}$ inches

3. In each of the above cases, design the flange section considering that $\frac{1}{8}$ of the web area is taken as effective flange area (For demonstration of the methods to be employed in the solution of this problem, see the succeeding text.)

While the section of a plate-girder is composite—that is, it consists of certain shapes joined together, and is not one solid piece—nevertheless these shapes are joined so securely that the section may be considered as a solid one and its moment of resistance computed accordingly. Let Fig. 142 be considered.

The moment of resistance of the section is:

$$M = \frac{S\left(A \times \frac{h^2_e}{2^2} + A \times \frac{h^2_e}{2^2} + \frac{th^3}{12}\right)}{\frac{h}{2}}$$

in the derivation of which the moment of inertia of the flange about its own neutral axis is considered as zero, and A equals the net area of one flange. Now, as the values of h_e and h seldom differ by more

than one inch, for all practical purposes they may be considered as equal. The above expression then reduces to:

$$M = S \times h \left(A + \frac{th}{6} \right)$$

$$= S \times h \text{ (net area of flange + one-sixth gross area of web)}$$

Since the rivet-holes decrease the moment of resistance of the web, one-sixth of the gross area cannot be considered, as is theoretically indicated in the above formula. It is common practice to take one-eighth, instead of one-sixth, of the gross web area. Substituting this value in the above equation, and transposing, there results:

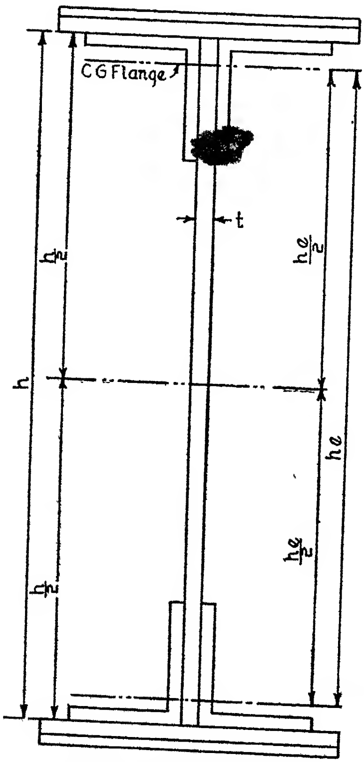


Fig 142. Section of Plate-Girder.

$$\text{Area of flange} + \tfrac{1}{8} \text{ gross web area} = \frac{M}{Sh}.$$

The flange section will now be designed for the moments previously given, considering $\frac{1}{8}$ of the gross web area as efficient in withstanding the moment.

The gross area of the web is $74 \times \frac{3}{8} = 27.75$ square inches; and $\frac{1}{8}$ of this is 3.47 square-inches. The total approximate amount of flange area required is, as in the first case, 24.48 square inches.

According to the above formula, $\frac{1}{8}$ of the web area, or 3.47 square inches, may be considered as flange area, and therefore $24.48 - 3.47 = 21.01$ square inches, is the approximate area of the angles and cover-plates of the flange. The approximate area of one angle is then $21.01 \div (2 \times 2) = 5.25$ square inches. A 6 by 6 by $\frac{9}{16}$-inch angle gives the gross area of 6.43 square inches and, two rivet-holes being deducted, a net area of 5.305 square inches (see "Steel Construction," Part I, Table VIII, or Carnegie Handbook, p. 117). As this is quite close to the approximate area determined above, this angle will be taken. The ap-

proximate area of the cover-plates is 21.01 − 2 × 5.305 = 10.40 square inches. As before, the gross width of the cover-plate will be taken as 14 inches. The thickness is then $\frac{10.40}{14 - 2\left(\frac{7}{8} + \frac{1}{8}\right)}$ = 0.867 inch—say ⅞ inch.

The gross area of the angles being 12.86 square inches, and that of the cover-plates 12.25 square inches, the center of gravity of the section is found, by a method similar to that previously employed, to be 1.10 inches from the center of the cover-plate, or 1.10 − 0.438 = 0.662 inch from the back of the flange angles. This makes the effective depth 72.93 inches.

For this section, the true live-load flange stress is (1 340 000 × 12) ÷ 72.93 = 221 000 pounds, and the actual dead-load flange stress is (275 000 × 12) ÷ 72.93 = 45 400 pounds. The actual areas required for the live and dead load are 22.10 and 2.27 square inches, which are obtained by dividing the above flange stresses by 10 000 and 20 000 pounds, respectively. The total required area is the sum of the two areas above, and is equal to 24.37 square inches. The total area required in the flange angles and cover-plates is therefore 24.37, less ⅛ the gross area of the web, 3.47, which leaves 20.90 square inches. The same angles as decided upon before will be used. This gives a required area for the cover-plates of 20.90 − 10.61 = 10.29 square inches. The required thickness is then 10.29 ÷ (14 − 2) = 0.857—say ⅞ inch. The following section of the flange will therefore be decided upon:

Shape	Net Section	Gross Section
2 Angles 6 in. by 6 in. by $\frac{9}{16}$ in.	10.61 sq. in.	12.86 sq. in.
1 Cover-plate 14 in. by ⅜ in.	4.50 "	5.25 "
1 Cover-plate 14 in. by ½ in.	6.00 "	7.00 "
Total =	21.11 "	25.11 "

As the total net area above is within 2½ per cent of the required net area, that section will be taken (see Specifications, Article 149). Note that in this case, the thickness of the cover-plates in the final design is the same as that determined in the preliminary design. Also note that the total net area is about 4 square inches, or 20 per

cent, less than in the flange as first designed, in which case none of the area of the web was considered as withstanding the bending moment.

The $\frac{1}{2}$-inch cover-plate on the top flange will extend the entire length of the grider, and is therefore 62 feet 9 inches long. The lengths of the other cover-plates are:

$$\text{For } \tfrac{1}{2}\text{-inch plate at the bottom, } L = 61.75\sqrt{\frac{10.50}{21.11}} = 43.5 \text{ feet.}$$

$$\text{For each } \tfrac{3}{8}\text{-inch plate, } L = 61.75\sqrt{\frac{4.50}{21.11}} = 28.5 \text{ feet}$$

One foot should be added to each of the above lengths at each end, thus making the total lengths 45 feet 6 inches and 30 feet 6 inches, respectively.

EXAMPLES FOR PRACTICE

1. If the span is 63 feet center to center, compute the length of the cover-plate. The section consists of two angles 6 by 6 by $\frac{5}{8}$ in.; one cover-plate 14 by $\frac{1}{2}$ in.; and one cover-plate 14 by $\frac{7}{16}$ in.; two rivet-holes being taken out of each angle and each cover-plate.

2. If the span is 87 ft. 9 in. center to center, compute the length of the cover-plate if the flange consists of two angles 6 by 6 by $\frac{3}{4}$ in., and four cover-plates 16 by $\frac{9}{16}$ in., two rivet-holes being taken out of each angle and each cover-plate.

In determining the area of plates, the tables in the Carnegie Pocket Companion, pp. 86 to 88, will be found very convenient.

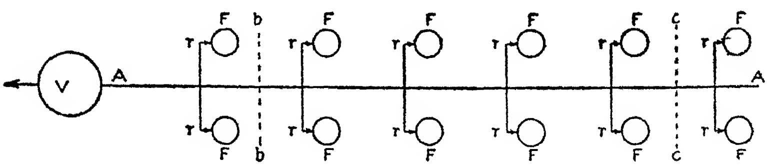


Fig. 143. Diagram Illustrating Transference of Shear from Web to Flanges by Rivets.

Another presentation of the subject of plate areas considerably more comprehensive and covering questions of design of riveted girders may be found in Steel Construction, pp. 134 to 153.

The spacing of the rivets in the flanges is a matter of considerable importance; the shear is transferred from the web to the flanges, where it becomes flange stress. This is done by the rivets, each rivet taking as much flange stress as is allowed by the Specifications. The conditions are similar to those shown in Fig. 143, where V represents

an object exerting a pull on a long, thin plate $A-A$ which has, at various points along this length, small objects F attached to it by means of pegs $r-r$. These small objects F hold the plate $A-A$ in equilibrium. Here V represents the shear which tends to cause the movement; $A-A$, the web; $r-r$, the rivets; and $2F$ the amount of flange stress taken by each rivet.

At section $c-c$ the total amount in the web to be transferred is $2F$; at section $b-b$ it is $10F$. From this it is seen that enough rivets $r-r$ must be put in between the sections $b-b$ and $c-c$ to take up $10F - 2F = 8F$; hence it is proved that the rivets between any two sections of the flange take up the difference in flange stress between those two sections.

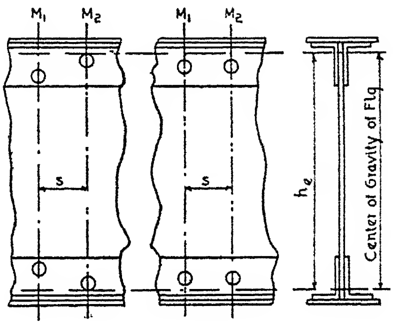


Fig. 144. Determination of Rivet Spacing

The discussion just given will be the means of giving us the number of rivets required between any two sections; but it does not give us the rivet spacing between these two sections. In order to determine the rivet spacing at any particular point, the following analysis is presented (see Fig. 144).

Let M_1 = Moment at one section;
M_2 = Moment at another section nearer center of girder than the section where M_1 occurs;
V = Shear at section where M_1 occurs,
v = Amount of flange stress one rivet can transfer, or it is the stress on one rivet;
s = Distance between the two sections,
n = Number of rivets between the two sections.

Then,

$\frac{M_1}{h_e}$ = Flange stress due to moment M_1;

$\frac{M_2}{h_e}$ = Flange stress due to moment M_2,

$\frac{M_2}{h_e} - \frac{M_1}{h_e}$ = Difference of flange stress between the two sections,

$\left(\frac{M_2}{h_e} - \frac{M_1}{h_e}\right) \div v = n$, Number of rivets required in space s . **(1)**

If the above sections be taken close enough together so that the number of rivets required will be 1 (that is, $n = 1$), then V can be considered as constant between the two sections, and then the moment $M_2 = M_1 + Vs$ (see Article 44, Part I). Substituting in Equation 1, above, there results:

$$\left(\frac{M_1 + Vs}{h_e} - \frac{M_1}{h_e}\right) - v = 1,$$

from which,

$$s = \frac{vh_e}{V},$$

which is the formula for the rivet spacing in the vertical parts of the flanges of any girder, providing the flange is not subjected to localized loading. It is to be used for the rivet spacing in both the top and bottom flanges of through girders, but not in the *top* flanges of deck plate-girders for railroad service. It is to be used, however, in the *bottom* flanges of deck plate-girders for railroad service. The distance h_e is not ordinarily used, the distance between rivet lines being used instead (see Fig. 145). The rivet spacing in the cover-plates and horizontal legs of the angles is made to stagger with that in the vertical legs, and usually the staggering is with every other rivet in the vertical flange. The term *stagger* signifies that the rivets in the top flange are not placed opposite the rivets in the vertical legs of the flange angles—or, that in case there are two lines of rivets in the vertical legs of the angle, a rivet near the outer edge of the cover-plate is placed in the same section where a rivet occurs near the lower edge of the vertical legs of the angle, and *vice versâ*.

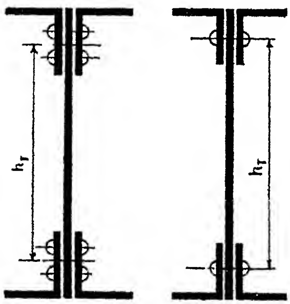


Fig. 145. Determination of Rivet Spacing

EXAMPLES FOR PRACTICE

1. Determine the rivet spacing at a section where the shear is 147 200 pounds, the value of one rivet 4 920 pounds, and the effective depth of the section 84¼ inches.

Ans. 2.82 inches.

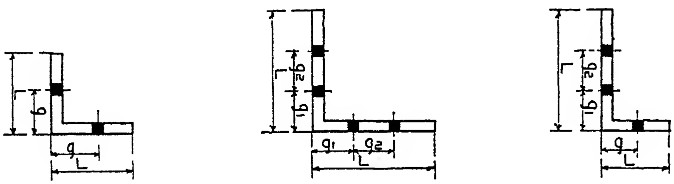
2. Determine the stress on a rivet at a section where the shear is 299 400 pounds, the spacing 2½ inches, and the effective depth of the girder 84¼ inches.

Ans. 8 870 pounds.

The rivet spacing is usually determined at the tenth-points, and a curve is plotted with the spacing as ordinates and the tenth-points as abscissæ. The rivet spacing at any intermediate point can be determined from this curve. When one-eighth the gross section of the web is considered as flange area, then only that proportion of the shear which is transferred to flanges is to be considered in computing the rivet spacing, on account of the fact that some of the shear is transferred directly to the bending moment in the web.

In order to determine the distance between rivet lines, the *gauge*, or distance out from the back of the angles to the place where the rivets must be placed, must be known for different lengths of legs. Table XXI gives the standard gauges, and also the diameter of the largest rivet or bolt which is allowed to be used in any sized leg. No gauges should be punched otherwise unless your large experience or instructions from one higher in authority demand it, and this should be so seldom that indeed it might be said never to be necessary.

TABLE XXI
Standard Gauges for Angles
(All dimensions given in inches)



L	*g*	Maximum Rivet or Bolt	*L*	*g*	Maximum Rivet or Bolt	*L*	*g*	Maximum Rivet or Bolt
8	$4\frac{1}{2}$	$\frac{7}{8}$	$3\frac{1}{2}$	2	$\frac{7}{8}$	2	$1\frac{1}{8}$	$\frac{1}{2}$
7	4	$\frac{7}{8}$	3	$1\frac{3}{4}$	$\frac{7}{8}$	$1\frac{3}{4}$	1	$\frac{1}{2}$
6	$3\frac{1}{2}$	$\frac{7}{8}$	$2\frac{3}{4}$	$1\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{8}$
5	3	$\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{8}$
4	$2\frac{1}{4}$	$\frac{7}{8}$	$2\frac{1}{4}$	$1\frac{1}{4}$	$\frac{5}{8}$	1	$\frac{9}{16}$	$\frac{1}{4}$

L	g_1	g_2	*L*	g_1	g_2
8	3	3	6*	$2\frac{1}{2}$	$2\frac{1}{4}$
7	$2\frac{1}{2}$	3	5	2	$1\frac{3}{4}$
6	$2\frac{1}{4}$	$2\frac{1}{2}$			

*When thickness is $\frac{3}{4}$ inch or over

The distance between rivet lines for the girder being designed (see Fig. 146), is, in the first case:

$$h_r = h - \left(2g_1 + \frac{2g_2}{2}\right)$$
$$= 74.25 - (2 \times 2\tfrac{1}{2} + 2\tfrac{1}{4})$$
$$= 67.00 \text{ inches}$$

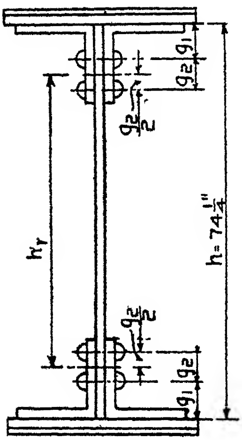


Fig. 146. Determination of Distance between Rivet Lines

In the second case, where $\frac{1}{8}$ of the web is considered, the above distance is 74.25 − $(2 \times 2\frac{1}{4} + 2\frac{1}{2})$ = 67.25 inches. The computations and the rivet spaces at the tenth-point, and at the ends of the cover-plates in the bottom chord of the plate-girder, are shown for each case in Table XXII. The value of v is the value of a $\frac{7}{8}$-inch rivet in bearing in a $\frac{3}{8}$-inch web (see Specifications, Article 40, and Carnegie Handbook, p. 195, second table). This value is 4 920 pounds.

In the first column, 7.98− indicates that the end of the cover-plate next to the flange is 7.98 feet from the end of the girder, and that this section is taken just to one side of that point, the side being that nearest the end of the girder. In a similar manner, 7.98+ indicates that the section is taken to that side of the point which is nearest the center of the girder. A like interpretation should be placed on 15.55− and 15.55+, the point under consideration in this case being the end of the outer or top cover-plate.

In the fifth column, values are given which indicate that portion of the shear which is transferred to the flanges. For example,

$$\frac{97\,700}{(14.08 \div 10.61)} = 74\,700,$$ and the difference between 97 700 and 74 700 represents that portion of the shear which is taken up directly by the web in the form of bending moment. An inspection of the headings of the third and fourth columns will tend to make this matter clearer.

Where there is local loading, as in the top flange, the rivets, in addition to the stress caused by the transferring of web stresses, are stressed by the vertical action of the angles being pressed downward by the ties and the consequent upward pressure of the web. Accord-

TABLE XXII

Rivet Spacing in Bottom Flange

Flange Taking All the Moment

Section	Total Shear (Pounds)	h_r (Inches)	v (Pounds)	Rivet Spacing (Inches)	Remarks
0	117 800	67	4920	2 80	
1	97 700	67	4920	3 38	
2	79 300	67	4920	4 16	
3	61 300	67	4920	5.38	
4	44 200	67	4920	7 46	See Specifica-
5	28 600	67	4920	11 52	tions, Art.54

One-Eighth of Web Area Considered

h_r = 67 25 inches, v = 4 920 pounds

Section	Total Shear (Pounds)	Net Flange Area plus ⅛ Web Area (Sq Inches)	Net Flange Area (Sq Inches)	Reduced Shear (Pounds)	Rivet Spacing (Inches)
0	117 800	14 08	10 61	88 800	3.75
1	97 700	14 08	10 61	74 700	4 45
7 98−	90 000	14 08	10 61	67 900	4 88
7 98+	90 000	20 08	16 61	74 800	4 42
2	79 300	20 08	16 61	65 800	5 02
15 55−	67 500	20 08	16 61	56 000	5 81
15 55+	67 500	24 58	21 11	58 100	5 69
3	61 300	24 58	21 11	52 700	6 28
4	44 200	24 58	21 11	38 000	8 71
5	28 600	24 58	21 11	24 600	13 42

ing to Article 15 of the Specifications, the weight of one driver is distributed over three ties (see Fig. 147).

Let $\frac{W}{l} = w$, the load per linear inch caused by one wheel W, which load is assumed to be uniformly distributed over the distance l;

$ws = v_1$, the vertical load or stress that comes on one rivet in the space s;

$v = \frac{Vs}{h_r}$, the stress on a rivet due to the distribution of flange stresses when s is a space, and V the shear at that point.

When these two stresses act on the rivet, the maximum stress will be v_u, the ultimate amount that it is allowed to carry, and this will act as shown in Fig. 147: Then,

$$v_u = \sqrt{v^2 + v_1^2}$$

$$= \sqrt{\left(\frac{V_s}{h_r}\right)^2 + (ws)^2}$$

$$= s\sqrt{\left(\frac{V}{h_r}\right)^2 + w^2},$$

from which,

$$s = \frac{v_u}{\sqrt{\left(\frac{V}{h_r}\right)^2 + w^2}},$$

which gives the spacing at any point in the girder flange under localized loading. Note that if w equals zero—that is, if there is no localized loading—there results:

$$s = \frac{v_u}{\sqrt{\left(\frac{V}{h_r}\right)^2 + 0}}$$

$$= \frac{v_u\, h_r}{V},$$

which is the same as previously deduced for flanges without localized loadings.

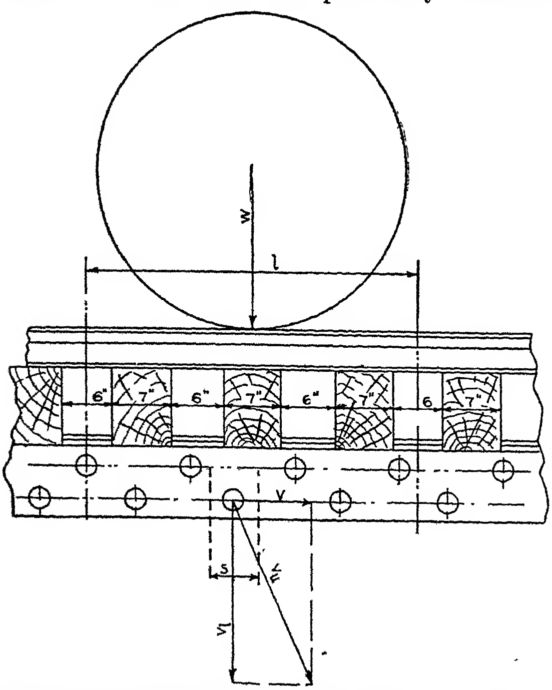


Fig 147. Rivet Spacing Determined by Stresses Distributed under Localized Loading.

The rivet spacing for the top flange of the girder which is being designed is given in Table XXIII. Here $W = 20\,000$; $l = (3 \times 7 + 3 \times 6) = 39$ inches; $w = \frac{20\,000}{39} = 513$; $h_r = 67$ inches; and $v_W = 4\,920$ pounds, which is the bearing of a $\frac{7}{8}$-inch rivet in the $\frac{3}{8}$-inch web. The top cover-plate is run the entire length of the span.

TABLE XXIII

Rivet Spacing in Top Flange

Flange Taking All the Moment

Section	u^2	Total Shear (Pounds)	$\left(\frac{V}{h_r}\right)^2$	$\sqrt{\left(\frac{V}{h_r}\right)^2 + u^2}$	Rivet Space (Inches)
0	262 600	117 800	3 080 000	1 825	2 70
1	"	97 700	2 140 000	1 550	3 17
2	"	79 300	1 390 000	1 285	3 83
3	"	61 300	840 000	1 050	4 68
4	"	44 200	435 000	835	5 88
5	"	28 600	181 500	660	7 45

One-Eighth of Web Area Considered

$w = 513$, $h_r = 67\frac{1}{4}$ inches, $r_u = 4\ 920$ pounds

Section	w^2	Reduced Shear (Pounds)	$\left(\frac{V}{h_r}\right)^2$	$\sqrt{\left(\frac{V}{h_r}\right)^2 + u}$	Rivet Space (Inches)
0	262 600	97 600	2 100 000	1 538	3 20
1	262 600	81 000	1 450 000	1 315	3 74
2	262 600	65 800	985 000	1 100	4 50
15 55 −	262 600	56 000	695 000	980	5 02
15 55 +	262 600	58 100	765 000	1 014	4 85
3	262 600	52 700	616 000	938	5 24
4	262 600	38 000	320 000	763	6 44
5	262 600	24 600	134 000	629	7.82

The points other than the tenth-points referred to in the first column are for sections taken just to the left and right of the top cover-plate. The values of the reduced shears given in the third column are obtained as has been previously explained. Although the rivet spacing in the lower flange is considerably greater than that in the upper flange, and accordingly a smaller number of rivets would be required, yet the spacing in the lower flange is made the same as that in the upper. Convenience in the preparing of plans and facility in manufacture make this action economical. Theoretical spacing greater than 6 inches should be dealt with according to Article 54 of the Specifications.

The values of the rivet spacing given in Tables XXII and XXIII are plotted in Fig. 148. Note that the effect of the localized loading is to decrease the rivet spacing, and also note that the effect increases from the ends toward the center.

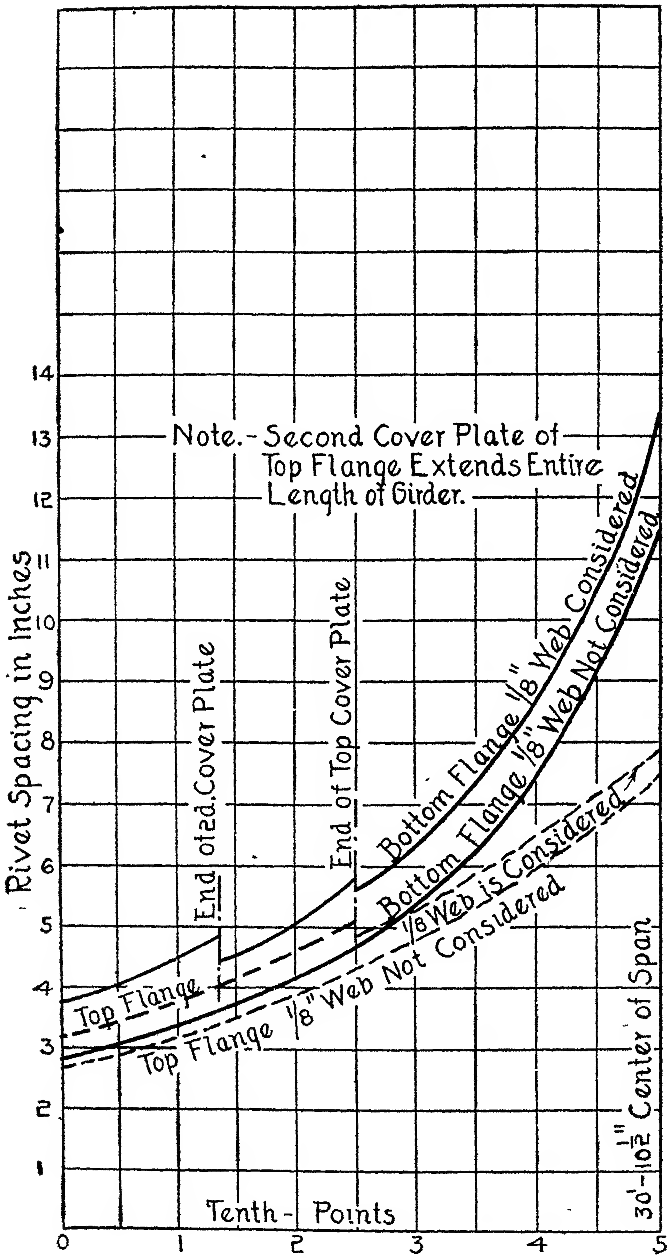


Fig. 148. Plotted Values of Rivet Spacing Given in Tables XXII and XXIII.

The size of the flange angles and the width of the cover-plates for different spans, are a matter of choice. Once the size is determined, the thickness can be computed. The sizes very generally adopted in practice are as follows:

Spans	Angles	Width of Cover-Plate
15 to 20 feet	5 x 3½ inches	none
20 " 25 "	6 x 4 "	"
25 " 40 "	6 x 6 "	"
40 " 75 "	6 x 6 "	14 inches
75 " 100 "	6 x 6 "	16 "
100 " 120 "	8 x 8 "	20 "

For another method of the presentation of this subject, see "Steel Construction," Part IV, pp. 264 to 268.

EXAMPLE FOR PRACTICE

1. Determine the rivet spacing for the top chord of a plate-girder, loading E 40, and 7 by 9-inch ties being used. The web is $\frac{3}{8}$ inch thick, distance from back to back of angles, 6 feet 6½ inches, flange angles, 6 by 6 by ½-inch, and cover-plate, 14 by $\frac{3}{8}$-inch, two $\frac{7}{8}$-inch rivets being out of each. First, consider the flange as taking all the bending moment, and second, consider one-eighth the gross area of the web. The total unreduced shear is 80 000 pounds in both cases.

Ans. 3.21 inches, 3.76 inches.

74. Lateral Systems and Cross-Frames. There are two methods in use in common practice in determining where the panels of the lateral bracing shall fall—namely, (1) To choose the number of panels so that the panel points come opposite the stiffeners, and (2) to choose the number of panels so that the placing of the panel points is independent of the stiffener spacing. The lateral systems should be of the Warren type; and in both of the above cases the angles that the diagonals make with the girder should not be greater than 45 degrees. Also, it is best to have all panels the same length and to have an even number of panels. This latter condition will simplify the drafting very much, since one-half of one girder can be drawn and the other half will be symmetrical, the opposite girder being similar to the one drawn, but being left-handed.

The members of the lateral systems will take tension or compression according to the direction the wind blows. Cross-frames are placed at intermediate points to stiffen the girders. These are

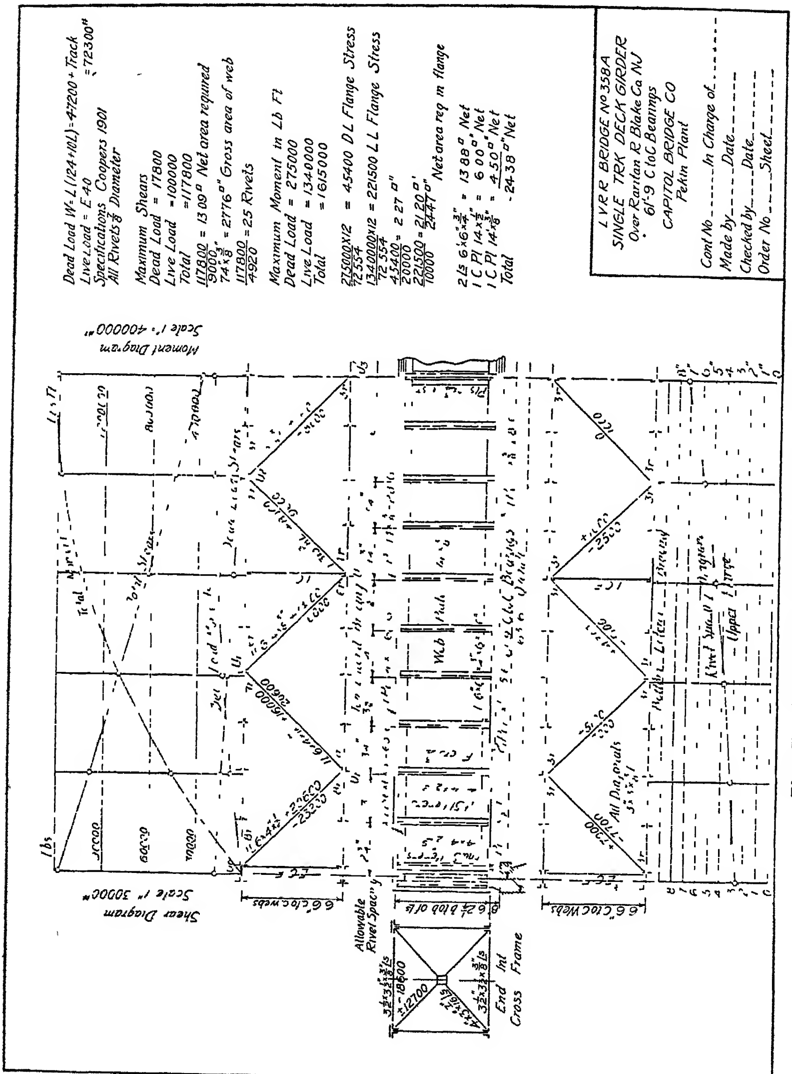


Plate II Stress Sheet of a Single-Track Deck-Girder Span

diagonal bracings (see Plate II), and are placed at certain intervals according to the judgment of the engineer. Good practice demands that their number should be about as indicated below:

SPAN OF GIRDER	NUMBER OF CROSS-FRAMES
0 to 20 feet	2
20 to 35 "	3
35 to 70 "	4
70 to 85 "	5
85 to 110 "	6

The above is not intended to serve as a hard and fixed rule. Variations from the limits given are to be made as the case demands. In all cases they are put at the panel points of the bracing, the top and bottom parts acting as sub-verticals in the lateral system. Also, the cross-frames should divide the span into equal parts if possible. In cases where that is not possible, the shortest divisions should come near the ends of the spans.

If the panel points are to be located at the stiffeners, the number of panels is a function of the depth of the girder (see Specifications, Articles 47 and 48). In this case the number of panels is given by:

$$N = \frac{\text{Span in inches}}{\text{Depth of girder in inches}},$$

no fraction being considered. As an example, let it be required to determine the number of panels in a girder 85 feet center to center of bearings, the depth being $90\frac{1}{4}$ inches back to back of angles. Then,

$$N = \frac{85 \times 12}{90.25} = 11.3, \text{ or, say, } 11 \text{ panels}$$

Each panel will then be 92.8 inches long. This, according to Article 47 of the Specifications, being greater than 5 feet, would not be allowed as a space between two stiffeners; but one stiffener can be placed in between, and then the panel points will come at every other stiffener. The cross-frames should be five in number.

The arrangement of panels and cross-frames is shown in Fig. 149. Here the cross-frames are marked *C. F.*, and the broken lines represent the lower lateral system.

In case an even number of panels were desired, then ten would be the number chosen and the general arrangement would be as shown in Fig. 150. The length of a panel would be 85 × 12 ÷

10 = 102 inches, or 8 feet 6 inches, which would allow of one stiffener in between and still keep the stiffener spacing within the limit of 5 feet.

The cross-frames at the ends of the span are designated as *end cross-frames,* and those in between are designated as *intermediate cross-frames.*

In case the spacing of the stiffeners is not required to be such as to coincide with the panel points of the lateral bracing, the panel length will depend upon the spacing of the girders, being equal to or

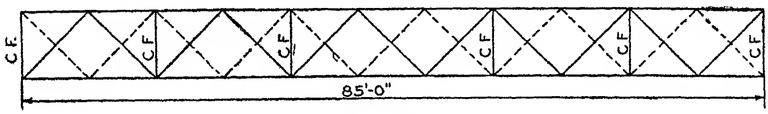


Fig 149

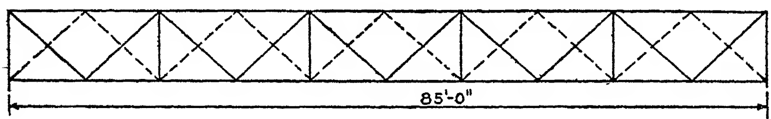


Fig 150

Arrangements of Panels and Cross-Frames.

greater than the spacing in order to keep the angle which the diagonals make with the girder less than 45 degrees. In this case,

$$N = \frac{\text{Span in feet}}{\text{Distance center to center of girders in feet}}$$

For the girder considered in Fig. 150, the number of panels would be $\frac{85}{7.5} = 11.3$—or, say, 11 panels—if odd numbers were to be used, and 12 if even numbers were to be desired.

For the case in hand, the panel points of the bracing will be taken at the stiffeners, and an even number of panels will be used. Then,

$$N = \frac{61.75 \times 12}{74.25} = 9.98 \text{ (say 10)}.$$

The arrangement of the panels and cross-frames, and also the maximum stresses in the diagonals, are shown in Plate II, the stresses being determined according to Article 50, Part I, "Bridge Analysis," and Article 24 of the Specifications. All the wind is taken as acting on one side of the bridge; and no overturning effect, either on the girder

or on the train, is considered. Also, note that the wind stresses in the flanges are not considered. Should the student determine these, he will find them too small to be considered according to Specifications, Article 39.

Before designing the lateral diagonals which consist of one or two angles, Articles 31, 33, 34, 35 (last portion), 38, 40, 63, and 83 of the Specifications should be carefully studied. The upper lateral bracing is to be designed first. Carnegie Handbook, pp. 109 to 119, is to be used.

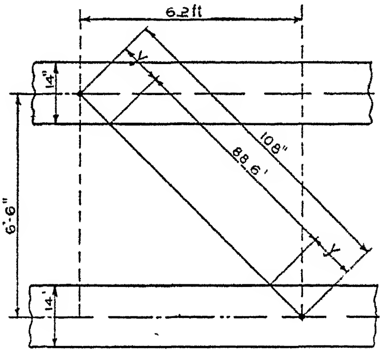


Fig 151 Determination of Length of Diagonal in Lateral Bracing.

The member U_0U_1 must be designed for a compressive stress equal to $23.20 + 0.8 \times 20.6 = -39.68$, and a tensile stress of $20.6+0.8\times20.6 = +37.08$. The length of the diagonal measured from center to center of girders is $\sqrt{6.5^2 + 6.2^2} = 9$ feet, or 108 inches. In reality the length is not 108 inches, as the cover-plate takes off a certain amount, as shown in Fig. 151. The true length, which is to be taken as a column length in designing, is $108 - 2y$, and y is readily computed to be 9.70 inches, thus making the true length 88.6 inches. The least allowable rectangular radius of gyration is obtained from the relation that the greatest value of $\frac{l}{r} = 120$, and therefore the least value of $r = \frac{l}{120} = 0.74$. It will be assumed that a 6 by 4 by $\frac{9}{16}$-inch angle with an area of 5.31 square inches will be sufficient. Here the length equals 88.60 inches, and the least rectangular radius of gyration is 1.14; hence,

$$P = 13\,000 - 60 \times \frac{88.60}{1.14}$$
$$= 8\,330 \text{ pounds per square inch.}$$

The required area is $\frac{39\,680}{8\,330} = 4.77$ square inches. As the angle assumed has an area of 5.31 square inches, which is considerably greater than the 4.73 square inches required, this angle cannot be used, and other assumptions must be made until the area of the angle

assumed and the required area as computed are equal or very nearly so.

A 6 by 4 by $\frac{1}{2}$-inch angle with an area of 4.75 square inches will now be assumed. The length is 88.60 inches as before, and the least rectangular radius of gyration is 1.15. The unit-load $P = 8\,340$ pounds per square inch, and the required area is $\frac{39\,680}{8\,340} = 4.72$ square inches. As the area of the angle assumed and the required area as computed are very close, this sized angle will be used.

The section must now be examined for tension, and in order that both legs of the angle may be considered as effective section, both legs must be connected at the end. The area required will be $\frac{37\,080}{18\,000} =$ 2 06 square inches. Considering one rivet-hole is taken out of the angle, the net area is $4\ 75 - 1 \times (\frac{7}{8} + \frac{1}{8}) \times \frac{1}{2} = 4\ 25$ square inches, which is amply sufficient.

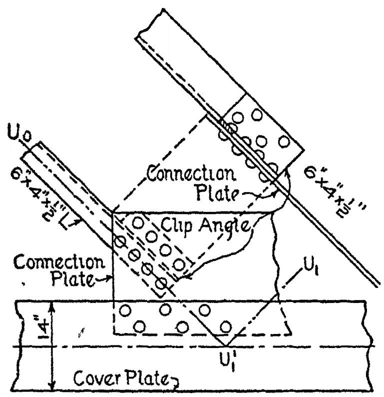


Fig. 152. Method of Connecting Angle Legs in Lateral Bracing and Cross-Frames

If the 4-inch leg only were assumed to be connected, the gross area would be $4 \times \frac{1}{2} = 2\ 00$ square inches, and the net area would then be $2.00 - 1 \times (\frac{7}{8} + \frac{1}{8}) \times \frac{1}{2}$-inch = 1.50 square inches, which is not sufficient. If the 6-inch leg were connected, the area would be sufficient. See Fig. 152 for method of connection and rivets.

The number of rivets required (see Specifications, Article 38 and 40) is computed as follows: If the member were not subjected to both tension and compression, the number of rivets required in single shear would be:

$$n = \frac{39\,680}{(9\,000 + 50 \text{ per cent of } 9\,000) \times 0.6013}$$
$$= \frac{39\,680}{8\,100}$$
$$= 4\ 86 \text{ (say 5)}.$$

But according to Article 38 of the Specifications, this number must be increased 50 per cent, and accordingly $4.86 \times 1\frac{1}{2} = 7.29$ (say 8)

shop rivets are to be used. In the above formula, 0.6013 is the area of the cross-section in square inches of a $\frac{7}{8}$-inch rivet. In order that both legs should be connected, a clip angle as shown in Fig. 152 is used; and the same number of rivets must go through both legs of the clip angle, since the stress in the vertical leg of the main angle is transferred to the clip angle and from there into the connecting plate.

The above number of rivets makes the joint safe so that it will not shear off in the plane between the connection plate and the horizontal leg of the angle. The joint must also be designed so that there will be sufficient rivets in bearing to prevent them from tearing out of the connecting plate. The number required is:

$$
\begin{aligned}
n &= \frac{39\,680}{(15\,000+50 \text{ per cent of } 15\,000)\times\frac{7}{8}\times\frac{3}{8}}\\
&= \frac{39\,680}{7\,380}\\
&= 5.38
\end{aligned}
$$

This 5.38 must be increased 50 per cent, making a total of $5.38 \times 1.5 = 8.07 =$ say, 8 shop rivets as before.

The above rivets are shop rivets, since it is assumed that the span, being a small one, will be riveted complete in the shop and shipped to the bridge site ready to place in position without any further riveting. In case the girders are shipped separately, then the lateral bracing must be riveted up in the field, and according to last part of Article 40 of the Specifications, the rivets, being field rivets, must have the allowed unit-stresses reduced one-third, which is equivalent to having the number of shop rivets increased 50 per cent. This will make the required number of field rivets $8 \times 1.5 = 12$.

As a rule, the connection plates are $\frac{3}{8}$ inch thick, seldom more. Also, the members of the upper lateral system are connected on the lower side of the connection plate in order not to interfere with the ties. Note that the use of the clip angles requires a smaller connection plate than would be necessary if these angles were not used, since in the latter case all the rivets must then be placed in one row in the horizontal leg of the angle.

The number of rivets required in the connection plate and the flange of the girder must be sufficient to take up the component of that member parallel to the girder. For the case in hand, the number (see Fig. 153) is:

$$\frac{x}{8} = \frac{6.2}{9.0},$$

from which,

$$x = 5.5 \text{ (say 6) rivets.}$$

Additional rivets should also be put in, in order to take up the component of the other lateral diagonal which meets at this point.

The member $U_1' U_1$ is to be designed for a maximum compressive stress of $20.6 + 0.8 \times 16.0 = -33.4$. A 6 by 4 by $\frac{7}{16}$-inch angle with an area of 4.18 square inches will be assumed. The least rectangular radius of gyration is 1.16. The unit allowable load is:

$$P = 13\,000 - 60 \times \frac{88.6}{1.16} = 8\,420$$

pounds per square inch

The required area is $\frac{33\,400}{8\,420} =$ 3.97 square inches. As this is very near the area assumed, and as trials with other angles do not give required areas which come any closer, this angle will be used.

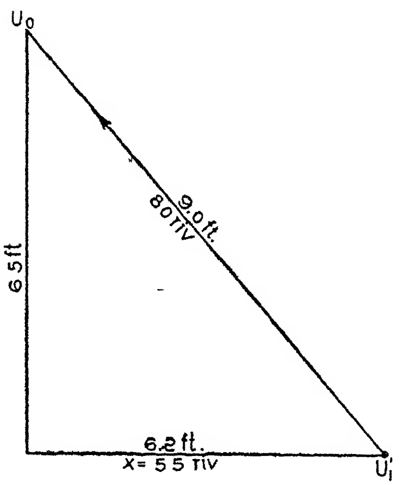


Fig. 153 Calculation of Rivets in Connection Plate and Flange of Girder

The rivets required in single shear are:

$$\frac{33\,400}{8\,100} \times 1.5 = 6.21 \text{ (say 7) shop rivets, and}$$

$$6.21 \times 1.5 = 9.3 \text{ (say 10) field rivets.}$$

The rivets required in bearing in the $\frac{3}{8}$-inch connection plate are:

$$\frac{33\,400}{7\,380} \times 1.5 = 6.78 \text{ (say 7) shop rivets, and}$$

$$6.78 \times 1.5 = 10.17 \text{ (say 11) field rivets.}$$

The above computations show the joint to be weakest in bearing, and therefore 7 shop or 11 field rivets must be used. It is not necessary to investigate this member for tension, as the computations for the first diagonal indicate that the area will be sufficient, both legs being connected.

The member U_1U_2' must be designed for a maximum compressive stress of $16.0 + 0.8 \times 14.1 = -27.28$. A 6 by 4 by $\frac{3}{8}$-inch

angle with an area of 3.61 square inches and a least rectangular radius of gyration of 1.17 will be assumed. The unit-stress P as computed from the formula in the Specifications, is 8 460 pounds per square inch; and the required area is $\frac{27\,280}{8\,460} = 3.23$ square inches. This angle will be used, as the given and required areas are close together, and as the next smaller angle—a 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angle with an area of 3.42 square inches—gives a required area of 3.58 square inches, thus being too small.

The rivets required in single shear are:

$$\frac{27\,280}{8\,100} \times 1.5 = 5.04 \text{ (say 5) shop rivets, and}$$
$$5.04 \times 1.5 = 7.6 \text{ (say 8) field rivets}$$

The rivets required in bearing in a $\frac{3}{8}$-inch web are:

$$\frac{27\,280}{7\,380} \times 1.5 = 5.54 \text{ (say 6) shop rivets, and}$$
$$5.54 \times 1.5 = 8.3 \text{ (say 9) field rivets}$$

In order to make the joints safe, 6 shop or 9 field rivets should be used.

The member $U_2'U_2$ must be designed for a maximum compressive stress of $9.6 + 0.8 \times 8.0 = -16.00$. A $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch angle with an area of 2.30 square inches and a least rectangular radius of gyration of 0.90 will be assumed. The unit-stress P is 7 090 pounds per square inch, and the required area is $\frac{16\,000}{7\,090} = 2.26$ square inches. As the required and the actual areas are very close together, this angle will be used.

The rivets required in single shear are:

$$\frac{16\,000}{8\,100} \times 1.5 = 2.96 \text{ (say 3) shop rivets, and}$$
$$2.96 \times 1\tfrac{1}{2} = 4.44 \text{ (say 5) field rivets}$$

By computation similar to the above, it is found that 4 shop or 5 field rivets are required in bearing. Since the bearing requires the most rivets to make the joint safe, 4 shop or 5 field rivets must be used.

If the Specifications would have allowed a $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{5}{16}$-inch angle with an area of 2.09 square inches, this angle would have exactly fulfilled the requirements, the required area being 2.09 square inches.

The member U_2U_3' must be designed for a maximum compres-

sive stress of $8.0 + 0 8 \times 4 1 = -11.28$. A 3 by 3 by $\frac{3}{8}$-inch angle with an area of 2.11 square inches and a least radius of gyration of 0.91 will be assumed. In this case the unit-stress is 7 160, and the area required is 1 58 square inches. The required area is considerably less than the area of the angle assumed; but it must be used, since it is the smallest allowed by the Specifications, which require that the material shall not be less than $\frac{3}{8}$-inch, and from Table XXI it is seen that 3 inches is the smallest size leg in which a $\frac{7}{8}$-inch rivet can be used.

The stresses in all the members of the lower lateral system are less than the stresses in the member just designed, and therefore all members of the lower lateral system will be made of one 3 by 3 by $\frac{3}{8}$-inch angle.

For the last member designed in the upper lateral system, and for all members in the lower lateral system, 3 shop or 5 field rivets will be required at the ends. These are more than sufficient to take up the stress, but it has been found that less than three rivets do not make a good joint.

The stress sheet, Plate II, shows the general arrangement of the lateral system, the number of rivets in the connections and also in the connection plates where they join the flanges.

The *intermediate cross-frames* do not lend themselves to a theoretical design, since the stresses which come upon them are not easily ascertained. It is good practice to require that all members be of the sizes as given below:

Span of Girder (in Feet)	Angles (in Inches)	Rivets	
		Shop	Field
30 to 65	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$	3	4
65 to 110	$4 \times 4 \times \frac{3}{8}$	4	5

The angles in the intermediate cross-frames will therefore be $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch.

The *end cross-frames* (see Fig. 154) act in a manner somewhat similar to the portal bracing in a bridge, since they transfer all the wind which comes on the top chord and on the train to the abutment. This load, which acts at the level of the ties, is in this case (see Article 24 of the Specifications):

$$P = \frac{600 \times 61 \text{ ft } 9 \text{ in}}{2} = 18\,525 \text{ pounds}$$

It is usually assumed that half of this is transferred to the point *a* by means of *a–b*, and from there down *a–b′* to the masonry. The other half goes directly down *b–a′* to the masonry. This causes stresses as shown in Fig 154. Note that the stress in *a–b* will always be compression; but the stresses in the diagonal will be either tension or compression according to the direction the wind blows. The member *a–b* will be a $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angle. To form the connections at its end, 3 shop or 5 field rivets will be used.

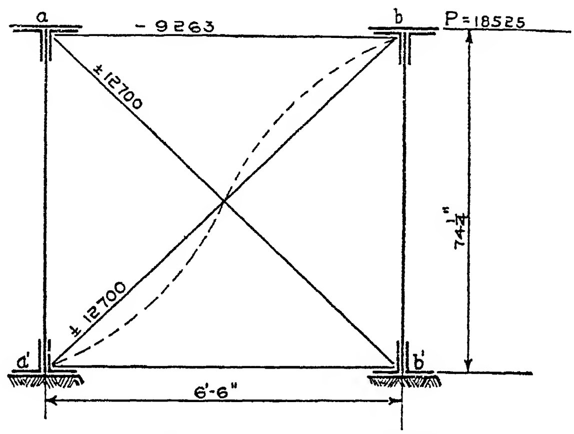


Fig 154 Action of End Cross-Frames

The maximum compressive stress for which the diagonals are to be designed is $12.70+0.8\times12.70 = -22.86$. Here the length is 108 inches if the angle tends to bend one way; but if it bends as shown by the broken lines in Fig. 154, the length will be one-half of this. For this reason, angles with unequal legs should be used, the longer leg extending outward. This allows the greatest rectangular radius of gyration to be used

A 4 by 3 by $\frac{7}{16}$-inch angle with an area of 2 87 square inches and a radius of gyration of 1 25 will be assumed. The unit-load *P* is computed to be 7 850 pounds, and the required area is therefore $22\,860 \div 7\,850 = 2.94$ square inches. This does not coincide very closely with the given area, but will be used since this angle comes nearer to fulfilling the condition than any of the other sizes rolled. The joint will require more rivets in bearing than in single shear. It is not necessary to perform the complete computations in order to determine this, since a comparison of the values of a rivet in single shear and in bearing shows that the value in bearing is less than that

in single shear, and therefore the number of rivets required in bearing will be greater than that number required in single shear. The number of rivets required in bearing is:

$$\frac{22\,860}{4\,920 \times 1.5} = \text{say, 4 shop rivets, and}$$

$$4.00 \times 1.5 = 6 \text{ field rivets.}$$

75. The Stiffeners. According to Article 47 of the Specifications, these should be placed at certain intervals whenever the unit-shear is greater than

$$S = 10\,000 - 75 \times \frac{74}{3/8} = 10\,000 - 14\,800 = -4\,800 \text{ pounds}$$

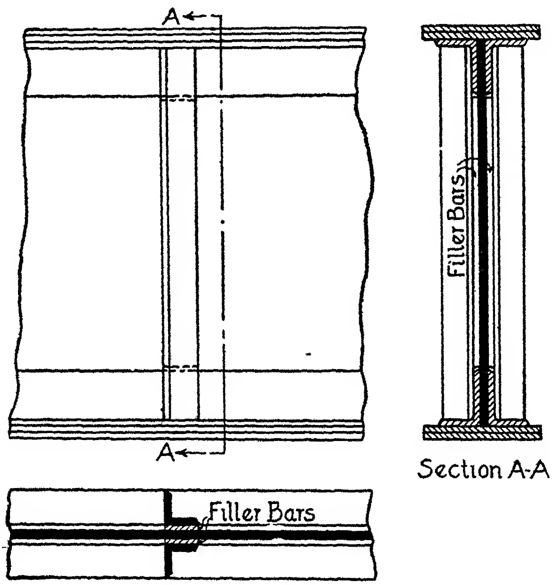


Fig. 155 Use of Straight Stiffeners, with Filler Bars

This negative sign signifies that whenever the unit shearing stress is greater than zero, the stiffeners must be placed throughout the entire length of the span at distances not to exceed 5 feet.

The intermediate stiffeners should have the outstanding leg long enough to give good support to the flange angle (see Fig. 155). The filler bars or fillers are put in so as to allow the stiffener angles to remain straight throughout their entire length; otherwise they will have to be bent as shown in Fig. 156. This bending is called *crimping*. Stiffeners must also conform to Article 48 of the Specifications. This would require a different sized stiffener at each point, and also a different number of rivets in each stiffener. This is not usually done in practice. In practice the stiffener for the first intermediate

point is designed, and the remainder are made the same in size and have the same number of rivets. An exception to this is where a stiffener comes at a web splice. In this case the size is usually kept the same, but the number of rivets is changed somewhat to conform to the requirements of the splice design.

The second intermediate stiffener comes at the first tenth-point, and is 6.175 (say 6.2) feet from the end, since it is at the first panel point, or opposite the first panel point, of the lateral system. The first stiffener will be 3.1 feet from the end; and scaling off the value of the shear at this point (see Fig. 134), it is found to be 98 000

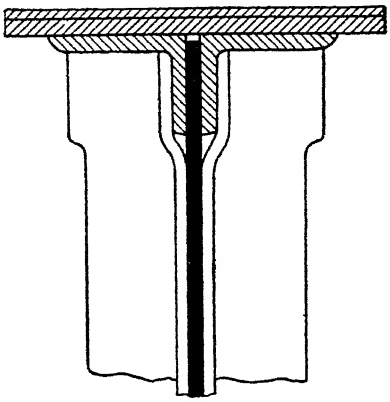


Fig. 156 Crimping of Stiffener Angles where No Filler Bars are Used

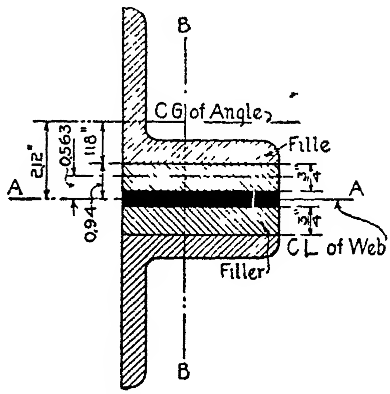


Fig. 157 Section of Intermediate Stiffener Construction

pounds. Here the length l to be used in the formula for the unit allowable compressive stress is $74\frac{1}{4} - 2 \times \frac{3}{4} = 72.75$ inches, the $\frac{3}{4}$ being the thickness of the flange angle. The section of the material which according to Article 48 of the Specifications is to be considered as a column, is shown in Fig. 157. The assumed column cannot bend about the axis $B-B$, but about the axis $A-A$, and therefore the radius of gyration about the axis $A-A$ must be computed. The moment of inertia of the fillers and the web plates about their own axes is considered as zero.

A 4 by 4 by $\frac{1}{2}$-inch angle with an area of 3.75 square inches will be assumed to be sufficient to withstand the stress. The moment of inertia of this and the filler bars and the web portion is

$$I_{A-A} = 2\,(5.55 + 3.75 \times 2.12^2 + 3.00 \times 0.563^2) = 46.70$$

The radius of gyration, then, is $\sqrt{\frac{46.70}{15.00}} = 1.764$, and the unit-stress computed with this value and a length of 72.75 inches is 8 140 pounds. The required area is now determined to be $98\,000 \div 8\,140 = 12.05$ square inches. The value 15.00 used in the above computation for the radius of gyration is the value of the area of the angles, the filler bars, and the web portion. A 5 by $3\frac{1}{2}$-inch angle with the 5-inch leg out would have given better support to the flange, but would not make so good a job, as it would have extended about $\frac{1}{8}$ inch beyond the curved part of the horizontal leg of the flange angle.

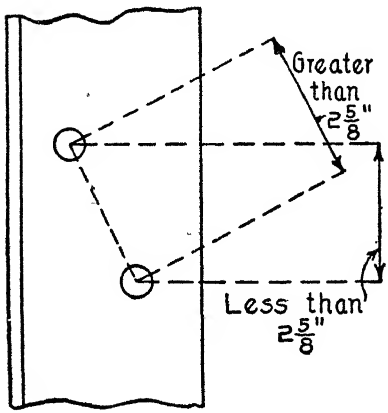


Fig. 158. Rivets Placed in Two Rows to Give Necessary Number and Spacing

The bearing determines the number of rivets in this case. The number is $98\,000 \div 4\,920 = 20$ shop rivets in the web.

The angle must now be investigated in order to see if these 20 rivets can go in one row without being closer together than $2\frac{5}{8}$ inches, which is three diameters of the $\frac{7}{8}$-inch rivet. The total length in which these rivets must be placed is 72.75 inches, and therefore we have $72.75 \div 20 = 3.6$ inches as a spacing. Since this is greater than $2\frac{5}{8}$ inches, 20 rivets can be placed in one row. If the spacing as determined above had been less than $2\frac{5}{8}$ inches, it would have been necessary to use two rows of rivets spaced as shown in Fig. 158; and then the distance center to center would be more than $2\frac{5}{8}$ inches, although the spacing in a vertical line would be less than that.

The four angles at the ends of the girders are called the *end stiffeners*. These are placed in pairs on opposite sides of the web (see Plate II, Article 74).

The total end shear is 117 800 pounds, and this is assumed to be carried by the two pairs of end-stiffener angles, each carrying one-half. This amount would require lighter angles than the angles used for intermediate stiffeners. It is the customary practice to make them the same size and thickness as the intermediate stiffeners,

additional strength being allowed in order to withstand the effects of the end cross-frame when in action.

The bearing determines the number of rivets required in each pair of stiffeners. The number required is $\frac{117\,800}{2 \times 4\,920} = 12$ shop rivets.

Some engineers arbitrarily choose the stiffeners regardless of the shear, enough rivets, however, being put in the end stiffeners to take up all the shear; and the spacing in the intermediate stiffeners is made the same. One noted engineering firm determines its stiffeners according to the following:

Flange Angle		Stiffeners	
Horizontal Leg	Thickness	End	Intermediate
4 in	Any	3 x 3 x ½ in	3 x 3 x ⅜ in.
5 in	Any	4 x 4 x ½ in.	3½ x 3½ x ⅜ in.
6 in	Over ¾ in	4 x 4 x ½ in.	3½ x 3½ x ⅜ in.
6 in	Less than ¾ in	5 x 3½ x ½ in	5 x 3½ x ⅜ in
8 in	Any	6 x 6 x ¾ in.	6 x 4 x ⅜ in

No rational method has as yet been determined for ascertaining the stresses in the stiffeners of plate-girders. Results obtained by placing extensometers on the stiffeners of actual plate-girders appear to indicate that the stresses are very small, in fact in most cases not being greater than 1 500 or 2 000 pounds per square inch.

PROBLEMS FOR PRACTICE

1. Design, according to Cooper's Specifications, the end stiffeners if the shear is 150 000 pounds, the distance back to back of angles is 6 feet 6½ inches, the web ⅜ inch thick, and the flange angle 6 by 6 by ½-inch. Use fillers.

2 Design the intermediate stiffeners for the girder of Problem 1, above, where the shear is equal to 75 000 pounds. Use crimped stiffener angles. Note that in this case the angles lie close against the web, no filler bars being used in between.

76. **The Web Splice.** Web splices are required because of the fact that wide plates cannot be rolled sufficiently long. Web splices should be as few as possible, and good practice demands that they be placed at the same points as the stiffener angle.

The tables in Carnegie Pocket Companion, p. 83, give the extreme length of plates which can be procured for any given width.

The length of plates for widths which are not given in these tables, should be taken equal to the length of the next plate given whose width is greater than that of the desired plate. From the first table it is seen that a 74 by $\frac{3}{8}$-inch plate can be rolled up to 400 inches, or 33 feet 4 inches, in length. Therefore, if the girder under consideration is spliced at the center, the web plates will be required to be $\frac{61 \text{ ft. } 9 \text{ in.}}{2} = 30$ feet $10\frac{1}{2}$ inches, which value does not exceed the 33 feet 4 inches as given above.

According to Articles 46 and 71 of the Specifications, a plate must be placed on each side of the web as shown in Fig. 159, and enough rivets placed in each side to take the total shear. The total thickness of both plates, and also their length, must be sufficient to stand the total shear, but must not be less than $\frac{3}{8}$ inch.

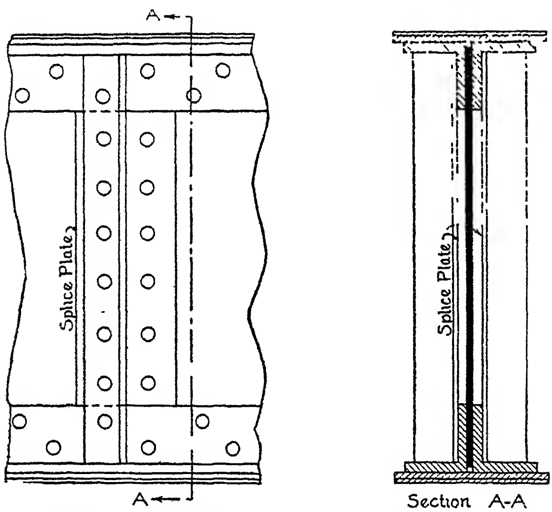


Fig 159. Splice Plates Placed on Each Side of Web.

The total shear at the center of the girder under consideration (see Fig. 134, p. 150) is 28 600 pounds. The area required in each of the two splice plates is $\frac{28\,600}{2 \times 9\,000} = 1.59$ square inches; and as their length is 62 25 inches, the thickness must be 1 59 ÷ 62.25 = 0.0255 inch, but they must be made $\frac{3}{8}$ inch thick according to the Specifications. The width should be somewhat greater than twice the width of the stiffener angle leg. This would make the width in this case about 10 inches.

The bearing governs the number of rivets required in this case, and there are 28 600÷4 920=5.81, say 6, shop rivets. More rivets than this will be required by practical considerations, as indicated by

Article 54 of the Specifications or in order to make the spacing in the stiffener angle the same as that in the other stiffeners. This detail is to be left to the draftsman, the required number only being put on the stress sheet

In case $\frac{1}{8}$ of the gross area of the web is considered as efficient flange area, then provision must be made in the splice for the bending moment which the web takes. A very economical and efficient splice is shown in Fig. 160. The horizontal plates take the stress due to the moment, and the vertical plates take the stress due to the shear.

The web equivalent is 3.47 square inches and the total moment is 1 615 000 pound-feet, which is composed of 275 000 pound-

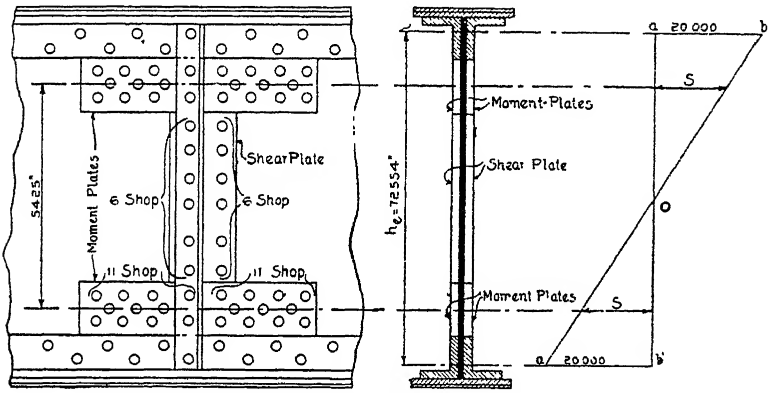


Fig. 160. Splice Consisting of Vertical and Horizontal Plates

feet due to dead load and 1 340 000 pound-feet due to live load. Therefore that proportion of the 3.47 which is taken up by the dead load is

$$\frac{275\ 000}{1\ 615\ 000} \times 3.47 = 0.59 \text{ square inch,}$$

and that proportion taken up by the live load is:

$$\frac{1\ 340\ 000}{1\ 615\ 000} \times 3.47 = 2.88 \text{ square inches}$$

The equivalent flange area is assumed to act at the center of gravity of the flange; and the bending moments equivalent to the above areas are, for dead load:

$$0.59 \times 20\ 000 \times 72.554 = 856\ 000 \text{ pound-inches,}$$

and for live load:

$$2.88 \times 10\ 000 \times 72.554 = 2\ 090\ 000 \text{ pound-inches.}$$

These bending moments must be taken up by the horizontal splice plates The stresses in these plates (see Fig. 160) are, for dead load:

$$\frac{856\ 000}{54\ 25} = 15\ 780 \text{ pounds,}$$

and for live load,

$$\frac{2\ 090\ 000}{54\ 25} = 38\ 500 \text{ pounds.}$$

While the unit-stresses are a maximum at the center of gravity of the flange and are those given by the Specifications, they decrease rapidly towards the center of the girder, being zero at the neutral axis of the entire section. The unit allowable stress at the center of the horizontal plates will not be so great as the maximum allowable, but will be proportional to the distance from center (see Fig. 160). The horizontal plates will be taken 8 inches in width. The unit-stresses are easily determined by means of the similar triangles *oab* and *oab′*. The dead-load stress is determined from the proportion:

$$\frac{S'}{20\ 000} = \frac{\frac{54.25}{2}}{\frac{72.25}{2}},$$

and is 14 950 pounds. For live load, the unit-stress will be one-half of this amount, or 7 475 pounds.

The area required for this plate is, for dead load, $\frac{15\ 780}{14\ 950} = 1.05$ square inches, and for live load $\frac{38\ 500}{7\ 475} = 5\ 16$ square inches, making a total of 6.21 square inches for both plates. Assuming two rivet-holes out of the section, the net width is $8 - 2\left(\frac{7}{8} + \frac{1}{8}\right) = 6$ inches; and the required thickness for one plate is $\frac{6\ 21}{2 \times 6} = 0.52$, say $\frac{9}{16}$ inch.

The joint will be weakest in bearing in the $\frac{3}{8}$-inch web. The number of rivets required is:

$$\frac{15\ 670 + 38\ 500}{4\ 920} = 11 \text{ shop rivets.}$$

The design of the shear plate is as follows: The shear is 28 600 pounds, and the required area is $\frac{28\ 600}{9\ 000} = 3.18$ square inches. As the length of the plate is $46\frac{1}{4}$ inches, the required thickness is $\frac{3.18}{2 \times 46.25}$

= 0 034 inch, but on account of the Specifications it cannot be less than $\frac{3}{8}$ inch thick. It will, however, be made $\frac{9}{16}$ inch thick, since it will then fill out even with the horizontal tension plates and no filler will be required. Bearing in the web plate decides the number of rivets, which is:

$$\frac{28\ 600}{4\ 920} = 6 \text{ shop rivets}$$

The width of this shear plate should be, as before, 10 inches. The same conditions limiting the spacing of the rivets apply here as in the case where the splice was designed for shear only. The length of the horizontal plates should be sufficient to get in all the rivets, and this is a detail which is left to the judgment of the draftsman.

PROBLEMS FOR PRACTICE

1 A plate-girder is 87 ft 9 in center to center of end bearings The dead-load moment is 9 125 000 pound-inches, and the live-load moment is 38 265 000 pound-inches, the total shear at the section being 202 700 pounds The web is 90 by $\frac{7}{16}$-inch, and the flange angles are 6 by 6 by $\frac{3}{4}$-inch Design the web splice when no part of the web is considered as taking bending moment

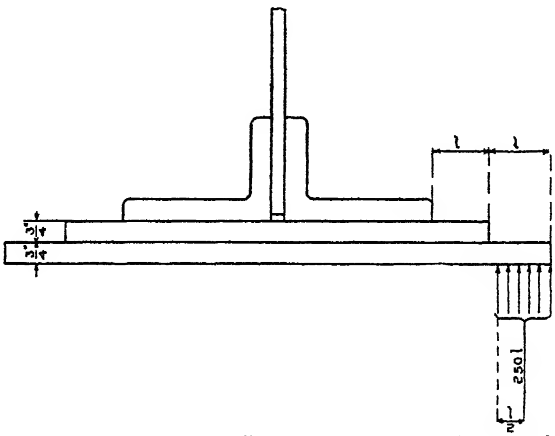


Fig 161 Proportions of Flange Angles, Bearing Plate, and Masonry Plate

2 For the girder of Problem 1, above, design the splice when $\frac{1}{8}$ of the gross area of the web is considered as effective flange area

77. The Bearings. Articles 113 to 119 of the Specifications should be carefully studied before proceeding; also Article 87. Article 70 should be referred to, and the remarks there made about bearings should be read. In case the length of bearing is such as to allow a simple $\frac{3}{4}$-inch plate, care must be taken that the bearing plate does not extend past the flange angles more than 2 inches, or that the masonry plate does not extend past the bearing plate over 2 inches. Reference to "Steel Construction," Part II, p. 130, to Fig. 161,

and to the discussion which follows, will explain the reason of this.

$$M = \frac{SI}{c} = 250 \times l \times \frac{l}{2},$$

$$I = \frac{bd^2}{12} = \frac{1 \times (\frac{3}{4})^2}{12},$$

$$c = \frac{d}{2} = \frac{\frac{3}{4}}{2} \qquad \therefore \frac{I}{c} = \frac{1 \times 9}{8 \times 12};$$

and as $s = 10\,000$,

$$250 \frac{l^2}{2} = \frac{10\,000 \times 9}{8 \times 12},$$

from which,

$$l = 2.76, \text{ say 2 inches}$$

In case it is desirable to have a simple masonry plate instead of a cast-steel pedestal, and to have the plate extend over the sides of the angles a distance greater than 2 inches, then some arrangement must be made for supporting the projecting portion. Fig. 162 shows one of the methods most commonly used. Notwithstanding the bracing of the gusset plates, the masonry plate is not adequately supported, the greater proportion of the stress coming upon the ends.

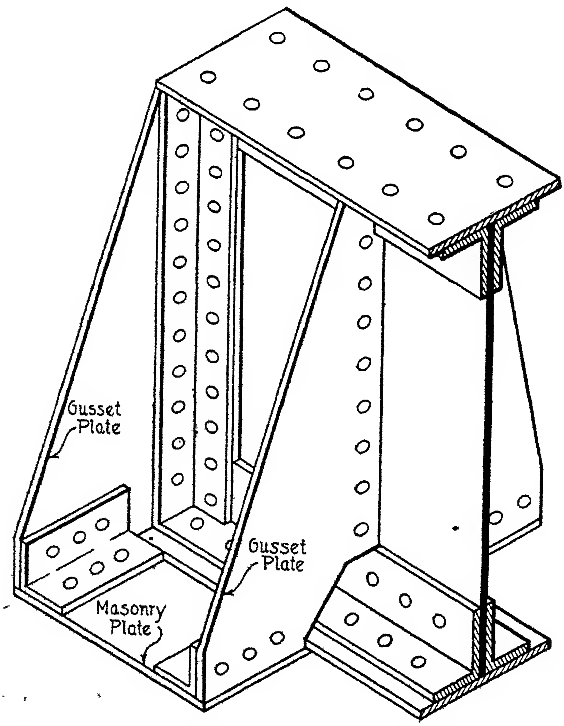


Fig 162 Arrangement where Masonry Plate is Used Instead of Cast-Steel Pedestal.

The disadvantage of having the masonry plate too long is plainly shown by Fig. 163. Here the girder is shown deflected under a live load, the rear end of the plate being tilted up and the greater part

of the pressure coming upon the forward end. The use of this style of plate is not to be recommended for spans over 40 feet.

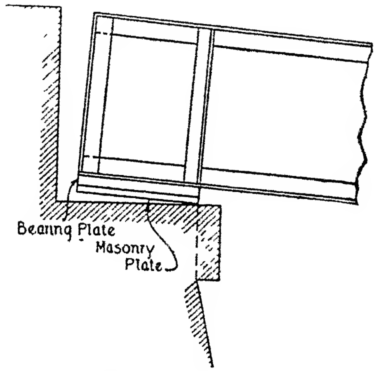


Fig 163 Effect of Having Masonry Plate Too Long

The design for the bearing of the girder under consideration will now be made. The total reaction of one girder must now be computed. This will be due to the weight of the steel in the girder, to the weight of the track, and to the reaction produced by the E 40 loading when wheel 2 is directly over the end support. This reaction is:

Weight of Steel, $\frac{(123.5 + 10 \times 61.75)\,61.75}{4}$	$=$	11 430 pounds
Weight of Track, $\frac{400}{2}\,(61.75 + 1.75)\,\frac{1}{2}$	$=$	6 350 "
Reaction Due to Engine Loading	$=$	99 700 "
Total	$=$	117 480 pounds.

The square inches of bearing surface required is $\frac{117\,480}{250} = 470$; and, as the length is 1 foot 9 inches, or 21 inches, the total width of the cast-steel pedestal will be $\frac{470}{21} = 22.4$, say 23 inches, or 1 foot 11 inches.

A bearing plate must be riveted to the lower flange where it rests upon the pedestal. The pedestal must be so constructed as to allow this bearing plate to set in it. Hand-holes should be provided in the casting in order to allow the bolts which connect the casting to the girder to be inserted. These bolts should be at least $\frac{3}{4}$ inch in diameter. Anchor bolts $\frac{7}{8}$ inch thick and at least 8 inches long should be provided and fox-bolted to the masonry. The thickness of the metal in all parts of the casting should be at least $1\frac{1}{4}$ inches. The details of the pedestal are given in Fig. 164, the length of the bearing being made 12 inches so as to allow one rivet to be driven in the flange angle in the space between the end stiffeners.

Allowance should be made for a variation of 150 degrees in tem-

perature. The coefficient of expansion for steel per unit of length is 0.0000065, and the amount of expansion for 150 degrees of temperature will be:

$$0.0000065 \times (61 \text{ ft } 9 \text{ in.}) \times 150 = 0.06 \text{ foot}$$

This is about $\frac{3}{4}$ inch, and therefore the holes in the flanges at one end of the girder should be made oblong and long enough to allow the

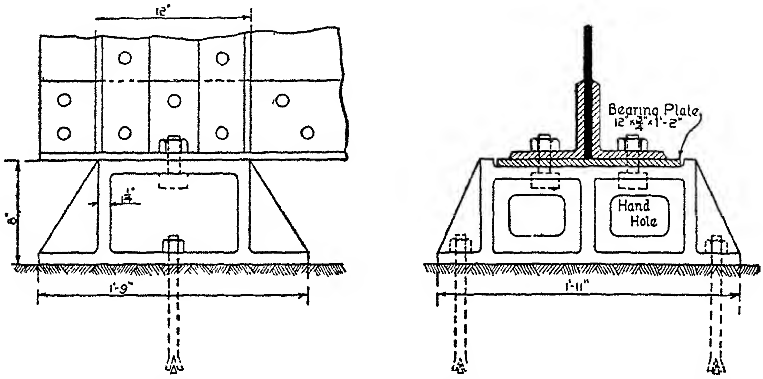


Fig. 164. Side and End Elevations Showing Construction of Pedestal and Connection to Bearing Plate

girder to move $\frac{3}{4}$ inch, or $\frac{3}{8}$ inch either backward or forward from a central position. In determining the length of this slotted hole (see Fig. 165), it must be noted that the $\frac{7}{8}$-inch bolt takes up part of this hole, and therefore its length should be $\frac{7}{8} + \frac{3}{4}$ = say $1\frac{3}{4}$ inches. The width of the hole should be sufficient to allow for any over-run in the diameter of the bolt. It should be at least $1\frac{1}{4}$ inches wide.

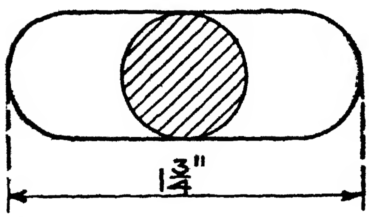


Fig. 165. Slotted Bolt-Hole in Flange at End of Girder to Allow for Contraction and Expansion Due to Temperature Changes.

PROBLEMS FOR PRACTICE

1. Determine the distance center to center of bearings, and the size of the masonry plate, for a plate-girder of 40-foot span under coping, the loading being E 40.

Ans. 41 ft. 4 in.; 350 square inches.

(Note—Interpolate values in Table I, Cooper, p. 30.)

2. If the girder span is 78 feet under coping, and the loading E 40, determine the maximum end reaction and the width of the masonry plate.

Ans. 147 130 pounds; $24\frac{1}{2}$ inches.

78. **The Stress Sheet.** Plate II (p. 172) shows the stress sheet for

the girder which has just been designed. It represents the best modern practice in that it gives in addition to the sizes of all the sections, the curves for the maximum shears and moments, the rivet-spacing curve, and the number of rivets required in the different parts of the structure This general form has been adopted by one of the largest bridge corporations in this country, and is to be recommended since it gives the draftsman all necessary data and thus prevents the loss of time by an inexperienced man in recomputing certain results The results just referred to are the shears, the moments, the rivet spacing, and the number of rivets required in the various parts Formerly it was not customary to give this information on the stress sheet, and the draftsman was therefore required to do all this computation which had previously been worked out by the designer but had not been placed on the stress sheet in available form. The laborious process of calculating girder sections and estimating costs may be avoided by using the tables in the Appendix, page 255 and following

DESIGN OF A THROUGH PRATT RAILWAY-SPAN

79. **The Masonry Plan.** The same remarks which are made in Article 67 apply here In this case the length of the masonry plate is usually determined by considerations relative to the number and length of the rollers in the bearing, and not by the bearing per square inch upon the masonry, the size of the plate as determined by the above considerations being usually much larger than if it had been determined by the unit bearing stress A preliminary design of the masonry plate is usually made in a manner similar to that done in the case of the plate-girder, or the length of the masonry plate may be approximately determined from the following:

Span	Length of Masonry Plate	
	Fixed End	Roller End
100 feet	23 inches	23 inches
125 "	26 "	26 "
150 "	28 "	28 "
175 "	31 "	31 "
200 "	35 "	35 "

The above masonry plates are for single-track bridges, with or without end floor-beams, the length being the same in either case.

80. **Determination of the Span.** The determination of the span is made in exactly the same manner as described in Article 68. Care should be taken, in case end floor-beams are not used, to allow for the pedestal stones, which are square stones resting directly upon the bridge seat, and upon the top of which rest the masonry plates of the stringers. Their height must, of course, be such as to keep the stringers level. In case these stones are used, their size must be determined; and if it is greater than that of the bearing or masonry plates, then their size determines the width of the bridge seat and the span center to center of bearings.

81. **The Ties.** In the design of the ties, as well as in all the design which follows, the Specifications of the American Railway

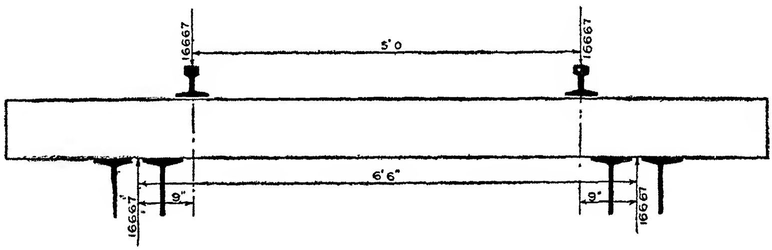


Fig. 166. Spacing of Stringers and Rails, and Position of Loads.

Engineering & Maintenance of Way Association will be followed. Whenever reference is made to these specifications, the number of the article will be enclosed in parentheses, as "(5)," which signifies that Article 5 of the Specifications is to be referred to.

The stringers in the bridge in question will be taken 6 ft. 6 in. center to center. The maximum loading (7) is such as to bring 8 333 pounds on one tie, and to this must be added 100 per cent for impact, making a total of 16 667 pounds. In order to illustrate the method of assuming the distance, center to center of rails, as 5 feet, that distance will be used in this case. The maximum moment will then be $9 \times 16\,667$ = say, 150 000 pound-inches. The size of the tie will be determined as in Article 71, the allowable unit-stress being 2 000 pounds per square inch (5). If a 7 by 9-inch tie is used, the unit-stress will be 1 590 pounds. If a 6 by 8-inch tie is used, the unit-stress will be 2 340 pounds. It is evident that a 7 by 9-inch tie must be used. See Fig. 166 for spacing of stringers and

rails, and for position of the loads. Note that, although impact is taken into account in this case, the size of the tie is the same as that designed for the plate-girder, although the unit allowable stress also differs.

82. **The Stringers.** The width, center to center of trusses, will be assumed as 17 feet, since this is sufficient to clear the clearance diagram in cases of single-track bridges of spans less than 250 feet.

The span which is to be designed in the following articles is a through-Pratt with 7 panels of 21 feet each, making a total span, center to center of bearings, of 147 feet 0 inches. See Plate III (p. 251). Rivets $\frac{7}{8}$ inch in diameter will be used throughout, except in channel flanges.

The length of the stringers end to end will be 21 feet, and according to Cooper's Specifications, p. 32, the maximum moment for the live load will be 226 000 pound-feet per rail. The coefficient of impact (9) will be $\left(\frac{300}{21 + 300}\right) = 0.935$, and therefore the moment due to impact will be $0.935 \times 226\,000 \times 12 = 2\,535\,000$ pound-inches, making a total of 5 247 000 pound-inches due to live load.

The section modulus for any particular beam is equal to the bending moment divided by the unit-stress, and this is equal to the moment of inertia divided by one-half the depth of the beam. This latter quantity is constant for any given beam, and for I-beams may be found in Carnegie Pocket Companion, pp. 142 and 143.

On account of the cheapness of I-beams, they will be used for stringers in this bridge; and sufficiently heavy shelf angles will be used to take up any distorting influences due to the eccentric connections which are unavoidable in this case. In case an I-beam had not been decided upon, the stringers would have been small plate-girders with a span of 21 feet and depth according to formulæ given. They would have been computed in exactly the same manner as a plate-girder span of 21 feet center to center of bearings.

Since the dead-load moment cannot be determined until the size of the stringer is known, an approximate design must first be made by using the live-load bending moment alone; and then, with the size determined in this manner, the extra section modulus required for the dead-load moment due to the weight of the beam and the track must be computed. If this extra section modulus, added to the one previously determined, is greater than that given by the beam in

question, a larger size beam must be used and a recomputation made.

The section modulus (17) required for live load only is $\frac{5\,247\,000}{16\,000}$ $= 327.9$. As this is too large for one beam, two beams will be used, thus giving a required section modulus of 164 for one beam. Two 24-inch 80-pound I-beams will be used, giving a total section modulus of $2 \times 174 = 348$.

Assuming the rails and ties to weigh 400 pounds per linear foot, the dead load per linear foot per stringer is $80 + \frac{400}{4} = 180$ pounds. The dead-load moment is therefore $\frac{180 \times 21 \times 21 \times 12}{8} = 119\,000$ pound-inches. This requires an additional section modulus of $\frac{119\,000}{16\,000} = 7.45$. This, added to the 164 as determined above, makes a total required section modulus of 171 25, which, being less than 174 (which is that for one I-beam), indicates that the above chosen beam is sufficient in strength, and it will therefore be used.

The number of rivets in the end connections will now be determined. The total end reaction for one I-beam is equal to the weight of one-half the beam, one-eighth the track in the panel, and one-half the maximum live-load reaction for one rail. These quantities are:

$\frac{1}{2}$ Live-Load Reaction $= \frac{51\,400}{2}$	$= 25\,700$ pounds
Impact $= 25\,700 \times 0\ 935$.	$= 24\,030$ "
Weight of Track $= \frac{400 \times 21}{8}$.	$= 1\,050$ "
Weight of Stringer $= \frac{21}{2} \times 80$.	$= 840$ "
	Total $= 51\,620$ pounds.

The coefficient of impact is that for a loaded length of 21 feet.

From p. 40, Carnegie Pocket Companion, column 6, it is seen that the longest connection angle which can be used with a 24-inch I-beam is $20\frac{3}{4}$, say 20 inches. In this case the thickness of the connection angles must be $\frac{51\,620}{10\,000 \times 20} = 0.26$ inch; but according to (38), $\frac{3}{8}$ inch will be used. The angles chosen will be 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch. The 6-inch leg will be placed against the web of the floor-beam in order to allow for sufficient room for rivets to be driven.

The rivets will tend to shear off at places between the webs of the stringer and floor-beam and the connection angles. They will also tend to tear out of the web of the stringer and out of the web of the floor-beam. As the thickness of this latter is not known, the determination of the rivets for this condition will be made under the next article. The bearing value of a $\frac{7}{8}$-inch rivet in a $\frac{1}{2}$-inch plate (19) is $\frac{7}{8} \times \frac{1}{2} \times 24\,000 = 10\,500$ pounds, and therefore $\frac{51\,100}{10\,500} = 5$ shop rivets are required in bearing in the web of the stringer. The value of a $\frac{7}{8}$-inch shop rivet in single shear (18) is $0.6013 \times 12\,000 = 7\,220$ pounds, and the number of rivets required to prevent shearing between the connection angles and the webs is $\frac{51\,100}{7\,220} = 7$ shop rivets. The value of a $\frac{7}{8}$-inch field rivet in single shear (18) is $0.6013 \times 10\,000 = 6\,013$ pounds, and therefore $\frac{51\,100}{6\,013} = 9$ field rivets are required to connect the connection angle to the web of the floor-beam. As mentioned above, the number of rivets in bearing in the web of the floor-beam will be determined in the next article; and if the number required for bearing is greater than 9, then that number must be used instead of 9. Fig. 167 shows the connection of the stringer to the floor-beam web, and also the number of rivets as determined above, in their proper positions. The distance between the webs of the stringers must be such as to prevent their flanges from touching at the top.

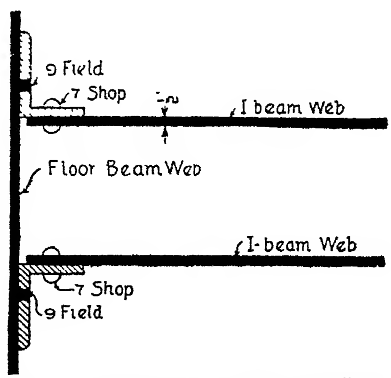


Fig. 167. Connection of Stringer to Floor-Beam Web, also Number of Rivets.

The stringers should be connected at the bottom by a system of lateral bracing of the Warren type. The size of these angles cannot be determined by theoretical considerations, but is usually chosen to be $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in. See Plate II (p. 172) for the general arrangement of this bracing.

83. **The Floor-Beams.** All floor-beams should be of sufficient depth to allow the use of small-legged connection angles at the ends

where they join the end-posts. The thickness of the web should also be greater than that which is theoretically computed, in order that sufficient bearing may be given so that the rivets for the stringer connections will not require the stringers to be of too great a depth. The depth of the floor-beam will, of course, vary somewhat with the length of the panel and with the loading, but should not be less than 36 inches in any case. A considerable variation in the depth will not affect the weight of the floor-beam or the bridge to an appreciable extent. A good plan is not to exceed a depth of 5 feet, with panel lengths of 25 feet and E 50 loading. In this bridge the depth of all intermediate floor-beams will be taken as 48 inches. It is good practice not to consider $\frac{1}{8}$ the web area when designing flanges of floor-beams and stringers, and the design here given does not consider the web as taking any bending moment.

The design of an intermediate floor-beam will now be made. The loads for which it is designed are the floor-beam reaction due to the live load (see Cooper, p. 32), the floor-beam reaction due to impact, the dead weight of the stringers and track, and the weight of the beam itself. The latter weight is distributed uniformly over the entire length of the beam, and the other loads act as concentrated loads spaced 6 feet 6 inches apart at equal distances from the center. The computation of the concentrated loads is as follows:

Live Load = . .	68 000 pounds
Impact $= 68\,000 \times \left(\frac{300}{37 + 300}\right)$.	60 500 "
Dead Load of Stringer $= 2\left(\frac{21 \times 2 \times 80}{2}\right)$	3 320 "
Dead Load of Track $= \frac{400}{2} \times 21$.	4 200 "
	Total = 136 020 pounds.

The moment at points under the loads (see Fig. 168) is 136 020 $\times$ 5.25 $\times$ 12 = 8 575 000 pound-inches. This is due to the concentrated loads only. The weight of one floor-beam may be approximately determined by the same formula as used to determine the weight of plate-girder spans; only, in place of the length of the span, the length of the panel must be substituted. The total weight of the above floor-beam, then, is:

$$W = 0.45 \times (123.5 + 10 \times 21) \times 21 = 3\,160 \text{ pounds.}$$

The dead-load moment at the center due to this weight will be:

$$\frac{W\,l}{8} = \frac{3\,160 \times 17 \times 12}{8} = 80\,700 \text{ pound-inches}$$

making a total moment at the center of the beam of 8 575 000 + 80 700

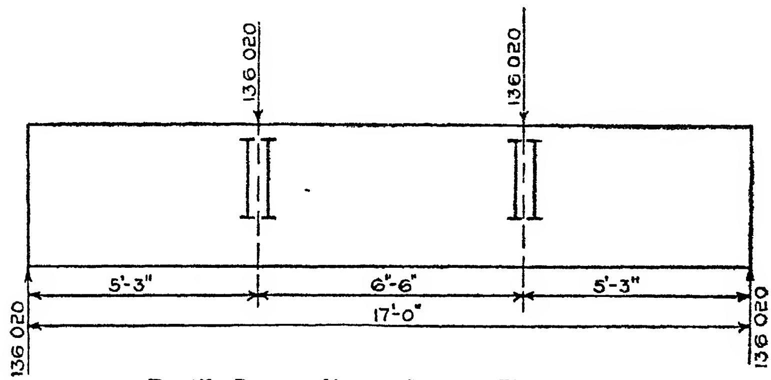


Fig 168 Diagram Showing Loads on Floor-Beam

= 8 655 700 pound-inches. Note that the dead-load moment at the center of the beam is added to the concentrated-load moment at the point where the concentrated load is applied. This will give the total moment at the center of the beam as shown by Fig. 169, since the concentrated-load moment is constant between the points of application. The end shear is readily computed to be 136 020 + 1 580 = 137 600 pounds. The curves of moments and shears are shown in Fig. 169.

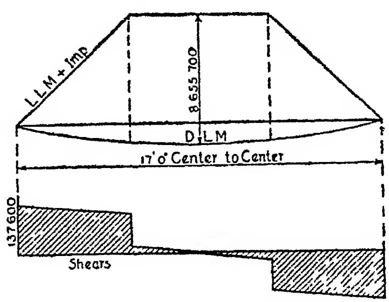


Fig 169 Shear and Moment Diagram

The total depth of the floor-beam, back to back of angles, is $48\frac{1}{4}$ inches; and the effective depth will, for approximate computation of the flange area, be taken as somewhat less, say $44\frac{1}{2}$ inches, since the flange angles will probably be 6 by 6-inch and the center of gravity of most of these angles lies about $1\frac{3}{4}$ inches from the back. The approximate flange stress is $\dfrac{8\,655\,700}{44.5} = 194\,500$ pounds, and the required net area (17) will be $\dfrac{194\,500}{16\,000}$

= 12.2 square inches. In assuming the size of the angle, it is to be remembered that when, as in this case, no cover-plates are used, no rivet-holes will be taken out of the top flange, and only one rivet-hole will be taken out of the vertical flange.

Two 6 by 6 by $\frac{5}{8}$-inch angles give a gross area of 7 11 square inches each, and a net section of 7 11 − 0 625 = 6.485 square inches each, or 12 97 square inches net for both. As this is near the required area, these angles will be taken; and a recomputation will now be made with the actual effective depth, in order to see if sufficient variation in the areas occurs to require another angle to be taken. The actual effective depth is now $48\frac{1}{4} - 2 \times 1\ 73 = 44.79$ inches; and making computations with this, it will be found that a net area of 12.10 square inches is required. As this is practically the same as was determined at first, no change will be made in the size of the angle.

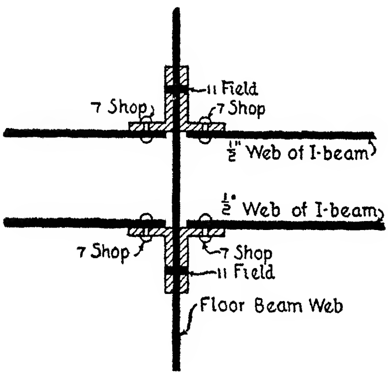


Fig. 170 Calculation of Number of Rivets through Connection Angles of Stringer and Floor-Beam Web

The web is to be designed for a total shear of 137 600 pounds. The required area (18) is $\frac{137\,600}{10\,000} = 13.76$ square inches, and the required thickness is $\frac{13.76}{48} = 0.286$ inch; but on account of the Specifications (38), $\frac{3}{8}$ inch must be used. The web will accordingly be 48 by $\frac{3}{8}$-inch.

The determination of the number of rivets which go through the connection angle of the stringer and the web of the floor-beam can now be made. The value of a $\frac{7}{8}$-inch field rivet in bearing in a $\frac{3}{8}$-inch plate (19) is $\frac{7}{8} \times \frac{3}{8} \times 20\,000 = 6\,560$ pounds, and the total number required in one connection angle will be $\frac{136\,020}{2 \times 6\,560} = 11$ field rivets (see Fig. 170).

The pitch of the rivets in the flange can in this case be determined by the use of the formula:

$$s = \frac{vh_e}{V}.$$

Since the flange is of the same cross-section throughout, the value of the effective depth will not change, and it can therefore be used in the above equation instead of considering the value of the distance between rivet lines. The shear being practically constant from the connection of the stringers to the end of the floor-beams, the rivet spacing will be constant in this distance. It will be:

$$s = \frac{7\,880 \times 41.79}{137\,600} = 2.51 \text{ inches},$$

the value of a $\frac{7}{8}$-inch shop rivet in bearing in the $\frac{3}{8}$-inch web being $\frac{7}{8} \times \frac{3}{8} \times 24\,000 = 7\,880$ pounds. This is seen to be less than $2\frac{5}{8}$ inches; but, as the angles have 6-inch legs, this spacing can be used in a horizontal direction; and the distance from center to center of rivets, which will be placed in rows on two gauge lines, will still be greater than $2\frac{5}{8}$ inches

The shear between the stringer connections is practically zero, and therefore the spacing will be very large. Being over 6 inches, it will be subject to (39).

The connection angles at the ends of the floor-beams will be taken as 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, the 6-inch legs being against the web of the floor-beam. The other legs are chosen small in order that they may fit into the channels which will very likely be required for the posts; and according to column 6, p. 54, Carnegie Pocket Companion, only $8\frac{1}{4}$ inches is available for this purpose. This $8\frac{1}{4}$ inches is taken from a 10-inch channel, since this is the smallest channel that can be used which will give room for connection and yet be in accordance with the Specifications. This is due to the fact that its web (38) is greater than $\frac{3}{8}$ inch. The rivets which connect the end angles to the floor-beam web are shop rivets, and those which connect the end angles to the posts are field rivets. Since the size of the post is not known, the thickness of its metal, of course, cannot be used, and therefore the number of rivets required in bearing in the post cannot be determined at this time.

The number of shop rivets required through the end angles and the floor-beam web is governed by the bearing of the rivets in the $\frac{3}{8}$-inch web of the floor-beam. The value of a $\frac{7}{8}$-inch shop rivet in bearing in a $\frac{3}{8}$-inch web (19), as has just been computed, is equal to 7 880 pounds, and the number of rivets required is $\frac{137\,600}{7\,880} = 18$.

The number of field rivets required in single shear to connect the end angles with the posts is $\frac{137\,600}{6\,013} = 23$. An even number of rivets will, of course, have to be used, one-half going into each of the $3\frac{1}{2}$-inch legs. See Fig 171 for the position and the number of rivets. It must be remembered that more than these numbers may be used by the draftsman on account of rivet spacing which may be required by conditions other than those of design.

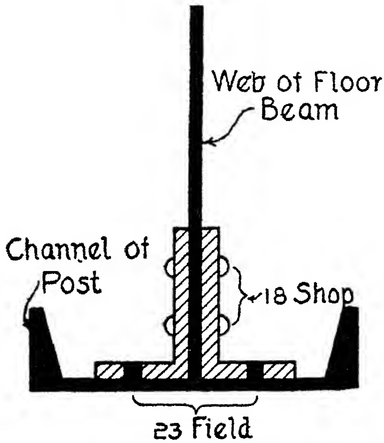


Fig. 171 Position and Number of Rivets to Connect End Angles with Posts.

The design of the end floor-beam is somewhat different from that of the intermediate floor-beams in that the load which comes upon it is considerably lighter, since this floor-beam takes the dead load of only one-half the panel and the live load due to the *maximum end reaction* for a stringer length instead of the floor-beam reaction for the stringer length (see Cooper, p. 32).

The maximum end shear is computed as follows:

End Shear for 21-foot Span .	51 400	pounds.
Impact $= 51\,400 \times \left(\frac{300}{21 + 300}\right)$	48 000	"
Dead Load of Stringers $= \frac{80 \times 2 \times 21}{2}$	1 680	"
Dead Load of Track $= \frac{400 \times 21}{2 \times 2}$	2 100	"
Total	103 180	pounds.

The maximum moment due to the above load is $103\,180 \times 5.25 \times 12 = 6\,500\,000$ pound-inches. The weight of the beam may be assumed as 3 160 pounds. This is the same as was computed for the intermediate floor-beam, but will be used for this beam, since the size of the web will be the same as in the others; and, although the flange area will be less, the end connections will be somewhat heavier owing to the connection of the beam to the end-post and the roller bearing,

and this additional weight will cause the total weight of the end floor-beam to be about the same as that of the intermediate ones. The total moment at the center will then be 6 500 000 + 80 700 = 6 580 700 pound-inches.

The depth of the end floor-beam will be somewhat greater than the depth of the intermediate floor-beams. This is due to the fact that it extends downward a greater distance, resting upon the bearing plate, which comes directly upon the top of the rollers. The exact depth cannot, of course, be determined until after the roller bearings are designed; but it may be safely assumed as four or five inches deeper than the intermediate floor-beams, and in case this is not enough, the draftsman can easily fill in the remaining distance with filler plates, as this distance will not be very great. In case this depth is too great, the flange angles may be bent upward at the end, or a re-design may be made.

The depth will be assumed as 52 inches in this case. The effective depth will be assumed as 48 inches, and this gives an approximate flange stress of

$$\frac{6\,580\,700}{48} = 137\,000 \text{ pounds,}$$

and an approximate net flange area required of

$$\frac{137\,000}{16\,000} = 8.57 \text{ square inches}$$

A 6 by 6 by $\frac{7}{16}$-inch angle gives a gross area of 5.06 square inches, and a net area of $5.06 - (\frac{7}{8} + \frac{1}{8})\ \frac{7}{16} = 4.62$ square inches. A recomputation with the true effective depth requires 8.42 square inches net. Two of these angles give 9.24 square inches; and as this coincides very closely with the required area, it will be used. The size of the web plate is 52 by $\frac{3}{8}$-inch.

The pitch or spacing of rivets in the flanges is:

$$\frac{7\,880 \times 48}{104\,760} = 3.62 \text{ inches}$$

The maximum end shear as above computed is taken by two stringers; and therefore the number of rivets required in bearing to form the connection between the stringers, connection angles, and the floor-beam web is, for each angle:

$$\frac{104\,760}{2 \times 6\,560} = 8 \text{ field rivets}$$

The value 6 560 in the above equation is the value of a $\frac{7}{8}$-inch field rivet in bearing in the $\frac{3}{8}$-inch web.

The number of rivets required in the end angles on the floor-beam is:

$$\frac{104\ 760}{7\ 880} = 14 \text{ shop rivets}$$

These rivets go through the web of the floor-beam. The connection of the floor-beam to the end-post is made by means of field rivets and a large gusset plate. This gusset plate is usually $\frac{3}{8}$ inch in thickness. The number of rivets through the end connection angles and this gusset plate is governed by single shear, since the rivets will shear off between the angles and the gusset plate before they will tear out of the gusset plate, as the value of a rivet is greater in bearing than in shear. The number required is:

$$\frac{104\ 760}{6\ 013} = 18 \text{ field rivets}$$

The general arrangement of the intermediate floor-beams is shown in Fig. 172. The ends of the lower flange are bent up as shown, in order to allow the I-bar heads or any other section of the lower chord to have clearance. This makes it necessary for the floor-beam web to be spliced at the ends, as shown. The distance which this plate should extend above the floor-beam proper depends upon the distance which the lower chord is bent up. In any case the length of the connection on the post should be at least equal to the depth of the floor-beam. Two splice plates, one on either side of the web, are placed here in a manner similar to that of a splice as designed in the plate-girder when shear only was considered. Here shear only is considered, and the number of rivets which must be on each side of the splice will be:

$$\frac{137\ 640}{7\ 880} = 18 \text{ shop rivets.}$$

The 7 880 which occurs in the above equation is the value of a $\frac{7}{8}$-inch rivet in bearing in a $\frac{3}{8}$-inch plate (19). Inspection of Plate II (p. 172) will make this design clearer. Plate II also shows the shape of the end floor-beams.

The small shelf angle shown in Fig. 172 should have sufficient rivets to prevent any twist of the stringers due to their being connected on one side of their web only. This number is a matter of

judgment. Experience seems to indicate that enough rivets to take up one-third of the total reaction of the stringers will be sufficient. This will require shop rivets, and the number will be

$$\frac{103\ 180}{3 \times 7\ 220} = 5 \text{ shop rivets in single shear}$$

84 **The Tension Members.** Tension members usually consist of long, thin, flat plates with circular heads forged upon their ends

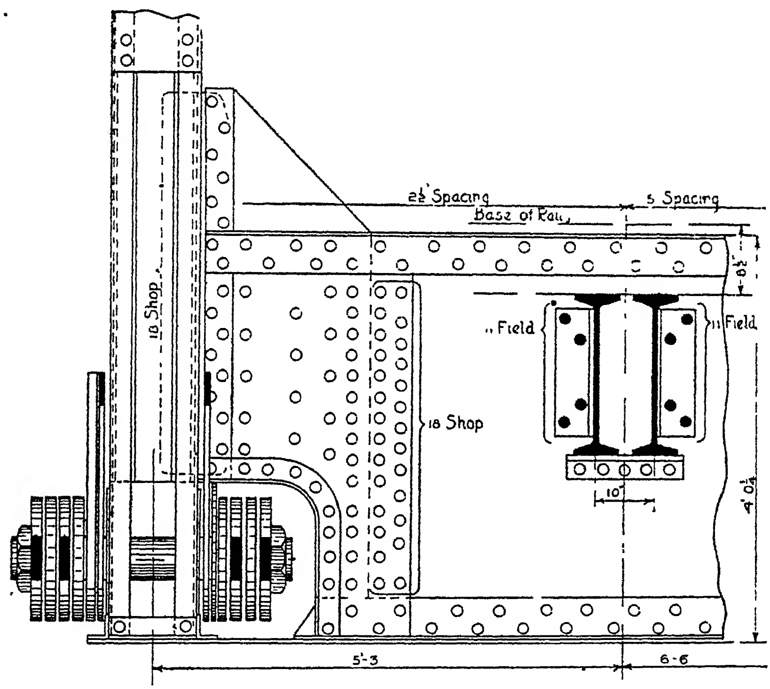


Fig 172 General Arrangement of Rivets, Splices, Connections, etc , for Intermediate Floor-Beams.

These circular heads have holes punched through their centers and then very carefully bored. Through these holes are run cylindrical bars of steel called *pins*. These pins connect them with other members of the truss. See Carnegie Pocket Companion, p. 118, for I-bars. The I-bars given are standard I-bars; and while departure from these widths and *minimum* thicknesses may be made, it may be done only at great cost to the purchaser. Note that there are no standard 9-inch I-bars. The thicknesses given are the *minimum* thicknesses

for that width of bar, and do not indicate that thicker bars of that width cannot be obtained; but on the contrary thicker bars of that width can be obtained, and this should be done, the minimum thickness as given in the table being avoided if possible.

It has been found that bars which have a ratio of thickness to width of about one-sixth give good service and are easy to forge. This relation gives us a rough guide which will enable us to determine the approximate width and thickness of any bar of a given area. Once the approximate dimensions are determined, the actual dimensions can be chosen from the market sizes of the material (see Carnegie Pocket Companion, p. 118).

An expression for the approximate depth of the bar will now be derived by using the above relation.

Let A = Area of bar, in square inches;
d = Width of bar, in inches,
t = Thickness of bar, in inches.

Then,

$$td = A;$$

also,

$$t = \frac{1}{6}d.$$

Substituting the value of t in the expression for A, there results:

$$d = \sqrt{6.1}$$

The stresses in all the members in the truss under consideration are computed by the method described in Part I, and are placed on the stress sheet, Plate III (p. 251). In the succeeding design, the student should obtain his stresses from Plate III without his attention being again called to the matter.

Table XXIV gives the tension members and their dead-load, live-load, impact, total, and unit stresses (15), together with the required area, the number of bars, the approximate depth of bars, and the final sizes used.

The first seven columns in Table XXIV are self-explanatory. The number of bars to be used in any particular case is a matter of judgment. One fast rule is that *an even number of bars should always be used, except in the case of counters, where one is permissible.* This is due to the fact that the placing of one of the main members in the center of the pin would create a large moment, and

TABLE XXIV

Tension Members

Member	Stresses (in pounds)					Required Area*	No. of Bars	Area of One Bar	d 1 64	Section Used	Area of Section Used
	Dead-Load	Live-Load	Impact	Total	Unit						
L_0L_2	11 000	111 700	78 500	231 200	16 000	16 61	.				
L_1U_1	13 000	68 000	60 500	141 600	16 000	8 85	.				
L_2U_1	47 900	113 900	103 300	295 100	16 000	18 41	2	9.22	7.41	7 × 1$\frac{3}{8}$	9 61
L_2U_3	−24 000	26 800	23 100	25 900	16 000	1.62	1	1 62	. . .	1$\frac{3}{8}$ × 1$\frac{3}{8}$	1.93
L_3U_2	24 000	95 200	72 600	191 800	16 000	11.98	2	5 99	6 00	6 × 1	6 00
L_2L_4	68 400	183 700	126 100	378 200	16 000	23 60	1	5 90	5 95	6 × 1	6 00
L_3L_4	82 000	218 100	149 100	419 500	16 000	28 20	1	7 05	6 50	6 × 1$\frac{1}{4}$	7 50
L_4U_4		51 200	11 100	98 300	16 000	6 14	2	3 07	1 30	4 × $\frac{7}{8}$	3 50

* Note.—All areas in square inches, and all dimensions in inches

Observe that the above table is not completely filled out with respect to the first two members given. This is on account of the requirements of the Specifications (80).

therefore an excessively large pin would be required, and accordingly a very large head on the I-bar in proportion to its width—all of which are very undesirable and costly. In general the number of I-bars should be as small as possible, and they should be so chosen that the widths of the chord members *increase* from the ends toward the center of the truss, and the widths of the diagonals *decrease* from the ends toward the center of the truss.

The area of one bar is obtained by dividing the total stress by the number of bars and also by the allowable unit-stress. Thus, for the member L_2U_1, for example, the required area of one bar is:

$$\frac{295\,100}{2 \times 16\,000} = 9\ 22$$

square inches

The approximate depth of this bar is determined by taking the square root of 6 times the area as above determined. It is:

$$d = \sqrt{6 \times 9.22} = 7.44 \text{ inches.}$$

As this is nearer 7 than 8 inches, a 7-inch bar will be chosen; and consulting the Carnegie Pocket Companion, for an area which will be equal to or in excess of 9.22, it is found that a $1\frac{3}{8}$-inch bar satisfies this condition, and therefore the section of this member consists of two bars 7 by $1\frac{3}{8}$ inches.

According to (80), the first two sections for the lower chord are to be made of built-up members. This requires that instead of I-bars they are to be made of angles and plates, or, in case the stress is light, of channels. The depth of the section is limited by the size of the greatest I-bar head. As the diameter of the I-bar head depends upon the size of the pin, it cannot of course be determined accurately before the pin is designed. It is customary to assume the largest head, and to design the section so as to clear this. The size of the largest head for bars of given width is given in the Carnegie Pocket Companion, p. 118.

The design of the member L_0L_2 will depend upon the size of the largest head of the 7-inch I-bar of the member U_1L_2. This is $17\frac{1}{2}$ inches; and in order that the head may have some clearance, it will be necessary to add $\frac{1}{2}$ inch to the top and the bottom, making a total of $18\frac{1}{2}$ inches. Since the flange angle, as in the case of plate-girders, will extend over the plate about $\frac{1}{4}$ inch, the plate itself may be 18 inches wide and still give sufficient clearance.

The total stress is 234 200 pounds, and the allowable unit-stress (15) is 16 000 pounds per square inch. The required net area, then, is:

$$\frac{234\,200}{16\,000} = 14.64 \text{ square inches.}$$

According to the Specifications, the thickness of the plate cannot be less than $\frac{3}{8}$ inch. The gross area of two 18 by $\frac{3}{8}$-inch plates is 13.5 square inches, and the gross area of four $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angles, which are assumed to be sufficient, is 9.92 square inches, thus making a total gross area of 23.42 square inches. If 5 rivet-holes are assumed to be taken out of each web, and one rivet-hole taken out of each angle, this will require a certain number of square inches to be

deducted from the section, and this is computed as follows:

Out of webs, $2 \times 5 (\frac{7}{8} - \frac{1}{8}) \times \frac{3}{8}$ = 3 75 sq in
Out of angles, $4 (\frac{7}{8} - \frac{1}{8}) \times \frac{3}{8}$ = 1 50 " "

Total = 5 25 sq. in

The net area of the section is now determined to be 23.42 − 5 25 = 18 17 square inches. This is somewhat greater than the required net area, but must be used, for according to (39), these are the smallest and thinnest angles that may be used.

Figs. 173 and 174 show the cross-section and the general detail at L_2. The width of the member cannot be determined until after the section of the end-post is computed, since it must fit inside of the end-post, the horizontal legs of the angles being cut off to allow this. The end-post, Article 87, is $14\frac{1}{2}$ inches inside. If it is assumed that all the pin-plates on the end-post are placed on the outside, and all those at L_0 on L_0L_2 are on the inside, then the width of L_0L_2, back to back of plates, must be $14 - (2 \times \frac{1}{2} + 2 \times \frac{3}{8}) = 12\frac{1}{4}$ inches or less, $\frac{1}{4}$-inch clearance being allowed between the sides of the angles and the web plates of the end-post (see Fig. 173).

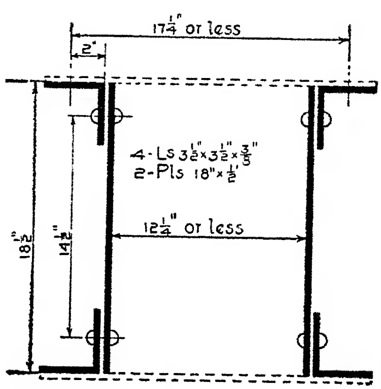


Fig 173 Cross-Section Showing Construction of Lower Chord Member

The total net section through the pin-hole at L_2 (26) must be $1\frac{1}{4} \times 18.17 = 22$ 7 square inches, or 11 35 square inches for one side. The plate which is to increase the section must be on the outside, since the intermediate post U_2L_2 and the two I-bars of member U_1L_2 must go inside. The gross width of this plate is $11\frac{1}{4}$ inches (see Fig. 174), and the net width is $2w = 11\frac{1}{4} - 5 = 6\frac{1}{4}$ inches. The net area through the pin is:

Two $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in. angles = 4.96 square inches.
One 18 by $\frac{3}{8}$-in plate = $(18-5) \times \frac{3}{8}$ = 4 88

Total = 9 84 square inches.

Since this is less than the 11.35 required, an extra plate will be required, or the $18 \times \frac{3}{8}$-inch plate increased to $\frac{1}{2}$ inch. This latter will be done.

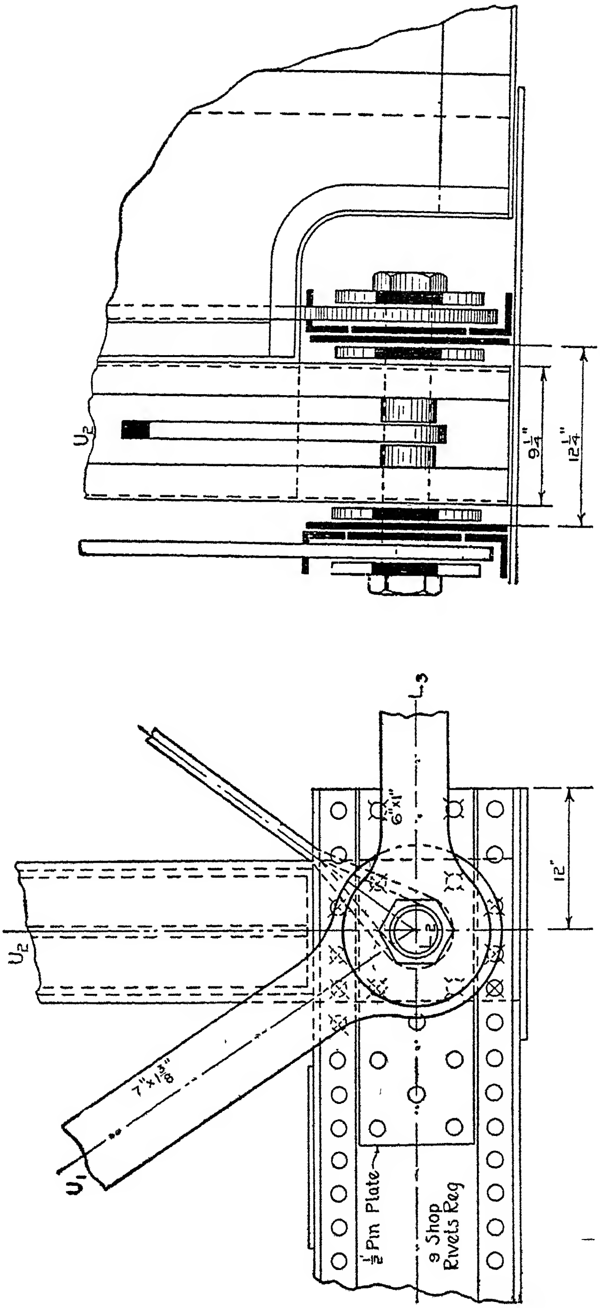


Fig. 174. Side Elevation and Transverse Section Showing General Details of Construction at Panel Point in Lower Chord.

Sufficient bearing area must be provided at this point. The total stress is 234 200 pounds, the total bearing area required is $\frac{234\,200}{24\,000} = 9.76$ square inches, and the total thickness for one side is $\frac{9.76}{2 \times 5} = 0.976$ inches. Since the thickness of the web is $\frac{1}{2}$ inch, the pin-plates must be $0.976 - 0.50 = 0.476$ inch (say $\frac{1}{2}$ inch) thick. A $\frac{1}{2}$-inch pin-plate must be used, and as the total thickness of the bearing area is now 1.00 inch, this pin-plate will take $\frac{0.50}{1.00} \times \frac{234\,200}{2} = 58\,550$ pounds. The joint is weakest in shear, and will therefore require $\frac{58\,550}{7\,220} = 8 -$ (say 9) shop rivets.

In case it is necessary to put the member U_1L_2 on the outer side of L_0L_2, then the outer legs of the upper angles must be cut off to allow U_1L_2 to pass. This will decrease the section by an amount $(3\frac{1}{2} - \frac{3}{8}) \times \frac{3}{8} = 1.20$ square inches. Considering the pin-plate, which is $(18\frac{1}{2} - 2 \times 3\frac{1}{2}) - \frac{1}{4} = 11\frac{1}{4}$ inches, the $\frac{1}{4}$ inch being allowed for clearance between the edges of its flange angles, the total net section through the pin-hole on one side will be:

One Angle	$3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in	= 2.48	square	inches
One Cut Angle	$3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in	= 1.28	"	"
One Web	$(18-5)\ \frac{1}{2}$ sq in	= 6.50	"	"
One Pin-Plate	$(11\frac{1}{4}-5)\ \frac{1}{2}$ sq in	= 3.13	"	"
	Total	= 13.39	square	inches

This is greater than 11.35 as required, and is therefore safe.

The distance from the center of the pin to the end must now be determined (28). The total net section of the body of the member is 18.17 square inches, or 9.09 square inches for one side, and the thickness of the web and the pin-plate is 1 inch. The distance from the pin to the end of the member is then $\frac{9.09}{1} = 9\frac{1}{8}$ inches, and the distance to the center of the pin is $9\frac{1}{8} + \frac{5}{2} = 11\frac{5}{8}$, say 12 inches (see Fig. 174). Rivets should be countersunk where necessary to prevent interference with I-bars. For signs, see Cambria, p. 291 and Carnegie Pocket Companion, page 212.

At point L_0 of this member, the pin is $6\frac{1}{4}$ inches in diameter, and, as previously mentioned, the legs of the angles are cut (see Fig. 175). The total bearing area required for one side is $\frac{9.76}{2} = 4.88$, and the required thickness is $\frac{4.88}{6.25} = 0.781$ inch. Subtracting the thickness of the $\frac{1}{2}$-inch web from this gives 0 281 inch. A pin-plate $\frac{3}{8}$ inch thick must be used.

The net area through the pin (28) must be 11.35 square inches.

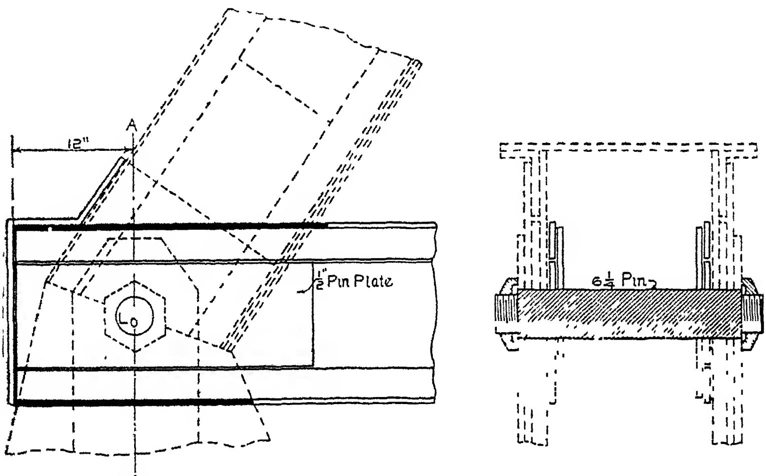


Fig. 175 Elevation and Section Showing Pin Connection at End of Truss.

This net area, remembering that the angle legs are cut and therefore their area is that of a bar $3\frac{1}{2}$ by $\frac{3}{8}$-inch, computed for one side, is as follows:

Two Angles, legs cut, $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in.	=	2 62	square inches
One Web $18 \times \frac{1}{2} - 6\frac{1}{4} \times \frac{1}{2}$ sq in.	=	5 88	" "
One Pin-Plate $11\frac{1}{4} \times \frac{3}{8} - 6\frac{1}{4} \times \frac{3}{8}$ sq. in.	=	1 87	" "
Total		10 37	square inches.

This shows the section to be deficient, and the thickness of the pin-plate must be made $\frac{1}{2}$ inch. This gives a net area through the pin of 11 62 square inches.

The distance between rivet lines (see Fig. 173) is $17\frac{1}{4}$ inches, and (46) the tie-plates must be $17\frac{1}{4}$ (say 18) inches long, and their thickness $\frac{17.25}{50} = 0.346$ inch (say $\frac{3}{8}$ inch).

The lattice bars (47) must be $2\frac{1}{2}$ inches wide, and (49) must be double. From (47) and Table XXV, page 219, the thickness must be $\frac{7}{16}$ inch, the distance c being $17.25 \times$ secant $45° = 2$ ft. $0\frac{7}{16}$ in.

The design of the hip vertical $= U_1L_1$ is also made in accordance with (82) of the Specifications. It will be assumed that the section consists of one 8 by $\frac{3}{8}$-inch plate, and four $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angles, since this is the lightest section that may be used according to the Specifications, the 8-inch plate being chosen as it gives some clearance between the inner edges of the legs of the angles.

The total stress in the member is 141 600 pounds, and the unit-stress is 16 000 pounds per square inch, thus requiring a net area of 8.85 square inches. The plate gives a net area of 2.25 square inches, and the four angles give a net area of 6.92 square inches, making a total of 9.17 square inches, two rivet-holes being taken out of each angle, and two out of the web, at any particular section. The net area is somewhat greater than that required, but must be used, as this is the minimum section allowed by the Specifications. Fig. 176 shows a cross-section of this member as above determined.

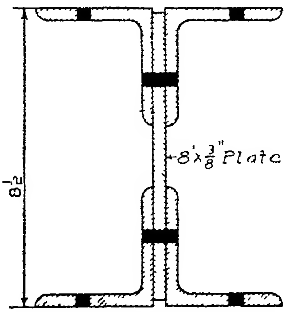


Fig. 176. Cross-Section of Hip Vertical

This member will be connected to the upper chord and end-post by means of a pin which is $6\frac{1}{4}$ inches in diameter, the diameter of the pin being determined later. The total stress is 141 600 pounds, and this will be taken by two plates, one on either side of the member. The net section of the member is 9.17 square inches, and the section through the pin (28) must be 25 per cent in excess of this, making a total of 11.46 square inches, or 5.73 square inches for each plate. The total width of these plates will be taken as 12 inches, and this (see Fig. 177) will make the required thickness:

$$\frac{5.73}{2c} = \frac{5.73}{12.00 - 6.95} = 1.00 \text{ inch.}$$

Fig. 177 shows the details of these pin-plates. Since the above thickness is too great to be punched in one single piece, the above thickness will be made up of two plates, each $\frac{1}{2}$ inch thick. The area at section A-A must be equal to that of the body of the bar. It

is $12.00 \times \frac{1}{2} = 6.00$ square inches for one side, or 12.00 square inches for both sides. As this is greater than the 8.85 square inches as above computed, the area at A–A is sufficient, as is also the width of the plate, which was assumed as 12 inches.

One of the plates will be riveted directly to the member, and the other will be riveted to it as a pin-plate. The section back of the pin (28) must be equal to the net section in the body of the member. The net section is 4 54 square inches for one side, and the total thickness

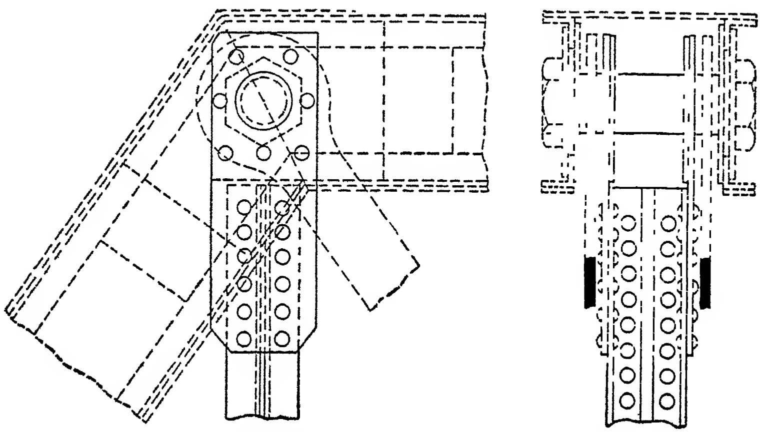


Fig 177. Connection of Hip Vertical to Upper Chord and End-Post

of the pin-plates is 1.125 inches, making the distance from the end of the member to the pin $\frac{4.54}{1.125} = 4\frac{1}{8}$ inches, and the distance to the center of the pin $4\frac{1}{8} + \frac{6\frac{1}{4}}{2} = 7\frac{1}{4}$ inches.

The joint between the plates and the main member will be weak in shear, the rivets tending to shear off between the $\frac{3}{8}$-inch angles and the plate, and also between the two plates themselves. As each side takes one-half of the above stress, the number of rivets required to connect the plates to the main member will be:

$$\frac{141\,600 - 2}{7\,220} = 10 \text{ shop rivets,}$$

and the number of rivets required to connect the inner $\frac{1}{2}$-inch plate to the outer one which is connected to the member itself will be:

$$\frac{141\,600 - 4}{7\,220} = 5 \text{ shop rivets.}$$

The distance from the center of the pin to the top of the main part must be greater than one-half the diameter of the largest I-bar head—that is, $17\frac{1}{2} \div 2 =$ say, 9 inches.

At the lower end, this member is connected to the bottom chord by means of a couple of clip angles and four or five rivets. Only sufficient rivets are required to prevent the sagging of the bottom chord, since the floor-beam is connected to the hip vertical above the lower chord, and hence no stress comes on the joint at the lower end (see Fig. 178).

The width of the plate has been assumed as 8 inches. This width is liable to be changed after the design of the intermediate

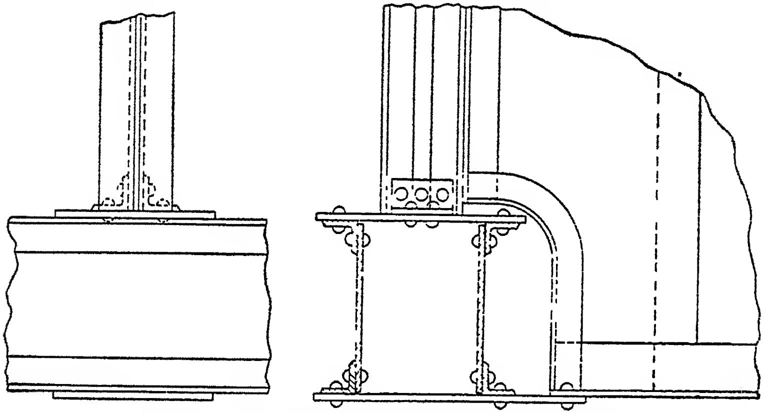


Fig. 178. Connection of Hip Vertical to Lower Chord.

posts has been made, since it will be economical to have all the intermediate floor-beams of the same length; and therefore the width of this plate will be changed so as to make the width of the hip vertical the same as the width of the intermediate posts.

85. **The Intermediate Posts.** The post U_2L_2 must be designed to stand a total stress of 163 600 pounds. Where possible, it is economical to make the intermediate posts out of channels, as this saves a large amount of riveting. As seen by the stress sheet, the length of these posts is 30.1 feet center to center of end pins. It is usually required that $\frac{l}{r}$ must not be more than 100, and this condition requires that the least radius of gyration cannot be less than $\frac{30.1 \times 12}{100} = 3.62$.

From Carnegie Pocket Companion, p. 144, it is seen that a 12-inch 30-pound channel has a radius of gyration of 4.28, and will fulfil the conditions. The area of two of these channels is 17.64 square inches. The unit allowable stress (16) is:

$$P = 16\,000 - 70 \times \frac{30\ 1 \times 12}{4.28} = 10\,090 \text{ pounds per square inch.}$$

The required area is then determined to be $\dfrac{163\,600}{10\,090} = 16.2$ square inches; and as this coincides very closely with the area given, these channels are efficient and will be used.

Fig. 179 shows the cross-section of this post. The radius of gyration which was used above was the radius of gyration of the channels about an axis perpendicular to their web. The radius of gyration of the entire section about an axis perpendicular to the web will be the same as that of one channel. In order to have the sections safe, the radius of gyration about the axis B–B must be equal to or greater than the other. The radius of gyration about the axis B–B can be increased or decreased by spacing the channels. The exact distance which will make the two rectangular radii of gyration equal may be determined by the methods of "Strength of Materials," or it may be found in columns 14 and 15 of the Carnegie Pocket Companion, p. 257. For any particular case it is equal to the value given in column 14, *plus* four times that given in column 15. For the channels under consideration, it is equal to $7.07 + 4 \times 0.677 = 9.78$ inches. Any increase in this distance will only tend to increase the radius of gyration about the axis B–B, and will make the post safer about that axis.

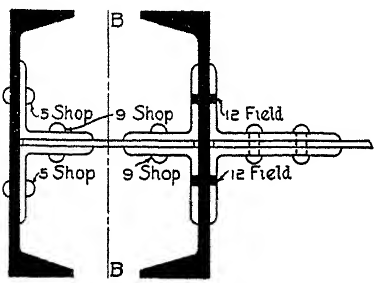


Fig 179 Cross-Section of Intermediate Post, Showing Diaphragm.

Fig. 179 shows a diaphragm. The web of this diaphragm cannot be less than $\frac{3}{8}$ inch, and the size of the angles cannot be less than $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, as this is the least allowed by the Specifications. The function of this diaphragm is to transfer one-half of the floor-beam reaction to the outer side of the post. The rivets which connect the

angles to the diaphragm web are shop rivets, and (see design of floor-beam) must be $\frac{137\ 600}{2 \times 7\ 880} = 9$ in number. The rivets which connect the diaphragm angle with the outer channel of the post are also shop rivets, and are $\frac{137\ 600}{2 \times 7\ 220} = 10$ in number, 5 on each side. The same rivets which connect the floor-beam to the post go through the diaphragm angle on that side of the diaphragm next to the center of the bridge, and must therefore be field rivets and take the entire floor-beam reaction These must be $\frac{137\ 600}{6\ 013} = 23$ in number, 12 on each side. The exact distance, back to back of the channels of the post, cannot be determined until after the top chord has been designed, since the post must slide up in the top chord and also leave room on each side for the diagonal members of the truss. The width is determined by the packing of the members at joint L_2 (see Fig. 174), and is found to be $9\frac{1}{4}$ inches. Since this is less than that required above, the post must be examined for bending about an axis parallel to the web of the channels.

According to the methods as discussed in the textbooks on "Mechanics" and "Strength of Materials," the moment of inertia about this axis is found to be 286.42, and the radius of gyration 3.96. The unit allowable compressive stress is then computed to be 9 580 pounds per square inch, and the required area $\frac{163\ 600}{9\ 580} =$ 17.10 square inches, which, being less than 17.64, shows the section to be safe.

This member is connected to the top chord at its upper end by a 5-inch pin. The total stress is 163 600 pounds, and the total bearing area required is $\frac{163\ 600}{24\ 000} = 6.8$ square inches, or 3.4 square inches for each side (19). The total thickness of the bearing area for each side is $\frac{3.4}{5} = 0.68$ inch. The thickness of the web of a 12-inch 30-pound channel is 0.513 inch, which leaves $0.68 - 0.513 = 0.167$ inch as the thickness of the pin-plate, but it must be made $\frac{3}{8}$ inch according to the Specifications. Fig. 180 shows the arrangement of the plates and the rivets.

The sum total of the pin-plates and the channel web is 0.888 inch, and therefore on one side the stress transferred to the pin by means of the pin-plate, which is 0 375 inch, is $\frac{1}{2} \times \frac{0.375 \times 163\,600}{0.888}$ $= 34\,600$ pounds. This plate will tend to shear off the rivets between it and the channel web, and therefore $\frac{34\,600}{7\,220} = 5$ shop rivets are required.

The stress that is shown on the stress sheet is the stress in the post above the floor-beam. The stress in that part below the floor-

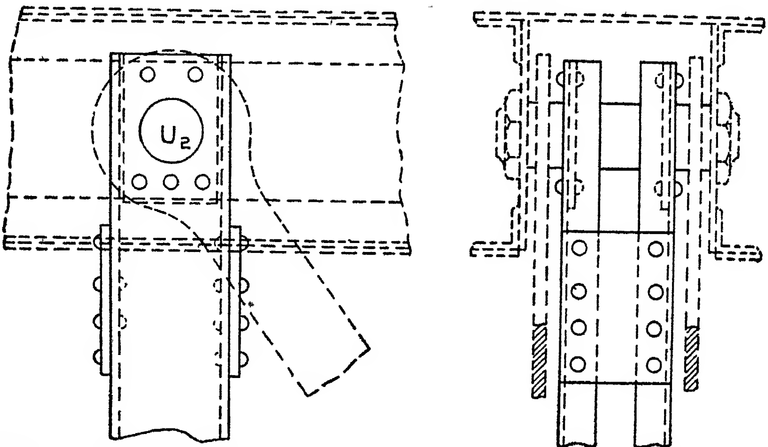


Fig. 180 Arrangement of Plates, Rivets, Pin, etc , at Connection of Intermediate Post to Top Chord.

beam is equal to the vertical component of the diagonal in the panel ahead of the post in question. In this case it is the vertical component of the stress in U_1L_2, and is equal to 242 000 pounds, and this requires a total bearing area of $\frac{242\,000}{24\,000} = 10.1$ square inches, and a total thickness of $\frac{10.1}{2 \times 5} = 1.01$ inches on each side, the pin being 5 inches in diameter. From this total thickness must be subtracted the thickness of the web of the channel, and this leaves $1.01 - 0.513 = 0.497$ inch as the total thickness of the pin-plates required. This shows that we must use one ½-inch plate. The total thickness of the bearing area is now $0.513 + 0.50 = 1.013$ inches.

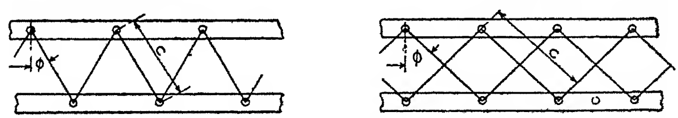
Each plate takes a total stress of $\frac{0.50}{1.013} \times \frac{242\,000}{2} = 59\,700$

pounds; and the joint being weak in shear, the number of rivets required will be $\frac{59\,700}{7\,220} = 9$ rivets in single shear. The detail will be similar to that in Fig. 180.

The distance, back to back of the channels in this post, will probably not be greater than 12 inches, and this will make the distance between rivet lines about 9 inches. According to (46), the end tie-plates must be at least 9 inches long and of course 12 inches wide. The thickness cannot be less than $\frac{9}{50} = 0.18$ inch, but they will be made $\frac{3}{8}$ inch (38). Between the tie-plates the channels will be connected by means of lattices. The Specifications (47) require that they should not be less than $2\frac{1}{4}$ inches in width and $(1.414 \times 9)\frac{1}{40} =$ 0.318 (say $\frac{3}{8}$) inch in thickness. Table XXV gives the thickness of lacing bars for any distance between rivets.

TABLE XXV

Thickness of Lacing Bars



Single Lacing $\left(t = \frac{c}{40},\ \phi = 30°\right)$		Double Lacing $\left(t = \frac{c}{60},\ \phi = 45°\right)$	
t	c	t	c
$\frac{1}{4}$ in.	0 ft. 10 in	$\frac{1}{4}$ in.	1 ft. 3 in.
$\frac{5}{16}$ in.	1 ft 0½ in	$\frac{5}{16}$ in.	1 ft 6¾ in.
$\frac{3}{8}$ in.	1 ft. 3 in.	$\frac{3}{8}$ in.	1 ft. 10½ in.
$\frac{7}{16}$ in	1 ft. 5½ in.	$\frac{7}{16}$ in.	2 ft 2¼ in.
$\frac{1}{2}$ in.	1 ft. 8 in	$\frac{1}{2}$ in	2 ft. 6 in.
$\frac{9}{16}$ in	1 ft. 10½ in.	$\frac{9}{16}$ in.	2 ft 9¾ in
$\frac{5}{8}$ in.	2 ft. 1 in.	$\frac{5}{8}$ in.	3 ft. 1½ in.

A width of $2\frac{1}{4}$ inches is chosen above, since according to Carnegie Pocket Companion, p. 212, a $\frac{3}{4}$-inch rivet is the largest which can be used in the channel flange.

The post U_3L_3 must be designed for a total stress of 87 000 pounds. It will be assumed that two 10-inch 20-pound channels

with a radius of gyration 3.66 and an area of 5 80 square inches each will be sufficient. The length, as before, is 30.1 feet, and the unit-stress is:

$$P = 16\,000 - 70 \times \frac{30\ 1 \times 12}{3.66} = 9\,080 \text{ pounds.}$$

The required area is $\frac{87\,000}{9\,080} = 9\ 60$ square inches. Since the total area of the two channels is 11 76 square inches, and the required area is 9.6 square inches, it is seen that they do not coincide very closely. These channels, however, will be used, since the thickness of the web is the thinnest allowed by the Specifications, and the width of the channels is the smallest that can be used and still give sufficient room to make the connections with the end connection angles of the floor-beams.

The lower end of this post also has a diaphragm which must transfer half of the stress to the outer channel of the post. The sides of the diaphragm are the same as in the posts previously designed; and the number of rivets required is computed in a similar manner and found to be as indicated in Fig. 181, which shows the cross-section of this post.

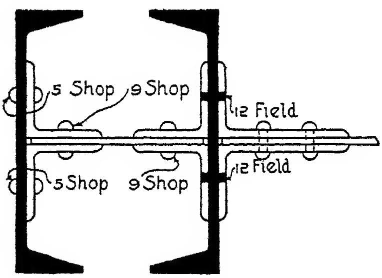


Fig. 181. Cross-Section of Intermediate Post

At the upper end the bearing area required on one channel is $\frac{87\,100}{2 \times 24\,000} = 1\ 814$ square inches, and the thickness required is $\frac{1.814}{5} = 0.363$ inch, a 5-inch pin being used. As the web of the channel is 0 382 inch thick, it will give sufficient bearing area without pin-plates.

At the lower end, the vertical component of U_2L_3 is 157 500 pounds. The bearing area required on each side of the post is $\frac{157\,500}{2 \times 24\,000} = 3.28$ square inches, and the thickness is $\frac{3.28}{5} = 0.66$ inch. The thickness of the channel web being 0 382 inch leaves 0.660 − 0.382 = 0 278 inch as the required thickness of the pin-plate;

but $\frac{3}{8}$ inch must be used, making a total thickness of $0.382 + 0.375 = 0.757$ inch. The plate will carry $\frac{0.375}{0.757} \times \frac{157\,500}{2} = 39\,000$ pounds, and this requires $\frac{39\,000}{7\,220} = 6$ shop rivets in single shear.

The distance, back to back of channels, will be the same as in

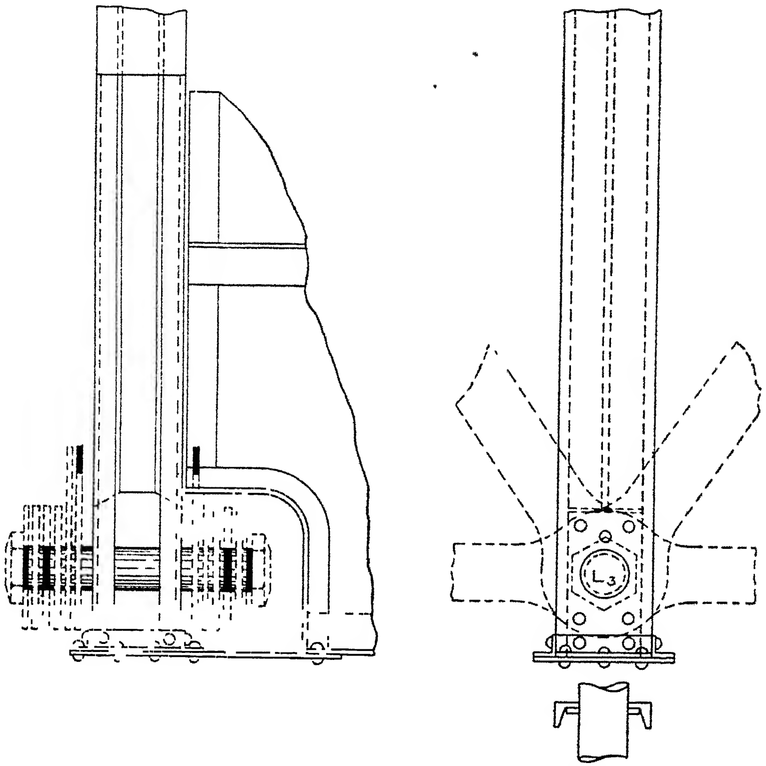


Fig. 182 End and Side Elevations Showing Detail of Construction at Lower End of Intermediate Post

U_2L_2, and therefore the tie-plates and lacing bars will be the same. Fig. 182 gives a detail of the lower end of U_3L_3.

86. **The Top Chord.** The top chords of small railway bridges may be made of two channels laced on their top and bottom sides. This is not very good practice, since it leaves the tops of the channels open and lets in the rain and snow, which tends to deteriorate the joints. It is better to add a small cover-plate, even if this does give

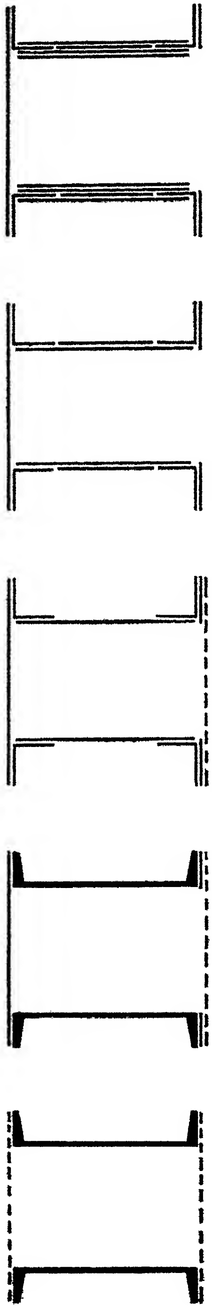


Fig. 183. Chord Sections, Showing Various Types of Construction

an excessive section. In case of stress such as is demanded, the chords may consist of two channels and a cover-plate. In this case it is necessary to place small pieces called *flats* upon the lower flanges of the channel, in order to lower the center of gravity of the section and to bring it near the center of the web. This section makes a very economical section in that it saves much riveting. On account of channels being made only up to 15 inches in depth, the use of this section is quite limited owing to the fact that it is not deep enough to allow the I-bar heads sufficient clearance, for the I-bar heads in bridges of even ordinary span will exceed this amount.

The most common section is that which consists of two side plates, four angles, and one cover-plate. Sometimes this section has flats placed upon the lower angle in order to lower the center of gravity, as explained above. According to (35), the section should be as symmetrical as possible, and the center of gravity should lie as near the center of the web as is consistent with economy.

In case the stress is great enough to demand a heavier section than that above described, additional plates are added upon the sides of the original plates, and heavier and larger cover-plates and angles are used. Fig. 183 shows different types of chord sections.

In addition to the cover-plate being designed to withstand the total stress, close attention must be paid to (44). This clause has been inserted on account of practical considerations, since it has been found out

that if plates are made much thinner than the proportions here required, they will crumple up and fail long before the allowable unit of stress as computed from the formula has been reached. In some cases—especially where the stress is light—the proportions laid down in (44) and (38) will govern the design of the section, instead of the required net area as determined by the formula for the allowable unit compressive stress.

The design of the first section of the top chord will now be made. Here, as in the case of the first sections of the lower chord, the diameter of the head of the greatest I-bar determines the width of the plates in the section The head of the 7-inch I-bar which constitutes the member U_1L_2 is $17\frac{1}{2}$ inches, and, allowing a clearance of $\frac{1}{2}$ inch on either side of the head, the total depth inside the chord should be $18\frac{1}{2}$ inches. As in the case of the lower chord, plates 18 inches wide may be used.

The size of the angles to be chosen is a matter of judgment. Usually any size should be chosen at first, and the preliminary design will indicate at once what size should have been taken. For this case, $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch will be assumed at first.

For sections of this character, the radius of gyration is approximately equal to $0.4h$, in which h is the height, or rather the width, of the side plate. The approximate radius of gyration is $r = 0.4 \times 18 = 7.2$ inches, and the length is equal to one panel length, or 21 feet. The allowable unit of stress (16) is:

$$P = 16\,000 - 70 \times \frac{21 \times 12}{7.2} = 13\,550 \text{ pounds.}$$

The required area is $\frac{449\,500}{13\,550} = 33.2$ square inches. The correct proportion for sections of this character is that 0.4 of the total area should be taken up by the web. The area of the web would then be $0.4 \times 33.2 = 13.28$ square inches, and the thickness would be $\frac{13.28}{2 \times 18} = 0.37$ inch. According to this, a $\frac{3}{8}$-inch plate should be used, but (44) requires that it shall be $\frac{14.5}{30} = 0.483$ inch or thicker. Therefore an 18 by $\frac{1}{2}$-inch plate must be used for the web.

The correct proportion for sections of this character is that the width between plates should be about $\frac{7}{8}$ the width of the side plates.

This will give the required width between plates equal to $\frac{7}{8} \times 18 =$ 15.75 inches. The cover-plate (44) must not be thinner than $\frac{1}{40}$ the distance between the connecting rivet lines. The rivet lines are, in this case, $15.75 + 2 \times 2 = 19.75$ inches apart, and therefore the thickness of the cover-plate cannot be less than $\frac{19.75}{40} = 0.494$ inch. The cover-plate will therefore be taken as $\frac{1}{2}$ inch thick. The width of the cover-plate (see Fig. 184) must be about $15.75 + 2 \times 3\frac{1}{2} + \frac{1}{2} = 23\frac{1}{4}$ inches (say 23 inches). The cover-plate will be taken 23 by $\frac{1}{2}$-inch.

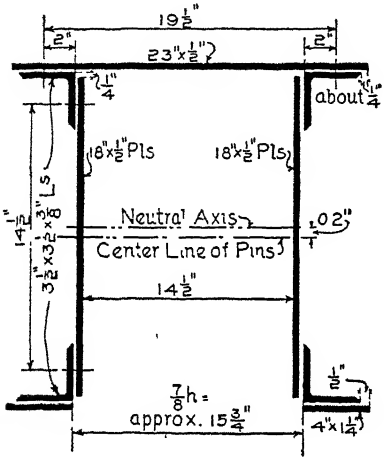


Fig. 184. Section of Top Chord.

The center line of pins will be taken at the center line of the web, and the center of gravity of the section will be assumed as $\frac{1}{2}$ inch above this. In order that the center of gravity may be near that assumed, the moment of the cover-plate about the assumed center of gravity axis should be about equal to the moment of the flats about the same axis. The moment of the cover-plate about the assumed axis is:

$$23 \times \tfrac{1}{2}(9.0 - 0.5 + 0.25 + 0.25) = \frac{23 \times 9}{2};$$

and the moment of the flats about the same axis is:

$$A(9.0 + 0.5 + 0.25 + 0.5) = 10.25\,A,$$

in which A is the area in square inches of both of the flats. Equating these two expressions, and solving for A, there results:

$$A = \frac{23 \times 9}{2 \times 10.25} = 10.1 \text{ square inches.}$$

Assuming the flats to be 4 inches wide, the thickness on each side will be 1.25 inches. As this is too thick to punch, the flats on each side will be composed of two 4 by $\frac{5}{8}$-inch plates.

The total area is:

One cover-plate	$= 23 \times \frac{1}{2}$	$= 11.5$ sq in
Two web plates	$= 2 \times 18 \times \frac{1}{2}$	$= 18.0$
Two flats $4 \times 1\frac{1}{4}$		$= 10.0$
	Total	39.5 sq in

But the required area is 32.2 square inches, which is considerably less than the area above given, and which does not include the angles and hence we can use the smallest size angles, which are those previously assumed. The area of each of these angles is 2.48 square inches, thus making the total area of the section $39.5 + 4 \times 2.48 = 49.42$ square inches. This is considerably in excess of the area as required according to the formula for compression; but it is the least allowed by the Specifications. Note that this is the case where (44), instead of the formula for compressive stress, is the ruling factor in the determination of the section.

The center of gravity of the approximate section must now be determined, the moment of inertia and the radius of gyration about the neutral axis must be computed, and the required area must be determined by using this radius of gyration as computed. If the required area as determined with the actual radius of gyration is less than the approximate area, then the thickness of the angles or the plates must be increased and the section then examined for its radius of gyration and required area. If the area is sufficient, the section is used; if not, another recomputation is in order.

In the determination of the center of gravity of the section, the moment is taken about the top of the cover-plate. The moments are computed as follows:

Cover-plate $(23 \times \frac{1}{2}) \times \frac{1}{4}$ . .	2.88
Webs $2\,(18 \times \frac{1}{2}) \times (9 + \frac{3}{4})$ . . .	175.60
Top angles $2\,(2.48) \times (1.01 + \frac{1}{2})$ . . .	7.50
Lower angles $2\,(2.48) \times (\frac{1}{2} + \frac{1}{4} + 18 + \frac{1}{4} - 1.01)$	89.30
Flats $2\,(4 \times 1\frac{1}{4}) \times 19\frac{5}{8}$ . . .	196.25
Total	471.53

The center of gravity is now found to be $\dfrac{471.53}{39.50 + 4 \times 2.48} = 9.55$ inches from the top of the cover-plate. The distance from the top of the cover-plate to the middle line of the web is $9 + \frac{1}{4} + \frac{1}{2} = 9.75$ inches, and this leaves a distance of $9.75 - 9.55 = 0.2$ inch from the center line of the web to the neutral axis. This distance is gen-

erally represented by the letter e, and it is known as the *eccentricity of the section.*

The moment of inertia about this axis must now be computed. The relation used is that the moment of inertia about any axis is equal to the moment of inertia about some other axis, *plus* the product of the square of the distance between the two axes by the area of the section whose moment of inertia is desired. The moments of inertia of the various parts of the section (see "Steel Construction," Part IV, pp. 292 and 293) are computed and are as follows:

Cover-plate	994.64
Webs. . .	486 72
Top angles.	325 74
Lower angles	359 74
Flats. . .	1 017 37
Total	3 184.21

The radius of gyration is equal to the square root of the quotient obtained by dividing the moment of inertia by the area. It is

$$r = \sqrt{\frac{3\ 184.21}{49.42}} = 8\ 04$$

Using this value of the radius of gyration in the formula for the compressive stress, there is obtained 13 800 pounds as the unit allowable stress in compression, and this requires an area of $\frac{449\ 500}{13\ 800} = 32.5$ square inches. Since this is considerably less than the actual area of the section, the section will not be changed but will be taken as first assumed.

In order that the section should be safe about both axes, the moment of inertia about the axis perpendicular to the cover-plate should be equal to or greater than that as above computed. By computing the moment of inertia about the axis perpendicular to the cover-plate, it is found to be 3 256.3, which gives a radius of gyration of 8.11; and since both of these are greater than those first computed, it is seen that the section is safer about the axis perpendicular to the cover-plate than it is about an axis perpendicular to the web plates.

There are small stresses in this member due to its own weight and to the fact that the pins are not placed directly upon the neutral axis (see "Strength of Materials," p. 82). These stresses are seldom more than 1 000 pounds per square inch in the extreme fibre; and

since the section has such an excess of area, they will not be computed, as it is evident that there is sufficient strength in the member to withstand them.

The section just designed is that for the top chord having the greatest stress; and since this is the minimum section allowed by the Specifications, it must be used in all the sections of the top chord.

The section as finally designed is:

One cover plate, 23 by ½ inch,
Two webs, 18 by ½ inch;
Four angles, 3½ by 3½ by ⅜-inch
Four flats, 4 by ⅜-inch.

A pin 6¼ inches in diameter will be used at the point U_1. The stress in the member U_1U_2 is 378 200 pounds, and the bearing area required is $\frac{378\,200}{24\,000} = 15.75$ square inches, or 7.875 for each side. This makes a total required thickness of $\frac{7.875}{6.25} = 1.26$ inches for one side. Since the thickness of the web plate is ½ inch, it will be necessary to provide pin-plates whose total thickness must be $1.26 - 0.5 = 0.76$ inch. Two ⅜-inch plates will give a thickness of 0.75 inch; and since this is less than the required thickness by an amount not over 2½ per cent, they may be used. The total thickness of the bearing area is now 1.26 inches. The stress transferred to the two ⅜-inch plates is:

$$s = \frac{2 \times \frac{3}{8}}{1.26} \times 189\,100 = 113\,500 \text{ pounds}$$

The rivets required to keep the outer plate from shearing off the other are $\frac{113\,500}{2 \times 7\,220} = 8$ shop rivets, and the rivets required to keep both of the ⅜-inch plates from shearing off the web of the chord section are $\frac{113\,500}{7\,220} = 16$ shop rivets in single shear. The bearing of a ⅞-inch shop rivet on a ½-inch plate is 10 500 pounds, and therefore the number of rivets required to keep these pin-plates from tearing the rivets out of the ½-inch web plates is $\frac{113\,500}{10\,500} = 11$ shop rivets in bearing. Fig. 185 shows the detail of this end of the top chord section. The pin-plates should extend well back on the member, and at least one pin-

plate should go over the angle, and enough rivets, as computed above, should go through the angles and this pin-plate. Experiments on full-sized bridge members go to show that unless the pin-plates cover the angles and extend well down on the member, the member will fail before the unit-stress reaches that value computed by the formula for compression.

Since the ends of the chord are milled at the splices, and therefore butt up against each other and allow the stress to be transmitted

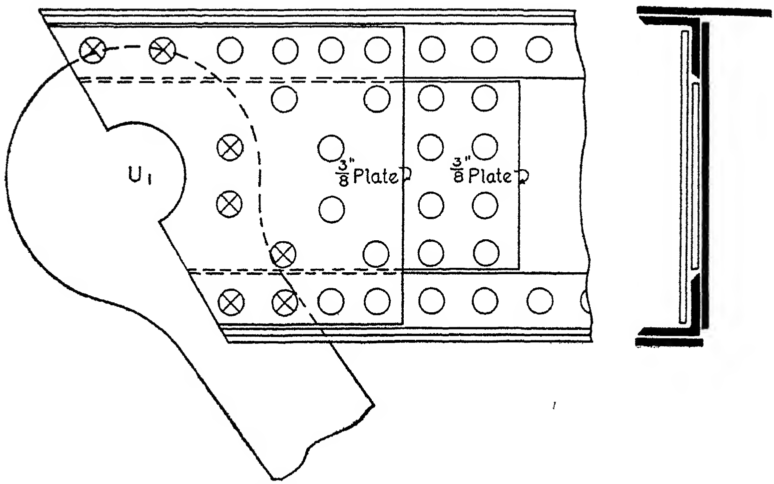


Fig. 185. Detail of Top Chord Section at Point U_1.

directly, only sufficient rivets need be placed in the splice to keep the top chord sections in line (57).

At the point U_2, it is not necessary to put in a pin-plate to take the stress in the upper chord; but it is only necessary to provide a pin-plate to take up the difference in stress between the two chord sections. This difference in stress is equal to the horizontal component of the maximum stress in the member U_2L_3. This is 110 000 pounds, and the area required on each side for bearing is 2.3 square inches; and as a 5-inch pin is used here, the thickness of the bearing area is $\frac{2.3}{5} = 0.46$ inch. As this thickness is less than the thickness of the web plate, no pin-plates will be required.

At the point U_3, a bearing area will be required to withstand the horizontal component of the member U_3L_4. This is 56 300, and the

bearing area required on each side is $\frac{56\,300}{24\,000 \times 2} = 1.18$ inches. The required thickness of the bearing area is $\frac{1.18}{5} = 0.24$ inch, as a 5-inch pin is used here also. As this thickness is less than the thickness of the web plate, no pin-plate will be required.

The under parts of these members must be stiffened by tie or batten plates, and these plates (46) must be equal in length to the distance between rivet lines. This is $19\frac{1}{2}$ inches. They will be made 20 inches long and 23 inches wide. The thickness of these plates (46) must be $\frac{19.5}{50} = 0.39$ inch (say $\frac{7}{16}$ inch). The size of the tie-plates will then be 20 in. by $\frac{7}{16}$ in. by 1 ft. 11 in.

Since the distance between the rivet lines is greater than 15 inches, double latticing must be used (49); and according to Table XXV the lacing must be $\frac{1}{2}$ inch thick; also, according to (47), it must be $2\frac{1}{2}$ inches wide, as the rivets used are $\frac{7}{8}$ inch in diameter. The lattices will then be $2\frac{1}{2}$ by $\frac{1}{2}$-in.

87. The End-Post. Since the minimum section as chosen for the top chord is about 50 per cent in excess of that required by the compression formula, it will be assumed to be sufficient for the section of the end-post, and it will now be investigated to see if it is safe.

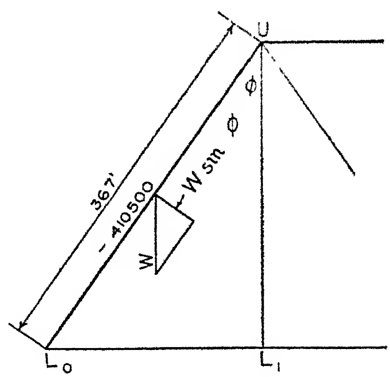


Fig. 186 Calculation of End-Post.

In addition to the stress due to direct compression, the end-post is stressed by its own weight, by eccentric loading due to the pin being in the center of the web instead of at the center of gravity of the section, and to a bending moment at the place where the portal brace joins it. This is due to the bending action of the wind on the top chord. These different stresses will now be computed; and since the post is in all cases stressed by a combination of bending and compressive stresses, this fact should be considered in the design. In determining the stress in the end-post due to its own weight, the entire

weight must not be used in computing the bending action, but only that component of it which is perpendicular to the end-post. The length of the end-post is readily computed, and is as shown in Fig. 186. The general formula for accurately computing stresses due to bending when the member is also subjected to compression, is:

$$S = -\frac{My_1}{I - \frac{Pl^2}{10E}},$$

in which,

S = Stress in pounds per square inch in the extreme upper fibre of the beam,
M = Exterior moment causing the stress, and is considered positive if it bends the beam downward, and negative if it bends the beam upward,
y_1 = Distance from the neutral axis to the extreme upper fibre,
I = Moment of inertia of the section,
P = Direct compressive stress, in pounds,
l = Total length, in inches;
E = Modulus of elasticity of steel, which is usually taken as 28 000 000 pounds per square inch.

In this case the force causing the bending is that component of the weight perpendicular to the end-post. This is $W \sin \phi$, in which W is the weight of the steel in the end-post; and this is computed and is as follows:

Cover-plate.	1 435 lbs.
Web plates	2 245 "
Angles	1 250 "
Flats...	1 245 "
	6 175 lbs.
Add 25 per cent for details	1 544 "
Total.	7 719 lbs.

Substituting in the above formula the various values, there results:

$$S = -\frac{\frac{1}{8} \times 7\,719 \times 36.7 \times 0.572 \times 12 \times 9.55}{3\,185 - \frac{410\,500 \times (36.7 \times 12)^2}{10 \times 28\,000\,000}}$$

= 800 pounds per square inch compression in the upper fibre due to bending

In the above equation, the stress in the member is 410 500 pounds; the distance y_1 is the distance from the neutral axis to the top of the cover-plate, and the coefficient of elasticity of steel is taken as 28 000 000.

In computing the stress due to the eccentric loading, the moment is equal to the product of the total stress in the member by the distance from the neutral axis to the center of gravity axis causing a negative moment. Substituting in the above formula for combined stresses, there results:

$$S = -\frac{-410\,500 \times 0.2 \times 9.55}{3\,185 - \dfrac{410\,500 \times (36.7 \times 12)^2}{10 \times 28\,000\,000}}$$

$= 270$ pounds per square inch tension in the upper fibre

In order to find the compression in the lower fibre, it is only necessary to notice that the stresses are proportional to the distances from the neutral axis. Accordingly (see Fig. 187), the stress in the lower fibre due to the weight is 895 pounds tension, and the stress in the lower fibre due to the eccentric loading is 302 pounds compression.

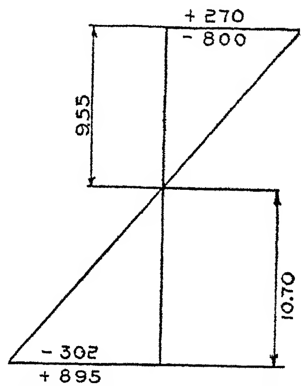


Fig. 187. Calculation of Stress in Chord.

Before computing the stress due to the bending moment caused by the wind on the upper chord, it is necessary to investigate the post to see if it is fixed or hinged at its lower end. This is very important, since, if the post is found to be hinged, the bending moment will be twice that which will occur when the post is not hinged.

An end-post is considered fixed when the product of one-half of the total stress *times* the distance between the web plates is greater than the product of the wind load acting at the hip, or joint U_1, *times* the length of the end-post. In this case the first value is $\frac{410\,500}{2} \times 15 = 3\,075\,000$; and the product of the latter (see Article 89) is $12\,600 \times 36.7 \times 12 = 5\,550\,000$. Since the latter is greater than the former, the post is hinged, and the bending moment at the foot of the portal strut, which joins the end-post 28.2 feet from the end, is $6\,300 \times 28.2 \times 12 = 2\,130\,000$ pound-inches. The stress in the extreme fibre due to this bending moment is:

$$S = \frac{2\,130\,000 \times 11.5}{3\,256.3 - \dfrac{410\,500 \times (36.7 \times 12)^2}{10 \times 28\,000\,000}}$$

= 8 250 pounds per square inch tension or compression

In computing this stress due to the wind moment, care must be taken to take y_1 equal to one-half the width of the cover-plate, and to take the moment of inertia as that about the axis perpendicular to the cover-plate.

In computing the total stress on the extreme fibre, it must be noted that the stresses due to weight and eccentric loading do not stress the same extreme fibres as the stress due to wind, the former stressing the extreme fibres on the top and bottom of the post, while the latter stresses those on the inner and outer sides. The total direct unit-stress is $\frac{410\,500}{49.42} = 8\,310$ pounds per square inch; and this, added to the 8 250 pounds per square inch due to the wind, gives a total of 16 560 pounds per square inch on the extreme fibre only.

The allowable unit-stress is $16\,000 - 70 \times \frac{36.7 \times 12}{8.11} = 12\,200$ pounds per square inch when wind is not taken into account, and (25) is $1\frac{1}{4} \times 12\,200 = 15\,250$ pounds per square inch when the wind is taken into account. The difference between this and the actual stress is $16\,560 - 15\,250 = 1\,310$ pounds per square inch, which shows that the section is not strong enough. The section can be increased by widening the cover-plate or by making the plates thicker; but as this excess is due to wind only, the section being amply sufficient under the other stresses, and is fixed to some extent by the floor-beam connection, no change will be made.

The pin at each end of the end-post will be the same—namely, $6\frac{1}{4}$ inches in diameter—and therefore the pin-plates will be the same at each end. The total stress in the post is 410 500 pounds, which makes a required bearing area of $\frac{410\,500}{24\,000} = 17.2$ square inches for both sides, or 8.6 square inches for one side, and the total required thickness of $\frac{8.6}{6.25} = 1.375$ inches for one side. Since the thickness of the web plates is $\frac{1}{2}$ inch, this leaves a remainder of $1.375 - 0.5 = 0.875$ inch for the thickness of the pin-plates. One plate $\frac{3}{8}$ inch thick and one plate $\frac{1}{2}$ inch thick will be used.

The proportion of the total stress which is taken by the $\frac{3}{8}$-inch

plate is $\frac{0.375}{1.375} \times \frac{410\,500}{2} = 56\,000$ pounds; and that taken by the $\frac{1}{2}$-inch plate is $\frac{0.50}{1.375} \times 205\,250 = 74\,600$ pounds. The number of rivets required to transfer the stress from the $\frac{3}{8}$-inch plate to the $\frac{1}{2}$-inch plate is $\frac{56\,000}{7\,220} = 8$ shop rivets in single shear; and the number of rivets required to transfer the stress from both pin-plates to the web is $\frac{56\,000 + 74\,600}{7\,220} = 18$ shop rivets in single shear. As in the case of the top chord, one pin-plate should extend over the angle, and the number of rivets required in that pin-plate should go through the pin-plate and the angles (see Fig. 188). The $\frac{3}{8}$-in. hinge plate is used for erection purposes, and is not considered as a pin-plate. It is omitted at L_0.

Since this section is the same as that of the top chord, the tie-plates and the lattice bars must be the same size.

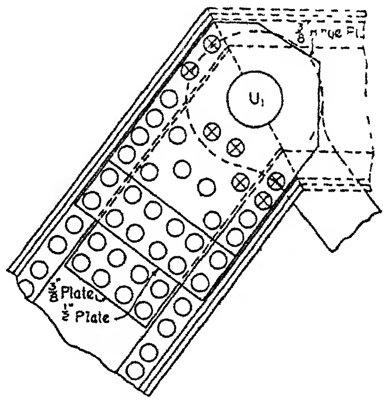


Fig. 188. Plates and Riveting at Upper End of End-Post.

88. **The Pins.** The design of the pins requires a simple but quite lengthy computation. Simple Pratt railroad trusses for single-track bridges usually have the same arrangement of tension and compression members; that is, the same tension members occupy relatively the same positions with respect to the compression members. Also, while theoretically a different sized pin will be required at every joint, it is not customary to make them so. In practice the pins at the joints U_1 and L_0 are made of the same diameter, and those at the remainder of the joints are also made in diameter equal to each other but different from those at U_1 and L_0, the pins at U_1 and L_0 usually being larger in diameter. On account of the above conditions and facts, it is unnecessary to design the pins in spans under 200 feet, since usually they are the same for any given span and loading. Table XXVI gives the diameters of pins for spans of 100 up to 200 feet for loading E 50.

TABLE XXVI

Pins for Single-Track Bridges

Loading E 50

Span	Diameter of Pin	
	U_1 and L_0	All Others
100 feet	$4\frac{1}{2}$ inches	4 inches
125 "	$5\frac{1}{2}$ "	5 "
150 "	$6\frac{1}{2}$ "	$5\frac{1}{4}$ "
175 "	$6\frac{3}{4}$ "	$5\frac{3}{4}$ "
200 "	7 "	6 "

For E 40 loading, decrease the above values by $\frac{1}{4}$ inch, for E 30 loading, decrease them by $\frac{3}{8}$ inch. The diameter of pins for spans not given in the table can be interpolated from the given values. No pin should be less than $3\frac{1}{2}$ inches in diameter

The span of this bridge is 147 (say 150) feet, and the diameter of the pins at U_1 and L_0 is $6\frac{1}{2} - \frac{1}{4} = 6\frac{1}{4}$ inches; and the diameter of the pins at the other panel points is $5\frac{1}{4} - \frac{1}{4} = 5$ inches. It should be noted that no pin is required at point L_1, as the two members which join here are built-up members and are riveted together.

The above table is for single-track bridges only. The diameters of pins for double-track bridges are given in Table XXVII. These values are for E 50 loading; and for E 40 and E 30 loading, deductions must be made as required in the case of Table XXVI.

TABLE XXVII

Pins for Double-Track Bridges

Loading E 50

Span	Diameter of Pin	
	U_1 and L_0	All Others
100 feet	6 inches	$5\frac{1}{4}$ inches
125 "	8 "	$6\frac{1}{2}$ "
150 "	9 "	$7\frac{1}{2}$ "
175 "	$9\frac{1}{4}$ "	$8\frac{1}{4}$ "
200 "	$9\frac{1}{2}$ "	$8\frac{1}{2}$ "

No pin in a double-track bridge should be less than $4\frac{1}{2}$ inches in diameter.

Pins for highway bridges are usually much less in diameter than those for railway bridges, except in the case of first-class trusses for heavy interurban traffic or for city bridges carrying paved streets,

where they should be taken equal to those given for E 30 loading Table XXVIII gives the diameters of pins for different length spans of simple highway bridges designed for 16-ton road-rollers or farm wagons and 100 pounds per square foot of roadway

TABLE XXVIII

Pins for Country Highway Bridges

Span	Diameter of Pin	
	[illegible] and Lower Chord	Upper Chord
50 feet	2½ inches	2 inches
100 "	3 "	2½
150 "	3½ "	2¾ "
200 "	4 "	3 "

89. **The Portal.** In order to have a clearance of 21 feet (2) above the top of rail, it is necessary that the portal be as shown in Fig. 189. The stresses are found by methods of Article 54, Part I, the wind load being computed according to (10). It must be remembered that the column is hinged. , p. 231.

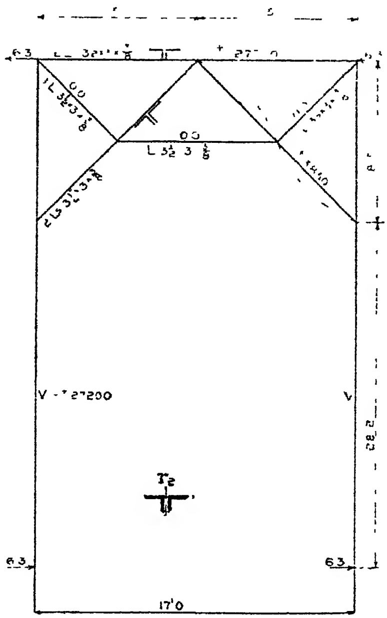


Fig 189 Portal Dimension and Stress Diagram

In case the members of the portal braces bend about one axis, their length will be equal to the distance from one end to the other. In case they bend about the other axis as indicated by the broken line in Fig. 189, their length will be one-half of what it was in the first case.

The portal struts or diagonals will be designed first. Their length is $8.5 \times 1.414 =$ 12 feet, or 144 inches. This is the total length. Although the Specifications do not mention it, the ratio of the length to the radius of gyration should not exceed 120. This means that the radius of gyration in this

case should be greater than $\frac{144}{120} = 1.2$. The section of the strut will be composed of two angles placed back to back.

Two angles $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch, (74), with an area of 4.6 square inches and r_2 equal to 1.72, will be assumed to be sufficient to take the stress, and the properties of these angles must now be examined to see if the assumption is correct.

The allowable unit-stress (25) is 25 per cent greater than in the case of live or dead loads. This makes the unit-stress as computed from the formula:

$$P = \left(16\,000 - 70 \times \frac{144}{1\ 72}\right) 1\tfrac{1}{4} = 12\,680 \text{ pounds per square inch.}$$

The required area is $\frac{38\,500}{12\,680} = 3\ 05$ square inches; and since this is less than the given area, the angle will be amply sufficient. The required area is over one square inch less than the given area, but this angle must be used, since it is the smallest angle allowed by the Specifications. Note that unequal legged angles should be used, as this will make the radius of gyration about one axis larger than about the other; and this will prove economical, since, when one axis is considered, the length of the member is greater than when the other is considered.

The above angle should also be examined for tension, it being considered that one rivet-hole is taken out of the section of each angle. The net section of the two angles will now be $4\ 60 - 2\left(\frac{7}{8} + \frac{1}{8}\right) \times \frac{3}{8} = 3\ 85$ square inches; and the area required for tension is $\frac{38\,500}{16\,000 \times 1.25} = 1.93$ square inches, which shows that the angle is amply sufficient. It should be noted that these Specifications do not require that only one leg of the angle shall be efficient unless both legs are connected. In case this strut had been designed according to Cooper's Specifications, two angles 5 by 3 by $\frac{1}{2}$-inch would have been required, and the 5-inch leg would have been placed vertically and the angle connected by this leg alone. While it is not within the province of this work to discuss the question of connecting angles by one or by both legs, yet it might be said that tests made on angles connected with one leg only, seem to indicate that the ultimate strength in tension is about 60 per cent of that obtained from the same angle when tested with both legs connected.

While according to (22) the alternate strains in the wind bracing do not have to be considered, since they do not occur very closely together, yet in framing the connections it is required that the sum of both positive and negative stresses shall be added. In this case the stress for which the connections must be designed is $2 \times 38\,500 = 77\,000$. It must be remembered that in this case also, the unit-stresses are increased 25 per cent over those allowed for live and dead loads.

The number of rivets required in the end connections will be governed by bearing in the connection plates, and these plates are usually made $\frac{3}{8}$-inch thick. The number of rivets required is $\frac{77\,000}{7\,880 \times 1\frac{1}{4}} = 8$ shop rivets, or $\frac{77\,000}{6\,560 \times 1\frac{1}{4}} = 10$ field rivets.

The portal bracing is riveted up in the shop and brought to the bridge site, where it is connected to the trusses by field-riveted connections at its end. Therefore the end of the portal struts which connect with the top piece will have 8 shop rivets, and the other end which connects with the end-post will have 10 field rivets. Since the angles are small, all the above rivets must go in one line, and this will cause the connection plate to be quite large. It will probably be better to connect both legs of the angle by means of clip angles and thus reduce the size of the connection plates.

The top part of the portal bracing will consist of two angles. Two angles $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch will be assumed and examined to determine if the area is sufficient. The length of this strut is the distance center to center of trusses, and is equal to $17 \times 12 = 204$ inches. The least radius of gyration is therefore $\frac{204}{120} = 1.70$. The radius of gyration of the two angles assumed is 1.72 when referred to an axis parallel to the shorter leg when the two angles are placed back to back and one-half inch apart. The unit-stress is now computed:

$$P = \left(16\,000 - 70 \times \frac{204}{1.72}\right) 1\tfrac{1}{4} = 9\,625 \text{ pounds per square inch.}$$

The required area is $\frac{27\,200}{9\,625} = 2.825$ square inches. This is considerably less than the area given by the two angles; but as these are the minimum angles allowable, they must be used. Since the stress in this case is less than in the previous case, and since the angles

used are the same, it is evident that these angles are safe in tension.

The number of rivets is determined by the bearing in the $\frac{3}{8}$-inch connection plates, and is:

$$\frac{2 \times 27\,200}{7\,880 \times 1.25} = 6 \text{ shop rivets, and}$$

$$\frac{2 \times 27\,200}{6\,560 \times 1.25} = 7 \text{ field rivets.}$$

As in the case of the lateral strut, this member should be connected by both legs of the angle in order to reduce the size of the connection plates. Fig. 190 gives the details of the portal bracing and its method of connection to the end-post. The full circles represent shop rivets, and the blackened circles represent field rivets. Some engineers connect the portal bracing to the top cover-plate of the end-post. This produces an excessive eccentricity in the end-post and is bad practice.

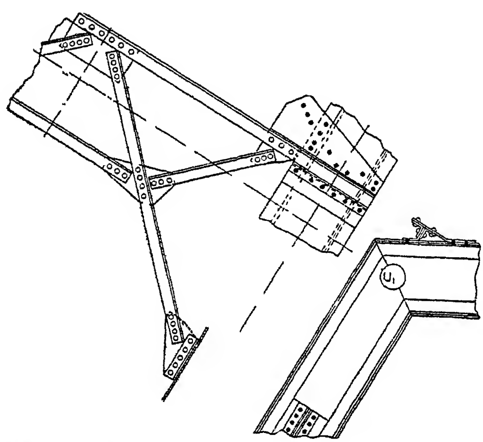


Fig. 190. Details of Portal Bracing and Connection to End-Post

Those members of the portal bracing which do not take any stress will be made of single angles, and the size of these angles will be taken $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch.

90. **The Transverse Bracing.** This bracing will be the same general style as the portal bracing, except that the top member will consist of two angles placed at a distance apart equal to the depth of the top chord, and these angles will be joined together by lacing. As in the case of portal bracing, those members which do not take stress will be made of one angle $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch.

The general outline is shown in Fig. 191, and the stresses are computed from (10) and by the methods of Article 54, Part I. In designing this top member, the top angle only is supposed to take the stress. The length in this case is 204 inches. Two $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch angles will be assumed as sufficient and will be examined. These angles

give a total area of 4.60 square inches In examining these it will be found that they are amply sufficient, in fact so much so that it will be better to see if one single angle at the top will not be better. According to the length, the smallest radius of gyration which can be used is 1.7. In looking over the tables of angles, it is seen that the

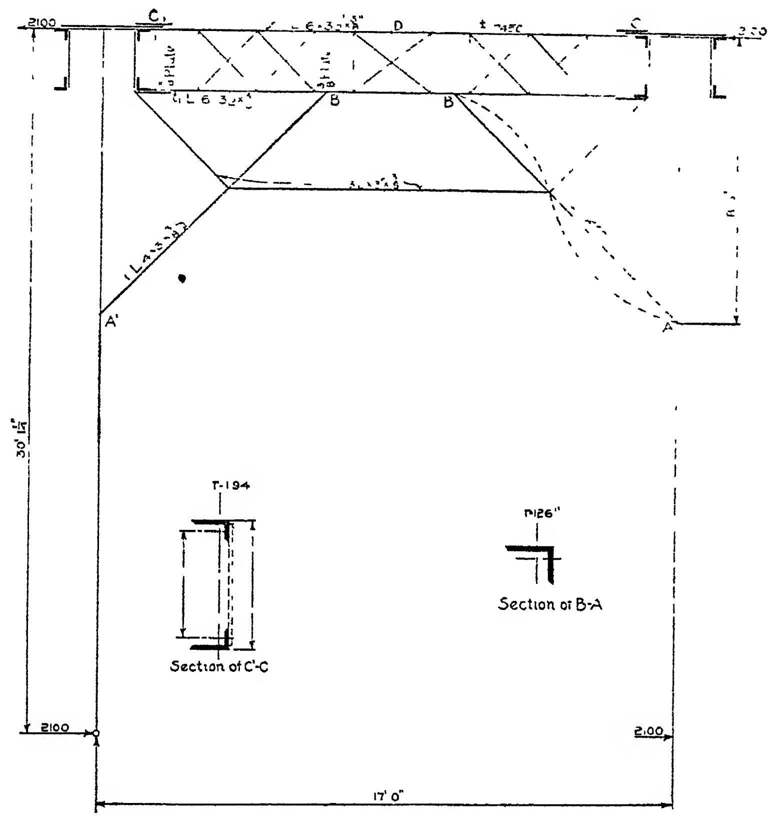


Fig. 191. General Outline of Transverse Bracing.

first angle to fulfil this condition is a 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, and it has a radius of gyration of 1.94. The allowable unit-stress is computed as follows:

$$P = \left(16\,000 - 70 \times \frac{204}{1.94}\right)1\tfrac{1}{4} = 10\,780 \text{ pounds per square inch,}$$

and the required area is $\dfrac{9\,100}{10\,780} = 0.85$ square inch. This is con-

siderably smaller than the area of the angle, which is 3 42 square inches; but since this is the smallest possible angle which will fulfil the conditions of the Specifications, and since it is much smaller than the two angles as first assumed, it will be used. Fig. 192 gives a cross-section of this member. Since this angle is joined to the cover-plate by one leg, the joints will be weak in single shear, and the number of rivets required will be:

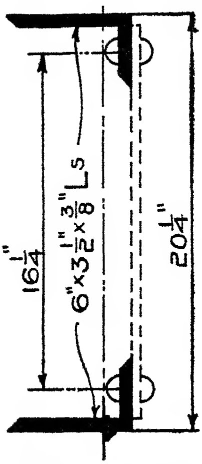


Fig 192. Cross-Section of Top Member of Transverse Bracing.

$$\frac{2 \times 9\,100}{7\,220 \times 1\frac{1}{4}} = 2 \text{ shop rivets, or}$$

$$\frac{2 \times 9\,100}{6\,013 \times 1\frac{1}{4}} = 3 \text{ field rivets.}$$

According to (47), the width of the latticing must be $2\frac{1}{2}$ inches; and according to Table XXV, the thickness must be $\frac{7}{16}$ inch, the distance c being 1 foot 11 inches.

The length of the knee-bracing is 144 inches; but on account of the small stress, one angle will be used. One 4 by 3 by $\frac{3}{8}$-inch angle, with an area of 2.48 square inches and a radius of gyration 1.26, will be assumed as sufficient. The radius of gyration is greater than the minimum allowable, which is 1.2. The allowable unit-stress is:

$$P = \left(16\,000 - 70 \times \frac{144}{1.26}\right) 1\tfrac{1}{4} = 10\,000 \text{ pounds per square inch.}$$

The required area is $\frac{12\,300}{10\,000} = 1.23$ square inches. The required area is much less than the given area; but this angle must be used, since it is the only one allowed on account of its radius of gyration. Two of the minimum sized angles might have been used; but their total area, 4.60 square inches, is much in excess of that of the angle used.

This angle must be examined for tension. The net area is 2.48 $- (\frac{7}{8} + \frac{1}{8}) \times \frac{3}{8} = 2$ 1 square inches. The required net area in tension is $\frac{12\,300}{16\,000 \times 1\frac{1}{4}} = 0.615$ square inch, which shows this angle to be amply sufficient.

The number of rivets required will be governed by the shear, since the angle is connected by one leg only; and it is:

$$\frac{2 \times 12\,300}{7\,220 \times 1\frac{1}{4}} = 3 \text{ shop rivets, and}$$

$$\frac{2 \times 12\,300}{6\,013 \times 1\frac{1}{4}} = 4 \text{ field rivets.}$$

91. **The Lateral Systems.** The stresses in these systems must be computed according to (10) and Article 54, Part I. They are given on the stress sheet, Plate III (p. 251). Since according to (70) these members must be constructed of rigid shapes, it is customary, in computing the stresses, to assume that one-half the shear is taken by each of the diagonals in any given panel; that is, one diagonal is in tension, and the other diagonal is in compression. The stresses given on the stress sheet are computed by making this assumption. Also, since both diagonals in each panel are considered as acting at the same time, the stresses in all the verticals are zero.

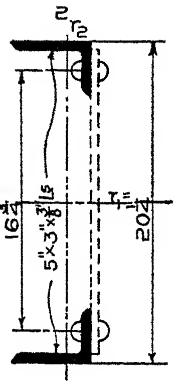


Fig 193. Section of Upper Lateral Member.

The section of the upper lateral members will be made up of two angles placed apart a distance equal to the depth of the top chord. Fig. 193 shows the section. The radius of gyration about the axis parallel to the long leg will be considerably larger than that about an axis parallel to the shorter leg. In fact, it is so much greater that the strut will not need to be examined with respect to this axis. The diagram of the first panel is given in Fig. 194. The radius of gyration is to be taken about the horizontal axis if the entire length is to be taken; and the radius of gyration is to be taken about the vertical axis if one-half the length is taken, in which case it will bend as shown by the broken line in Fig. 194. The members are designed for the latter conditions only, since they are amply safe in regard to the first condition if they satisfy the latter. The length in this latter condition is 13.5 feet, which requires a radius of gyration not less than

$$\frac{13.5 \times 12}{120} = 1.35.$$

Two angles 5 by 3 by $\frac{3}{8}$-inch, with a total area of 5.72 square

inches and a radius of gyration equal to 1.70, will be assumed and investigated to determine if they are sufficient.

The unit-stress is computed to be $P=(16\,000-70\times\frac{13.5\times12}{1.70})\times 1\frac{1}{4}=9\,320$ pounds per square inch, and the required area is $\frac{6\,700}{9\,320}=$ 0.72 square inch. The required area is very much less than the given area; but the angle chosen must be used, since this is the smallest one which conforms to the requirements of the Specifications.

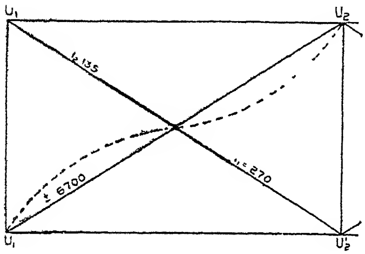


Fig. 194. Outline Diagram of First Panel in Upper Lateral System.

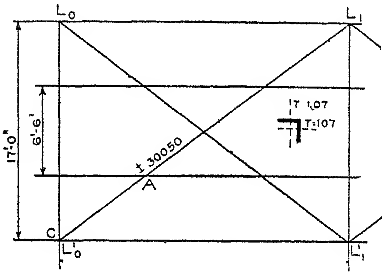


Fig. 195. Outline Diagram of First Panel in Lower Lateral System

The width of the lattices (48) must be $2\frac{1}{2}$ inches; and according to Table XXV, the thickness must be $\frac{7}{16}$ inch, the distance c being $24\frac{1}{4}$ inches.

Single shear governs the number of rivets required. In accordance with (22) and (25), their number is $\frac{2\times6\,700}{6\,013\times1\frac{1}{4}}=2$ rivets. Field rivets 3 in number are used in all places, since the lateral system is riveted up after the trusses are swung into place.

Since this is the minimum sized angle which will give a radius of gyration greater than 1.35, it must be used in the remainder of the panels of the top chord. Four angles of the minimum size might have been used, and would have been satisfactory, except that the area would have been excessive.

The stresses in the lower lateral system are computed according to (10), a similar assumption to that for the upper lateral system being made—namely, that both diagonals in each panel are stressed at the same time, one taking tension and the other taking compression. Fig. 195 shows the first panel of the lower lateral system. These

diagonals are connected to the stringers wherever they cross them, and also to each other where they cross in the center. This reduces the length which must be used in computing the cross-section of the member. In this case it is the distance C–A, and is equal to 90 inches.

Since the angle is free to move about either axis, angles with even legs should preferably be employed, since this will give greater economy. The radius of gyration must be greater than $\frac{90}{120} = 0.75$.

One angle $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, with an area of 2.48 square inches and a radius of gyration of 1.07, will be assumed and investigated. The allowable unit-stress is $P = (16\,000 - 70 \times \frac{90}{1.07})\ 1\frac{1}{4} = 12\,650$ pounds per square inch, and the required area is $\frac{30\,500}{12\,650} = 2.38$ square inches. This is nearly equal to the given area, and therefore the angle chosen will be taken for the section.

This angle must now be investigated for tension, one rivet-hole being taken out of the section. The net area is $2.48 - (\frac{7}{8} + \frac{1}{8})\ \frac{3}{8} = 2.10$ square inches. The required net area is $\frac{30\,500}{16\,000 \times 1\frac{1}{4}} = 1.53$ square inches, which shows the angle to be sufficiently strong.

Single shear determines the number of rivets to be required. These are:

$$\frac{2 \times 30\,500}{6\,013 \times 1\frac{1}{4}} = 8 \text{ field rivets.}$$

All rivets in the lower lateral system are field rivets, since this system also must be riveted up in the field after the trusses are swung into place.

The total stress in the second panel is 21 500 pounds, and a $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch angle, with an area of 2.30 square inches and a least radius of gyration of 0.90, will be assumed and examined. The allowable unit-stress in compression is $1\frac{1}{4}\ (16\,000 - 70 \times \frac{90}{0.90}) = 11\,250$ pounds per square inch, and the required area is $\frac{21\,500}{11\,250} = 1.91$ square inches. Since this is less than the given area, and since the size of the angle (74) is the smallest allowable, this angle must be used.

It is required that this member shall have a net area of $\frac{21\,500}{16\,000 \times 1\frac{1}{4}} = 1.08$ square inches in tension. The net area of the angle, one rivet-hole being taken out, is 1.92 square inches, which shows the angle to be safe in tension.

The number of rivets required is determined by single shear,

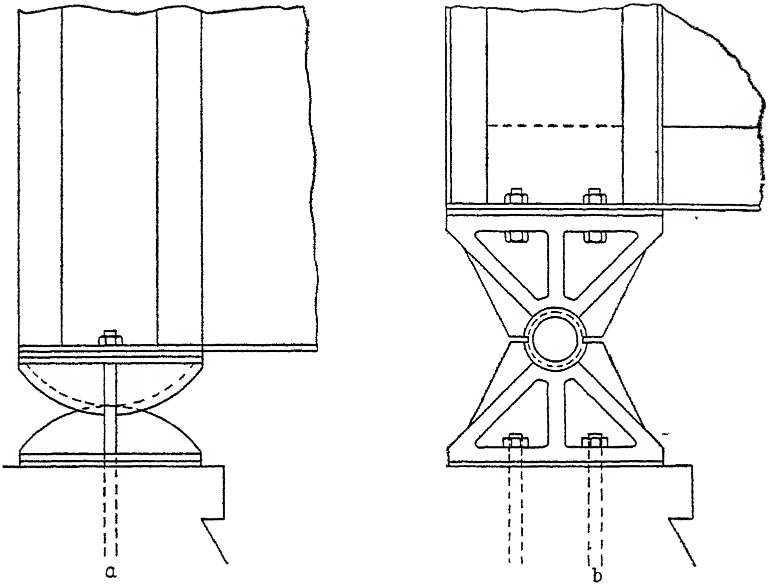


Fig. 196 Two Types of Bearings.

since they tend to shear off between the member itself and the connecting plates. The number required is $\frac{2 \times 21\,500}{6\,013 \times 1\frac{1}{4}} = 6$ field rivets.

Since the above angle is the smallest that can be used, and since the remaining angles of the panel of the lateral bracing have smaller stresses than the one just designed, it is evident that this size angle must be used in all panels of the lower lateral system other than the first.

92. **The Shoes and Roller Nests.** For bridges of short spans and for plate-girders whose spans require rocker bearings to be provided (60), several different classes of bearings are in use. Two such bearings are shown in Fig. 196 (*a* and *b*). The type illustrated by *a* is seldom used on any spans except plate-girders. That shown in *b*

may be used on either plate-girders or small truss spans; it is the invention of Mr. F. E. Schall, Bridge Engineer of the Lehigh Valley Railroad, who uses it on plate-girders. It has given very great satisfaction; and for simplicity of design and also for economy it is to be recommended. Some railroads have used a bearing which consisted of a lens-shaped disc of phosphor-bronze, the faces of which fitted into corresponding indentations in both the masonry and the bearing plates. One advantage of this bearing is that it allows movements due to the deflection of the girder, and also lateral deflection of the floor-beam. It is claimed to have given satisfaction.

A bearing which is used on both short-span and long-span bridges

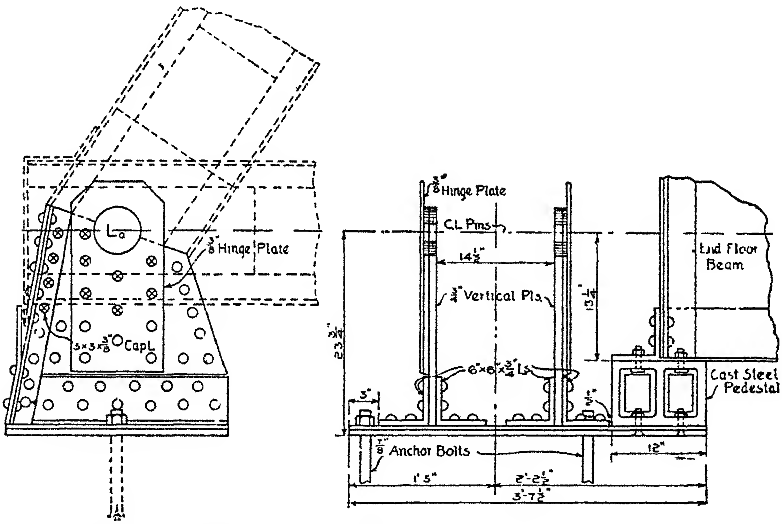


Fig. 197. Bearing Adapted to Bridges of Both Short and Long Span

is shown in Fig. 197. This class of bearing will be used. The end reaction of the bridge proper is equal to the vertical component of the stress in the end-post, and is $\frac{30\ 1}{36.7} \times 410\ 500 = 336\ 500$ pounds, which requires a bearing area (19) on the masonry, of $\frac{336\ 500}{600} = 561$ square inches. According to the table on page 193, the masonry plate will be 28 inches long.

The total bearing area for one of the vertical plates is:

$$\frac{336\ 500}{2 \times 24\ 000} = 7\ 0 \text{ square inches;}$$

and the total required thickness is:

$$\frac{7.0}{6.25} = 1.12 \text{ inches},$$

a $6\frac{1}{4}$-inch pin being used at L_0. Since the vertical plates will be made $\frac{3}{4}$ inch thick, this leaves a remainder of $\frac{3}{8}$ inch to be made up of pin-plates.

The amount of stress which is carried by the $\frac{3}{8}$-inch pin-plate is $\frac{0.375}{1.125} \times \frac{336\,500}{2} = 56\,100$ pounds. These plates will tend to shear off the rivets at a plane between the plates, and therefore $\frac{56\,100}{7\,220} = 8$ shop rivets will be required to fasten them to the vertical plate.

Since the length of the masonry plate is 28 inches, and the total area required is 561 square inches, the required width is $\frac{561}{28} = 20$ inches. The actual width will be greater than this, since it must be sufficient to allow for the connecting angles and also for the bearings of the end floor-beam. The connecting angles should be $\frac{3}{4}$ inch thick, and should not be less than 6 by 6 inches; and the plates to which they are connected should not be less than $\frac{3}{4}$ inch in thickness, and likewise they should not be greater, on account of the punching. The bottom plate should extend outward about 3 inches, in order to allow sufficient room for the anchor bolts, which should be $\frac{7}{8}$ inch in diameter and should extend into the masonry at least 8 inches.

In addition to the reaction of the bridge proper, the masonry plate must be of sufficient area to give bearing for the end reaction of the end floor-beam. The maximum end reaction (see Article 83, p. 197) is 104 740 pounds. The bearing area required on the masonry is $\frac{104\,740}{600} = 175$ square inches; and assuming that the base of the bearing will be 12 inches long (see Fig. 197), the required length will be 14.6 inches. Usually, however, the bearing is extended the entire length of the masonry plate, which is 28 inches in this case.

The distance from the center of the pin to the top of the masonry will be the same for both the fixed and the roller end. This distance should be such that the angles of the shoe will clear the bottom chord member and allow the floor-beam to rest upon the plate as shown. Since the first section of the bottom chord is $18\frac{1}{2}$ inches deep, the top of the angles of the two must be at least $9\frac{1}{4}$ inches from the center line

of pins This requires that the distance from the center line of the pin to the base of the angle shall be at least $(9\frac{1}{4} + 6) = 15\frac{1}{4}$ inches, or more.

The tops of all floor-beams are at the same height, and the bottoms of the intermediate floor-beams must be on a level with the bottom of the first section of the lower chord (see Fig. 174). This requires that the bottom of the intermediate floor-beams shall be $9\frac{1}{4}$ inches below the center line of pins, and this brings the top of the floor-beams $(48\frac{1}{4} - 9\frac{1}{4}) = 39$ inches above the center line of the pins. Since the end floor-beam is $52\frac{1}{4}$ inches deep, back to back of angles, the

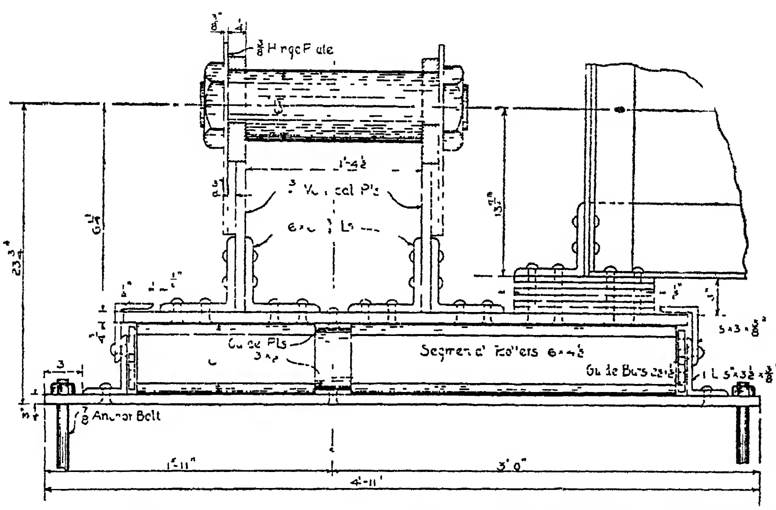


Fig 198 Type of Bearing Construction where End Floor-Beam Does Not Rest Directly on Bearing or Masonry Plate Grillage of Iron Bars Used instead of Cast-Steel Pedestal

lower flange will be $(52\frac{1}{4} - 39) = 13\frac{1}{4}$ inches below the center line of pins. In case the end floor-beam does not rest directly upon the bearing plate or the masonry plate, the intervening space is filled out with a grillage of iron bars or a cast-steel pedestal, as shown in Figs. 197 and 198.

The small plates upon the side of the shoe, going entirely around the pin, are called the *shoe hinge-plates.* These do not take any stress, and require only sufficient rivets to hold them in position. They are used during erection to keep the end-post in line; and after erection their function is to keep the end-post on the shoe, and to prevent it from having any upward motion due to the vibration of the structure.

The rivets through the vertical legs of the shoe angles are in double bearing in the $\frac{3}{4}$-inch angles, in single bearing in the vertical plate, and in double shear. A rivet in double shear has a less value than in bearing in the plates. This value is 14 440 pounds, and therefore the number of shop rivets required through the vertical legs of the angles is:

$$\frac{336\ 500}{2 \times 14\ 440} = 12 \text{ rivets.}$$

The rivets which go through the horizontal leg of the angle and through the cap plate and cap angles, do not take stress. The number of rivets put in is that demanded by the detailing, the rivets in the horizontal legs of the angles usually staggering with those in the vertical legs. The cap plate tends to keep the vertical plates in line, and to keep out the dust and dirt and other deteriorating influences of the elements.

Wherever the rivet-heads tend to interfere with other members or project beyond surfaces which are required to be flat—as, for example, the bottom of the masonry or bearing plates—they must be countersunk (see Carnegie Pocket Companion, p. 212, and Cambria, p. 291).

The space for the anchor bolts, that for the connection angles, and that for the bearing of the end floor-beam, require that the total width of the masonry plate for the fixed end shall be $2 \times \frac{3}{4} + 14\frac{1}{2} + 2 \times 6 + \frac{1}{2} + 3 + 12 = 3$ feet $7\frac{1}{2}$ inches.

The design of the roller end requires that the length of the masonry bearing, the size of the vertical plates and angles, and also the number of rivets shall be the same as that for the fixed end. The width of the masonry plate is determined by the length of the rollers and their connections at the end.

The rollers (62) are required to be 6 inches in diameter, and the unit-stress (19) per linear inch is $6 \times 600 = 3\ 600$ pounds, which requires:

$$\frac{336\ 500}{3\ 600} = 93\ 5 \text{ linear inches.}$$

This is for the reaction of the bridge alone; and in addition to this, there are required for the floor-beam reaction:

$$\frac{104\ 740}{3\ 600} = 29.0 \text{ linear inches.}$$

The total number of linear inches is $93.5 + 29.0 = 122.5$; and if 5 rollers are used, they must be at least $\frac{122.5}{5} = 24.5$ inches long. The masonry plate is only 28 inches long, and therefore cylindrical rollers cannot be used, since they would occupy a space 30 inches or over. Segmental rollers (see Fig. 199) must be used.

The determination of the sizes of the angles which go at the end of the rollers, and also of the guide-plates, is a matter of judgment and experience. Those sizes indicated in Fig. 198, represent good engineering practice, and will be used.

The distance from the center line of pins to the top of the masonry can now be determined, and is $16\frac{1}{4} + \frac{3}{4} + 6 + \frac{3}{4} = 23\frac{3}{4}$ inches.

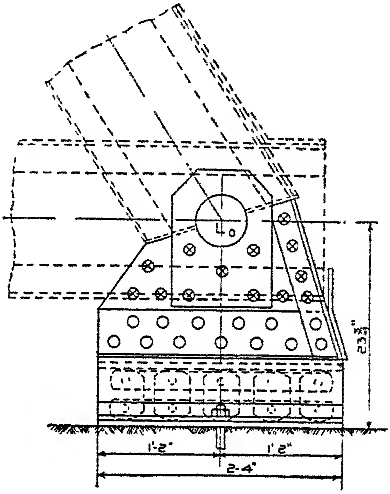


Fig 199. Segmental Rollers Used for Bearings in Space under 30 Inches.

On account of putting in sufficient connections and angles as shown in Fig. 198, the masonry plate must be considerably wider than that theoretically determined. According to Fig. 198, the total width must be as follows, and the width should be computed in two parts, as the plate is not symmetrical about the center line of the truss:

From center line to outer edge·

$$\frac{14\frac{1}{2}}{2} + \frac{3}{4} + 6 + \frac{1}{2} + \left(3 - \frac{3}{8} = 2\frac{5}{8}\right) + \left(3\frac{1}{2} - \frac{3}{8} = 3\frac{1}{8}\right) + 3$$
$$= 1 \text{ ft. } 11\tfrac{1}{4} \text{ in., (say, 1 ft. 11 in.).}$$

From center line to inner edge:

$$\frac{14\frac{1}{2}}{2} + \frac{3}{4} + 6 + 1 + 12 + \frac{1}{2} + 2\frac{5}{8} + 3\frac{1}{8} + 3 = 3 \text{ ft. } 0\tfrac{1}{4} \text{ in., say, } 3 \text{ ft. } 0 \text{ in.}$$

Total width 4 ft. 11 in.

Allowing guide-plates and guide-bars of dimensions as shown in Fig. 198, and assuming $\frac{1}{8}$ inch as clearance at the ends, the total length of the rollers is:

$$(4 \text{ ft } 11 \text{ in.}) - 2\left(3 + 3\tfrac{1}{2} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{2}\right) = 44.5 \text{ inches.}$$

This shows them to be amply long enough, as only 22 inches is

theoretically required. Here, as in most cases for single-track spans up to 200 feet in length, the width of the masonry plate is determined by the detail and not by the unit bearing stress.

The guide-plates are small bars riveted to the top of the bottom plates, and serve to keep the rollers in line. The guide-bars are connected to rollers at their ends, and serve to keep the rollers equidistant, therefore causing them to roll easier and keeping them from becoming worn by contact with each other.

The expansion (59) must be allowed for at the rate of $\frac{1}{8}$ inch for every 10 feet in length of span. This makes a total allowed for temperature of expansion of $\frac{147}{10} \times \frac{1}{8} = 1\frac{3}{4}$ (say 2) inches. No slotted holes are to be provided for the anchor bolts, since they do not go

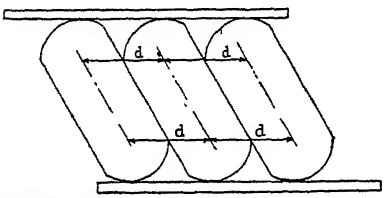


Fig. 200. Binding of Insufficiently Spaced Segmental Rollers

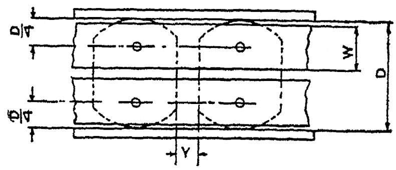


Fig 201 Computation of Spacing for Segmental Rollers

through that part of the bridge which slides. The shoe slides over the rollers, and is kept in place by the angles at the end, which are riveted to the masonry plate (see Fig. 198).

Unless sufficient room is allowed between the segmental rollers, they will tend to bind when the bridge has reached the extreme position for expansion or contraction (see Fig. 200). This distance can be computed from proportions as indicated in Fig. 201, and from the following formulæ*:

$$y = \frac{1}{\cos \phi};$$

$$\phi = \frac{e}{4} \times \frac{360}{3.14\ D},$$

in which e is the amount allowed for the change of temperature, and D is the diameter of the rollers, both being taken in inches. The angle ϕ is in degrees. In the present case, e is 2 inches; D is 6 inches;

* Derived by Prof. Frank B. McKibben of Lehigh University, and published in *Engineering News*, December, 1896.

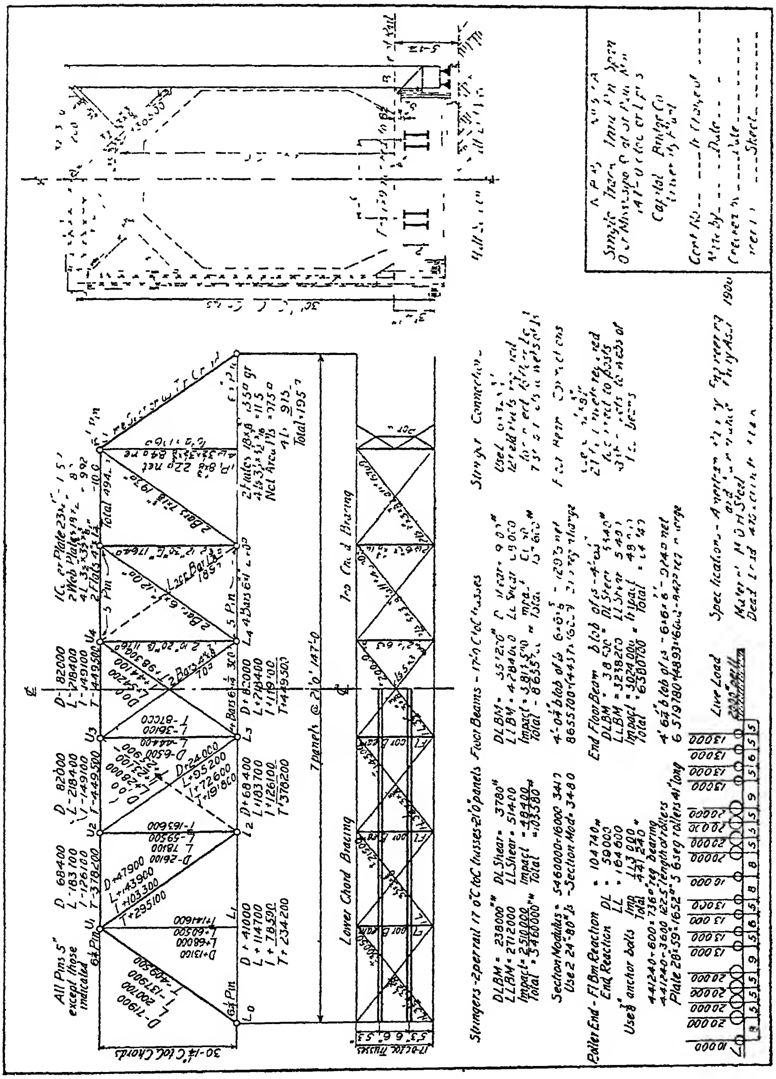


Plate III Stress Sheet of Through Pratt Single-Track Railway Span Designed in the Text.

and ϕ, computed from the above formula, is 9° 30′. Substituting in the equation giving the value for y, there is obtained 1.02 inches (say 1¼ inches) for the distance between rollers. Rollers must not be less in thickness than the total expansion allowed for temperature.

Since there are 5 rollers, there are 4 spaces between them. Also, since the rollers must occupy a space of 28 inches, the length of the masonry plate, each roller must be:

$$\frac{28 - 4 \times 1\frac{1}{4}}{5} = 4.6 \text{ inches (say } 4\tfrac{1}{2} \text{ inches) wide.}$$

The width of the guide-bars must be such as to allow freedom

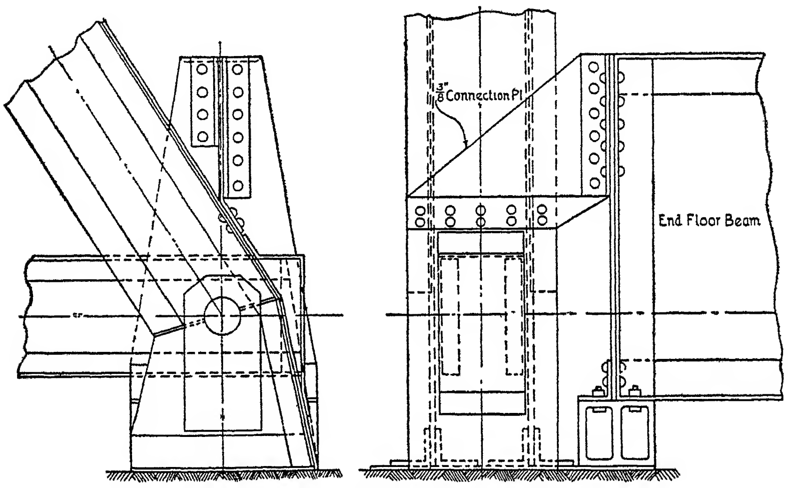


Fig. 202. Details of End Floor-Beam Connections.

of motion for the rollers. The maximum width allowable is given by the formula:*

$$W = \frac{D}{2} \cos \phi,$$

in which ϕ and D are indicated above. This requires the bar to be:

$$W = \frac{6}{2} \times 0.985 = 2.96 \text{ (say } 2\tfrac{1}{2}\text{) inches wide.}$$

93. **The Stress Sheet.** Plate III shows the stress sheet of the bridge which has been designed in the preceding articles. This sheet represents the best current practice among the larger bridge

* Derived by Prof Frank B McKibben of Lehigh University, and published in *Engineering News*, December, 1896.

corporations. It will be noted that very few details are given upon the sheet; also that few rivets are noted, and that sketches showing the manner in which the parts go together are entirely wanting. The shears and moments for the stringers and floor-beams, as well as the reactions and the number of rollers required, are given. This is to save the draftsman the trouble of recomputing values which have necessarily been determined by the designer.

The details of the various members, and also the manner in which the different members are connected, are left to the draftsman, who is under the direct supervision of the engineer in charge of the drafting room, upon whom rests the responsibility for good details. The figures given in the text indicate the best current practice. Figs.

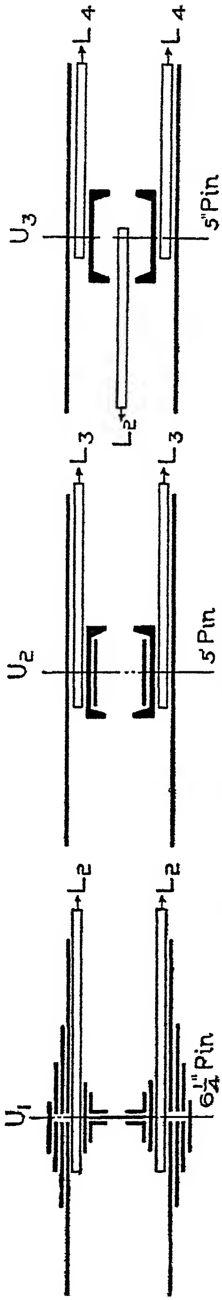


Fig. 203. Details of Packing of Upper Chord Members

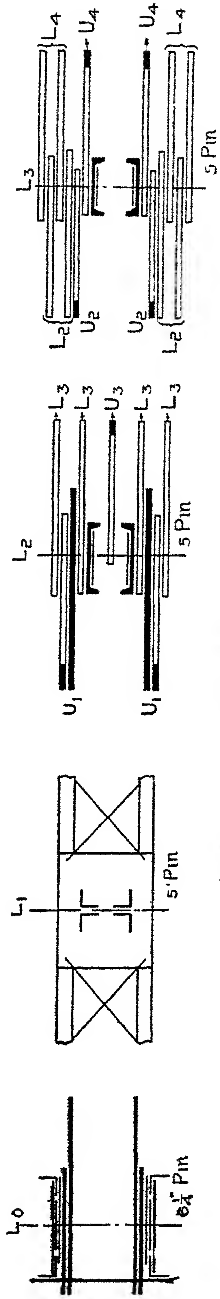


Fig 204 Details of Packing of Lower Chord Members

202 to 204 show details of the end floor-beam connections, and also the packing of the members of the upper and the lower chord. The arrangement here given may be said to be standard for single-track Pratt truss spans up to 200 feet in length.

BIBLIOGRAPHY

The following books are recommended to the student in case it is desired to pursue further the study of the subjects of Bridge Analysis and Bridge Design:

The Theory and Practice of Modern Frame Structures. JOHNSON, BRYAN, and TURNEAURE. John Wiley & Sons, New York, N Y

Roofs and Bridges. MERRIMAN and JACOBY. John Wiley & Sons, New York, N. Y.

Design and Construction of Metallic Bridges. BURR and FALK John Wiley & Sons, New York, N. Y.

Influence Lines for Bridges and Roofs BURR and FALK John Wiley & Sons, New York, N. Y.

Details of Bridge Construction FRANK W SKINNER. McGraw Publishing Company, New York, N. Y.

Steel Mill Buildings. Milo S Ketchum. Engineering News Publishing Company, New York, N Y

Statically Indeterminate Stresses HIROI Engineering News Publishing Company, New York, N. Y.

Stresses in Frame Structures. A. JAY DUBOIS. John Wiley & Sons, New York, N. Y.

Die Zusatzkrafte und Nebenspannungen eiserner Fachwerkbrücken. FR ENGESSER. Julius Springer, Berlin, Germany.

Bridge Drafting WRIGHT and WING. Engineering News Publishing Company, New York, N. Y.

It must not be presumed that the above is a complete list of the books which have been published relating to the theory and practice of Bridge Engineering; neither must it be presumed that the obtaining of information relative to bridges is limited to textbooks on the subject. One of the best sources of information is found in the current engineering periodicals and the "Proceedings" of the various technical societies. The great advantage of these sources is that they give the most up-to-date information, and usually they are very profusely illustrated.

APPENDIX

Use of Tables in Designing Deck Plate Girders. On account of the labor involved in calculating the girder sections for the varying conditions which arise in Bridge Engineering work, the series

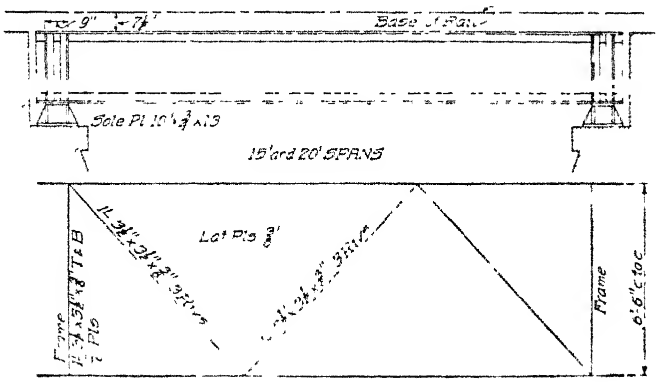


Fig. 205. Design Diagram for 15- and 20-Foot Spans

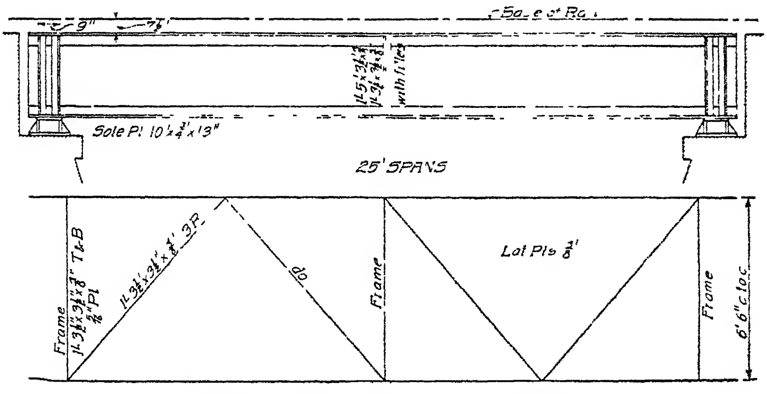


Fig. 206. Design Diagram for 25-Foot Spans

of tables for E 30, E 40, and E 50 loadings, pages 265 to 276 have been prepared. By the aid of these tables and the diagrams, Figs. 205 to 218, which give the sizes of members of end and intermediate cross frames and stiffeners for the various loadings and spans considered, the design may be immediately determined.

To illustrate the use of Table XXIX, let us assume that we are to prepare a stress sheet for a deck plate girder of 64-foot span center to center of bearings, loading E 50.

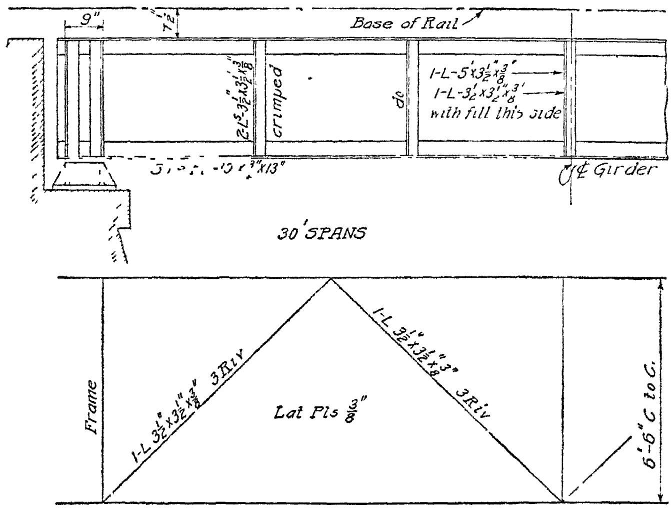


Fig 207 Design Diagram for 30-Foot Spans

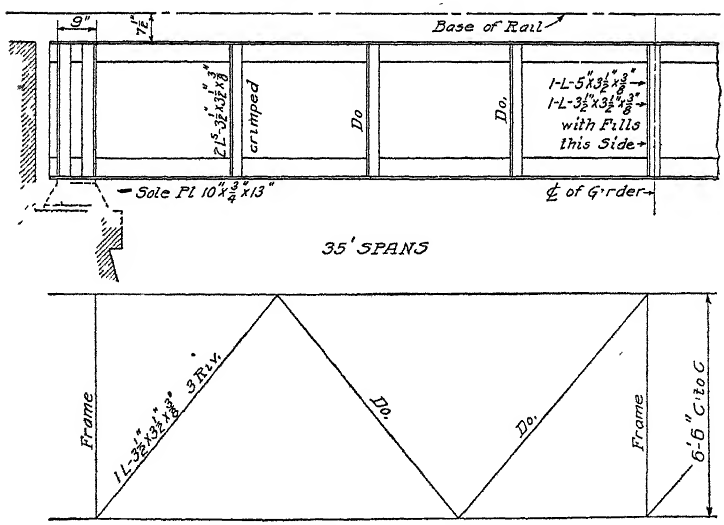


Fig 208 Design Diagram for 35-Foot Spans

On page 274, last column, we find the section to be:

Web. 78″×$\frac{3}{8}$″; Each flange—2 Ls 6″×6″×$\frac{5}{8}$″; 1 Pl. 14″×$\frac{1}{2}$″×48′; 1 Pl. 14″×$\frac{7}{16}$″×40′; 1 Pl. 14″× $\frac{3}{8}$″×31′.

On page 258, Fig. 212, we find a stress sheet without the sizes put on the web and flanges. Completing the operation we have

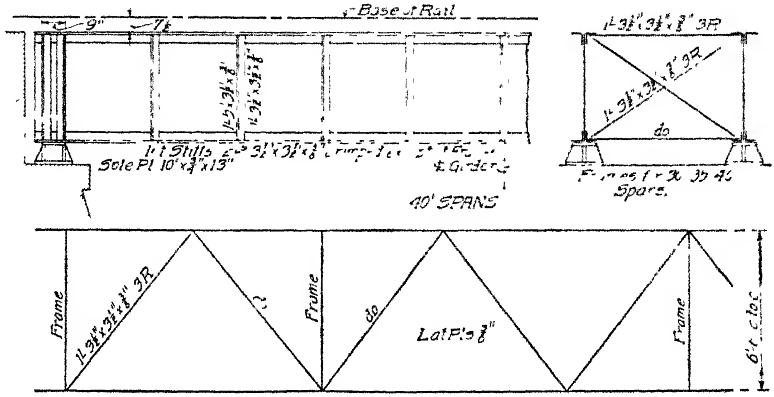


Fig 209. Design Diagram for 40-Foot Spans]

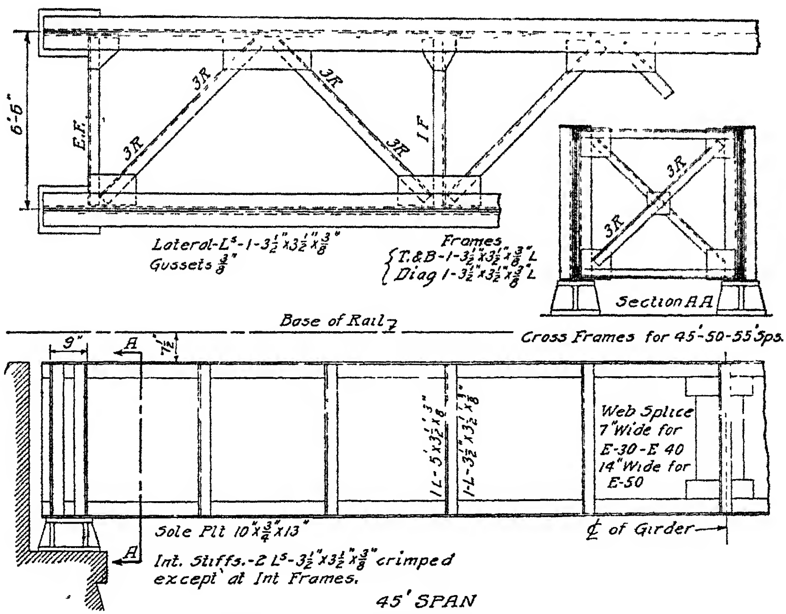


Fig. 210. Design Diagram for 45-foot Spans

on page 264 the completed stress sheet, Fig. 218, which may now be sent to the bridge company to bid on or to prepare the shop drawings.

Methods of Using Unit Cost Tables. The following unit prices will be found convenient for a preliminary estimate of the

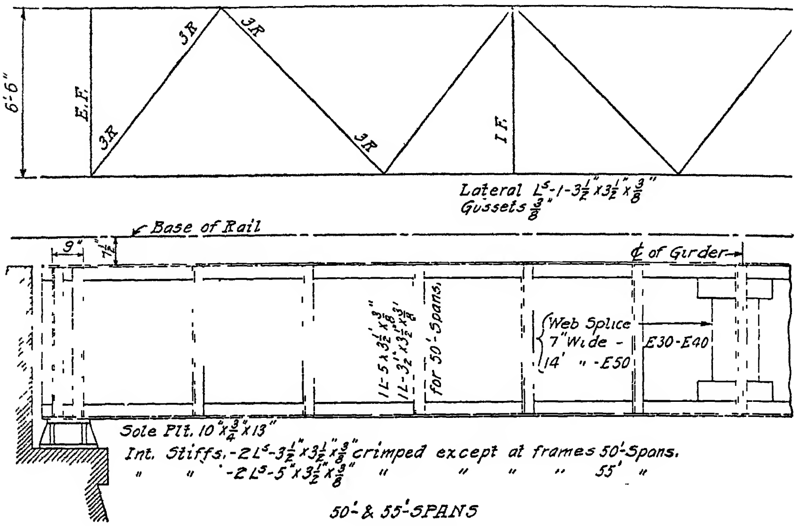


Fig 211. Design Drawing for 50- and 55-Foot Spans

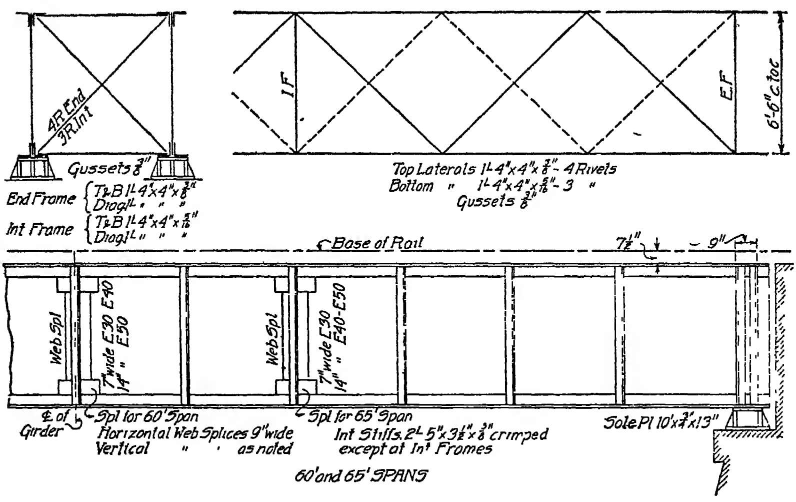


Fig 212. Design Diagram for 60- and 65-Foot Spans

costs of bridges. By making an estimate based on these unit prices one is able to tell what a fair charge should be and can therefore tell whether or not the bids submitted are reasonable.

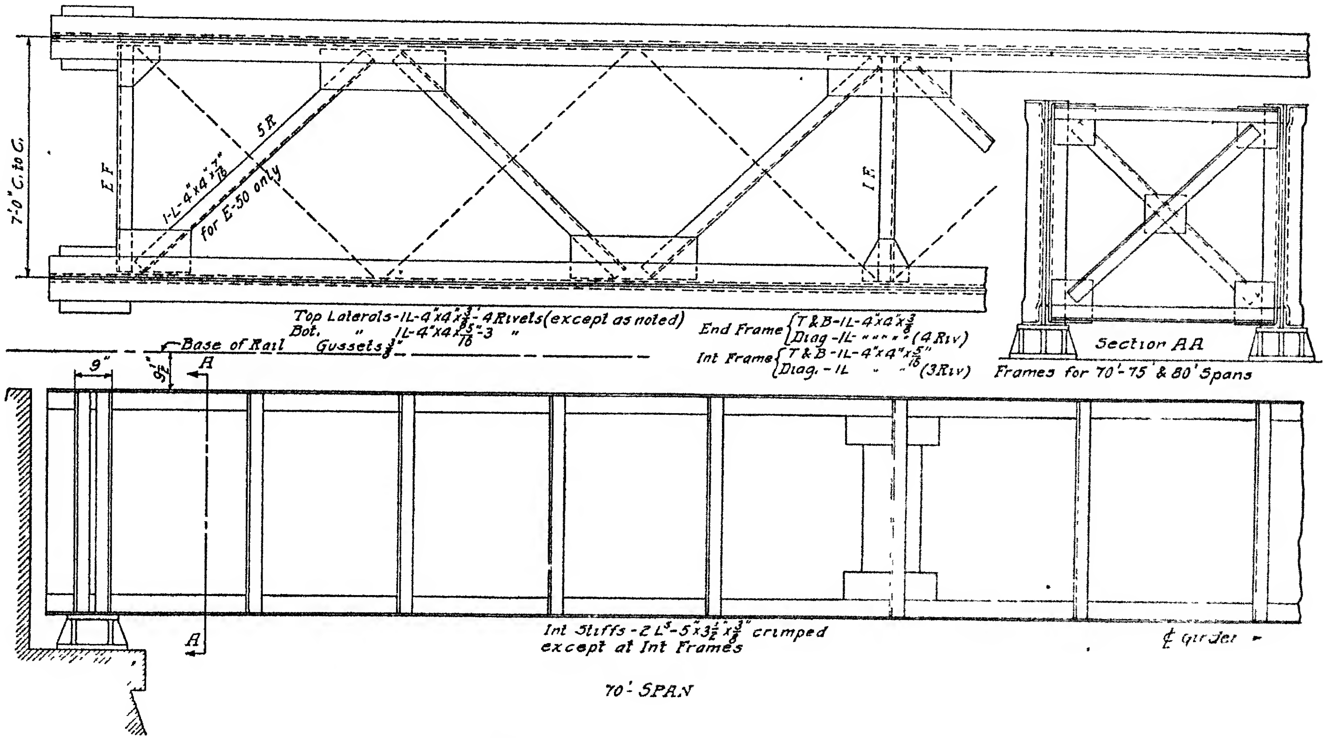


Fig 213. Design Diagram for 70-Foot Spans

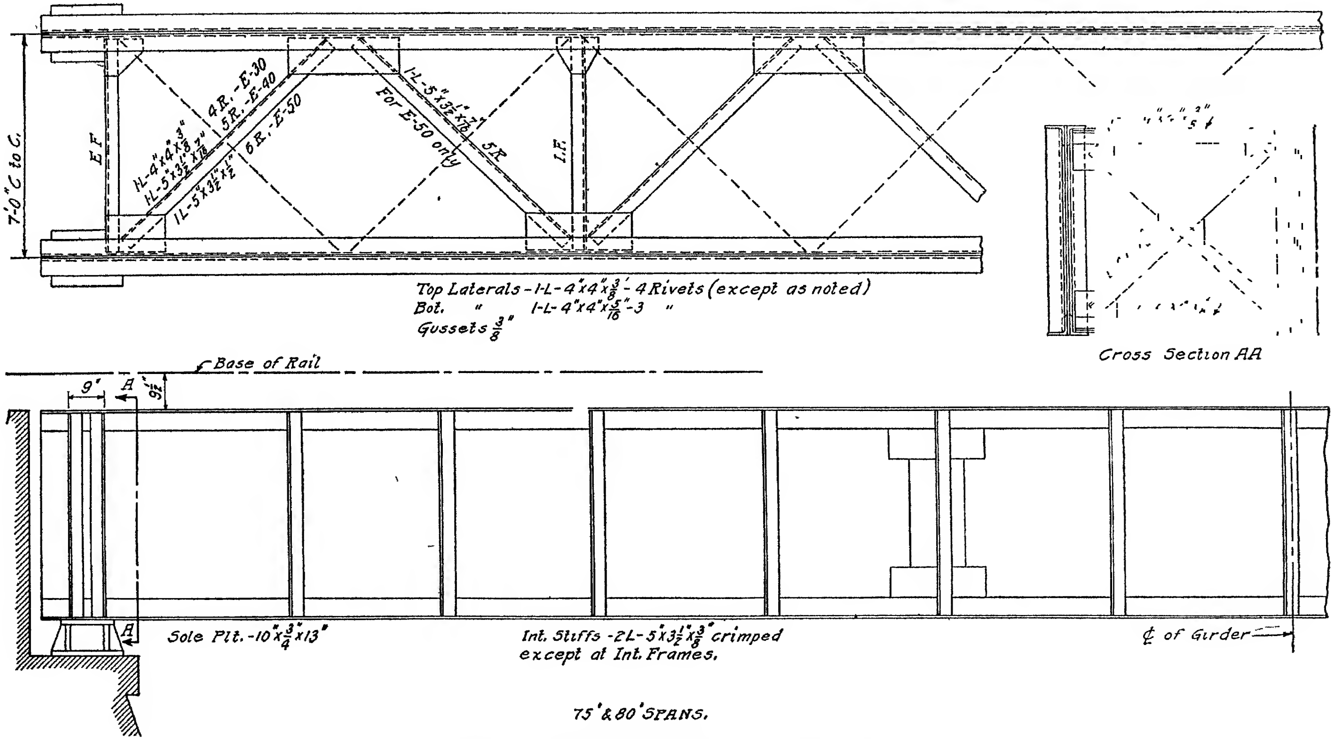


Fig. 214 Design Diagram for 75- and 80-Foot Spans

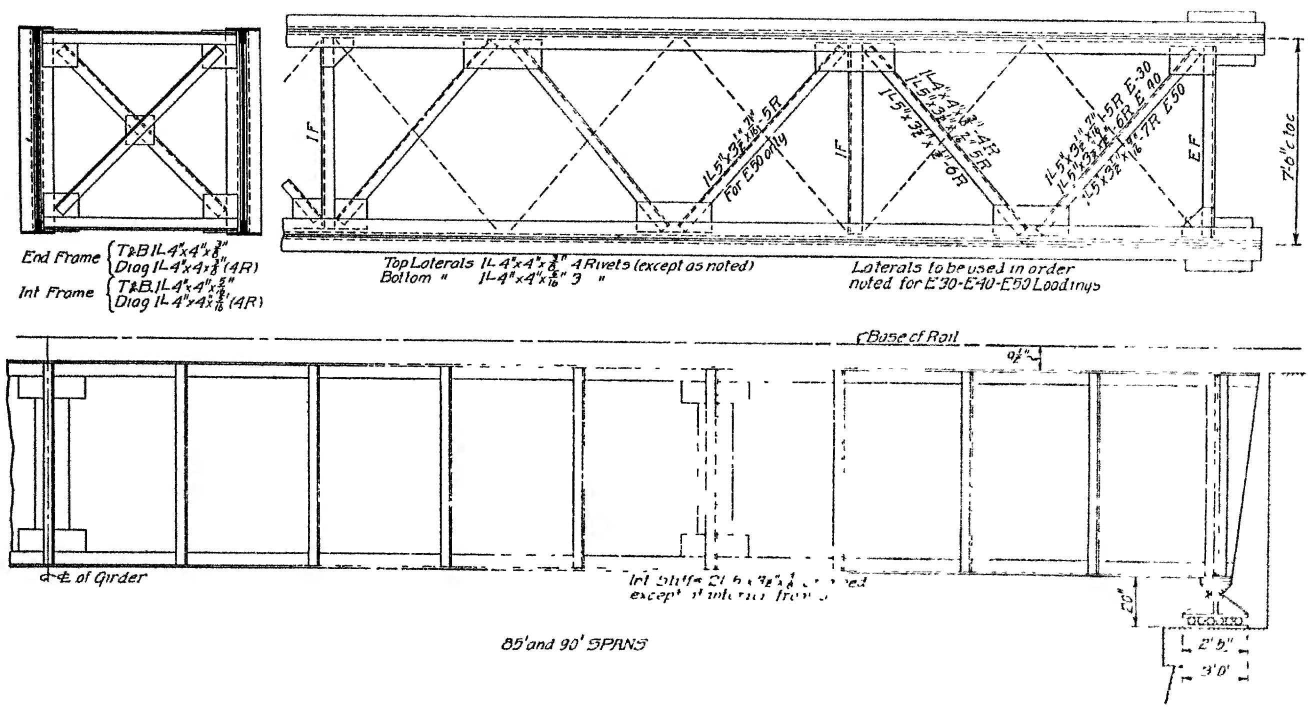


Fig. 215. Design Diagram for 85- and 90-Foot Spans

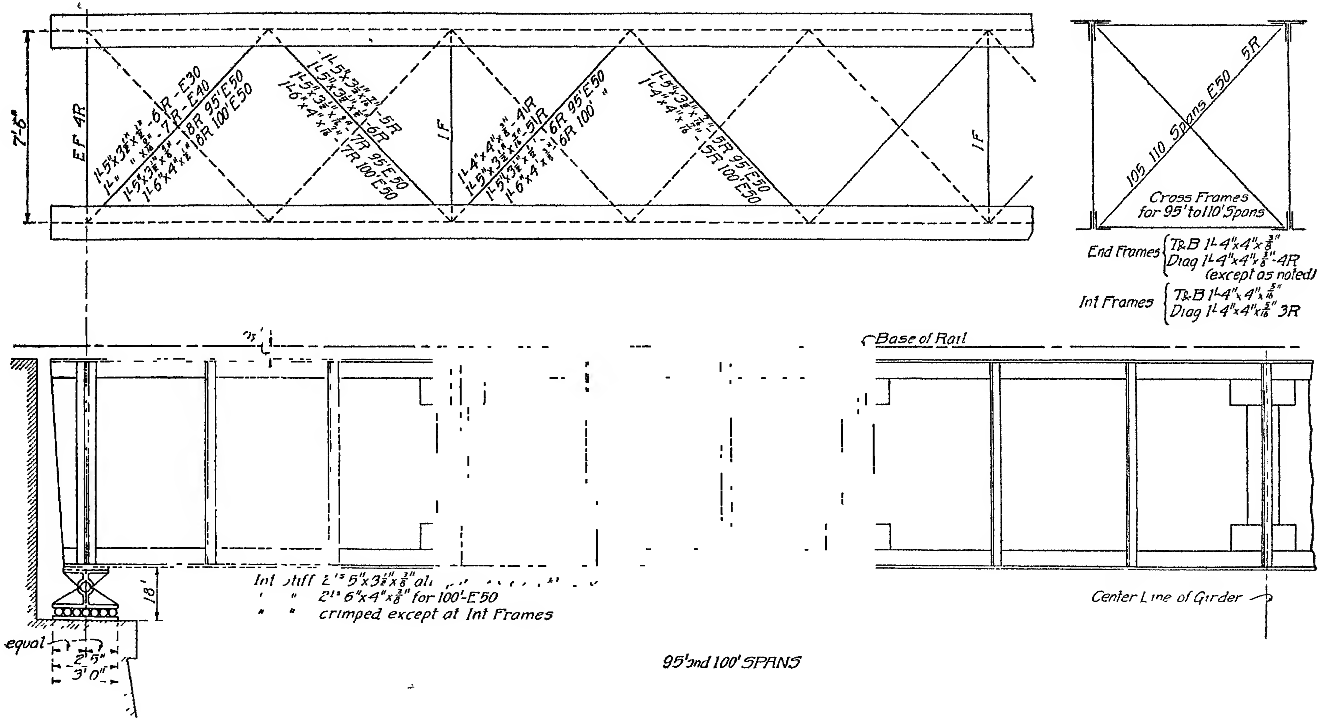


Fig. 216. Design Drawing for 95- and 100-Foot Spans

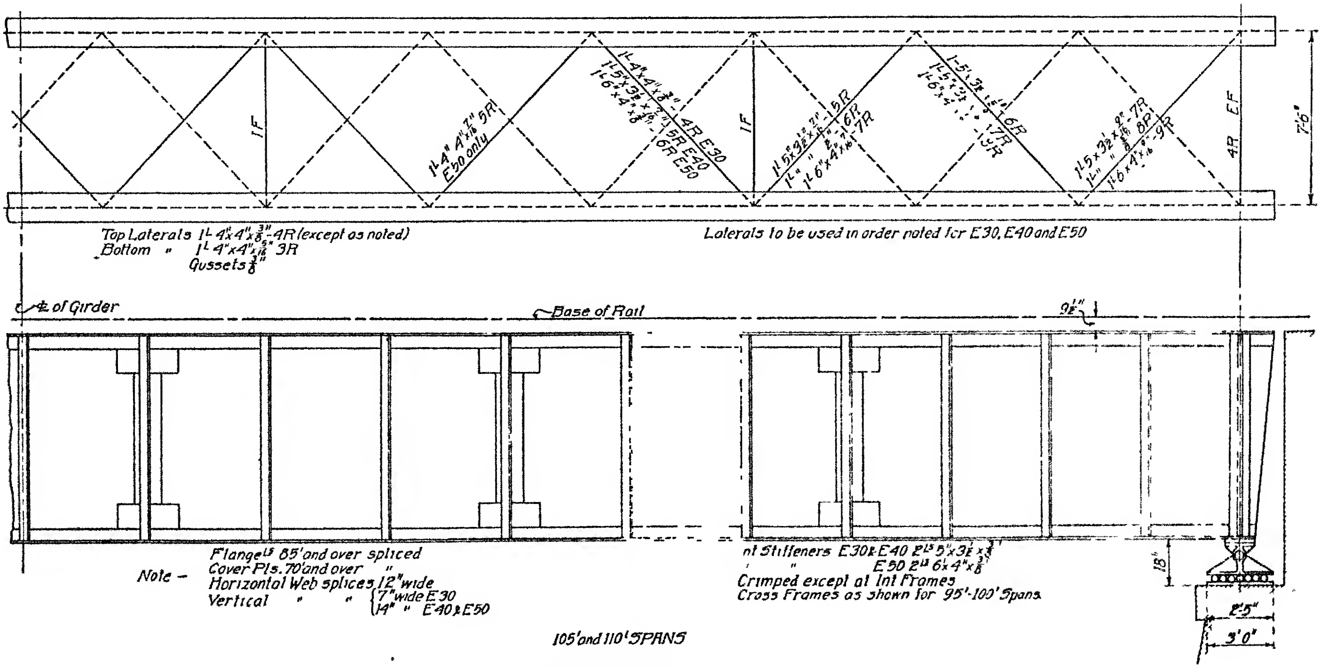


Fig. 217. Design Diagram for 105- and 110-Foot Spans

In order to illustrate the use of Tables XXIX and XXX, it is assumed that we are required to determine the approximate cost of a 45-foot deck plate girder span of E 50 capacity, the weight being 27,400 pounds, as determined from Table XXIX, page 274, and the other quantities are as given below:

Structural steel erected. .	27,400 lb	at	$ 0 04	$1,096 00
Dry earth excavation. . . .	105 C. Y.	at	0 50	52 50
Wet earth excavation. .	371 C. Y.	at	3 00	1,113 00
Cofferdam. . . .	2,192 ft. BM	at	50 00	109 60
Foundation piling. . . .	1,426 lin. ft.	at	0 50	713 00
Abutment concrete. . .	862 C. Y.	at	10 00	8,620 00
Guard timbers 6×8=. .	760 ft BM	at	40 00	30 40
24 ties 8″×10″×10′-0″ (14″ cts.). .	1,600 ft. BM	at	40.00	64 00
Probable cost. .				$11,798 50

If this bridge is laid out and supervised by anyone other than the contractor, a certain amount must be added for engineering

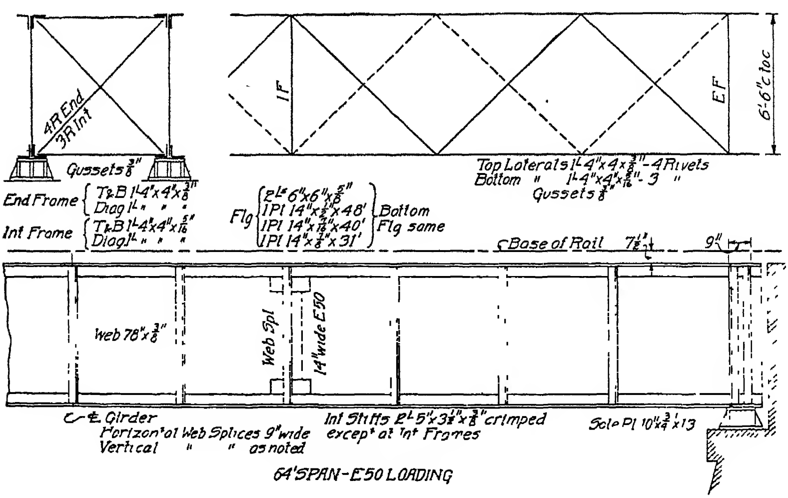


Fig. 218 Completed Stress Sheet for 64-Foot Span and E-50 Loading

and overhead expenses, usually 10 per cent, which would make the cost

Contract price.	$11,798 50
Engineering and overhead—10%	1,179 85
	$12,978 35

TABLE XXIX
Design of Deck Plate Girders

For spans in between those given, use next sizes For weights for spans not given, interpolate.

Assumed Live Loading	E. 30					
Length c to c of girders	15′	20′	25′	30′	35′	40′
Length c to c bearings	13′-9″	18′-9″	23′-6″	28′-6″	33′-6″	38′-6″
Depth b to b of Ls	1′-9½″	2′-3½″	2′-10½″	3′-6½″	3′-9½″	4′-9½″
Dead load steel # p l ft (effective)	200	240	290	340	370	410
Dead load moment, in -lb	85,000	169,000	286,000	451,000	648,000	931,000
Live load moment, in -lb	955,000	1,640,000	2,478,000	3,391,000	4,402,000	5,518,000
Impact load mom, in -lb	913,000	1,546,000	2,298,000	3,098,000	3,960,000	4,892,000
Total moment, in -lb	1,953,000	3,355,000	5,062,000	6,940,000	9,010,000	11,341,000
Effective depth in inches	19.8	25.6	31.2	39.2	42.2	54.2
Flange stress	99,000	131,000	162,000	177,000	214,000	209,000
Flange area required	5.8	7.7	9.5	10.4	12.6	12.3
Flange section	⅜″ web = 1.0 2 Ls 5×3½×⅜″ = 5.3	⅜″ web = 1.3 2 Ls 6×4×⅜″ = 6.4	⅜″ web = 1.6 2 Ls 6×6×⅜″ = 7.9	⅜″ web = 2.0 2 Ls 6×6×7/16″ = 9.3	⅜″ web = 2.1 2 Ls 6×6×½″ = 10.5	⅜″ web = 2.7 2 Ls 6×6×½″ = 10.5
Total net area, sq in	6.3	7.7	9.5	11.3	12.6	13.2
Dead load shear	2,100	8,000	4,100	5,300	6,500	8,100
Live load shear	28,700	35,900	40,900	45,700	50,600	55,300
Impact load shear	27,400	33,800	37,900	41,700	45,500	49,000
Total shear	58,200	72,700	82,900	92,700	102,600	112,400
Web area required, sq in	5.8	7.3	8.3	9.3	10.3	11.2
Web section	21×⅜″	27×⅜″	34×⅜″	42×⅜″	45×⅜″	57×⅜″
End pitch through flange	2¾″	3″	3″	3″	3″	3″
End stiffeners (with fillers)	4 Ls 3½×3½×⅜″	4 Ls 5×3½×⅜″	4 Ls 5×3½×⅜″	4 Ls 5×3½×⅜″	4 Ls 5×3½×⅜″	4 Ls 5×3½×⅜″
Base of end casting	12×21″	12×21″	12×21″	12×21″	12×21″	16×21″
Base of rail to masonry	3′-0″	3′-6″	4′-1″	4′-9″	5′-0″	6′-1″
Wt of steel per span (actual)	4,200	6,200	8,800	11,700	14,500	19,100
Dead load track 400 lb p l ft						

TABLE XXIX—(Continued)

Assumed Live Loading	E 30				
Length o. to o. girders	45′–0″	50′–0″	55′–0″	60′–0″	65′–0″
Length c. to c. bearings	43′–3″	48′–3″	53′–3″	58′–0″	63′–0″
Depth back to back Ls	5′–6½″	5′–9½″	6′–0½″	6′–0½″	6′–6½″
Dead load steel # p l ft. (effective)	480	500	540	610	650
Dead load moment in -lb.	1,233,000	1,572,000	1,981,000	2,550,000	3,128,000
Live load moment in -lb	6,749,000	8,060,000	9,427,000	11,010,000	12,740,000
Impact load moment in -lb.	5,899,000	6,947,000	8,103,000	9,228,000	10,524,000
Total moment in -lb	13,833,000	16,579,000	19,421,000	22,788,000	26,392,000
Effective depth in inches	63 1	66 0	69 0	70 0	76 2
Flange stress	220,000	251,000	282,000	326,000	346,000
Flange area required	13 0	14 7	16 6	19 2	20 4
Flange section	$\frac{1}{8}$″ web = 3 1 2 Ls 6 × 6 × ½″ = 10 5	$\frac{1}{8}$″ web = 3 2 2 Ls 6 × 6 × $\frac{9}{16}$″ = 11 7	$\frac{1}{8}$″ web = 3 4 2 Ls 6 × 6 × $\frac{5}{8}$″ = 13 0	$\frac{1}{8}$″ web = 3 4 2 Ls 6 × 6 × $\frac{5}{8}$″ = 11 7 Pl 14 × $\frac{3}{8}$ × 32 = 4 5	$\frac{1}{8}$″ web = 3 7 2 Ls 6 × 6 × $\frac{5}{8}$″ = 11 Pl 14 × $\frac{7}{16}$ × 36 = 5
Total net area sq inches	13 6	14 9	16 4	19 6	20 7
Dead load shear	9,500	10,800	12,600	14,600	16,500
Live load shear	59,800	63,800	67,700	71,600	76,000
Impact load shear	52,300	55,000	57,500	60,000	62,700
Total shear	121,600	129,000	137,800	146,200	155,200
Web area required sq. in.	12 2	13 0	13 8	14 6	15 5
Web section	66 × $\frac{3}{8}$″	69 × $\frac{3}{8}$″	72 × $\frac{3}{8}$″	72 × $\frac{3}{8}$″	78 × $\frac{3}{8}$″
End pitch through flange	3″	3″	3″	3″	3″
End stiffeners (with fillers)	4 Ls 5 × 3½ × $\frac{3}{8}$″	4 Ls 5 × 3½ × $\frac{3}{8}$″	Ls 5 × 3½ × $\frac{3}{8}$″	4 Ls 5 × 3½ × $\frac{3}{8}$″	4 Ls 5 × 3½ × $\frac{3}{8}$″
Base of end casting	16 × 24″	16 × 24″	16 × 24″	16 × 24″	16 × 24″
Base of rail to masonry	6′–10″	7′–1″	7′–4″	7′–4″	7′–10″
Actual weight of span (steel only)	22,800	27,000	30,300	37,000	43,500
Dead load track 400 # p l. ft					

TABLE XXIX—(Continued)

E 30				
70'-0"	75'-0"	80'-0"	85'-0"	90'-0"
68'-0"	72'-9"	77'-9"	82'-9"	87'-9"
6'-6$\frac{1}{2}$"	6'-6$\frac{1}{2}$"	7'-0$\frac{1}{2}$"	7'-6$\frac{1}{2}$"	7'-6$\frac{1}{2}$"
660	700	740	800	870
,023,000	4,763,000	5,671,000	6,676,000	7,912,000
,517,000	16,318,000	18,375,000	20,645,000	22,957,000
,832,000	13,136,000	14,609,000	16,166,000	17,746,000
,372,000	34,217,000	38,655,000	43,487,000	48,615,000
77 0	77 1	83 7	89 7	89 6
395,000	445,000	462,000	485,000	543,000
23 3	26 2	27 2	28 5	31 9
web =3 7	$\frac{3}{8}$" web =3 7	$\frac{3}{8}$" web =4 6	$\frac{3}{8}$" web =4 9	$\frac{3}{8}$" web =4 9
6×$\frac{9}{16}$"=10 6	2 Ls 6×6×$\frac{5}{8}$"=11 7	2 Ls 6×6×$\frac{1}{2}$"=9 5	2 Ls 6×6×$\frac{1}{2}$"=9 5	2 Ls 6×6×$\frac{5}{8}$"=11 7
×$\frac{3}{8}$x47'= 4 5	1 Pl 14×$\frac{1}{2}$×51'=6 0	1 Pl 16×$\frac{1}{2}$×59'=7 0	1 Pl 16×$\frac{1}{2}$×62'=7 0	1 Pl 16×$\frac{9}{16}$×65'=7 9
×$\frac{3}{8}$×35'=4 5	1 Pl 14×$\frac{7}{16}$×38'=5 3	1 Pl 16×$\frac{1}{2}$×44'=7 0	1 Pl 16×$\frac{1}{2}$×47'=7 0	1 Pl 16×$\frac{9}{16}$×49'=7 9
23 3	26 7	28 1	28 4	32 4
19,700	21,800	24,300	26,900	30,000
80,700	85,600	90,800	95,900	100,800
65,800	69,000	72,300	75,100	78,200
166,200	176,400	187,400	197,400	209,000
16 7	17 7	18 8	19 8	20 9
78×$\frac{3}{8}$"	78×$\frac{3}{8}$"	84×$\frac{7}{16}$"	90×$\frac{7}{16}$"	90×$\frac{7}{16}$"
s 5×3$\frac{1}{2}$×$\frac{3}{8}$"	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$"	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$"	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$"	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$"
21×25"	21×25"	21×25"	29×30$\frac{1}{4}$"	29×30$\frac{1}{4}$"
3"	3"	3"	3"	3"
8'-1"	8'-1"	8'-7"	10'-0"	10'-0"
47,300	53,800	60,800	73,800	84,000

TABLE XXIX—(Continued)

Assumed Live Loading	E 30			
Length o. to o. girder	95′–0″	100′–0″	105′–0″	110′–0″
Length c. to c. bearings	92′–3″	97′–3″	102′–3″	107′–3″
Depth b. to b. of Ls	8′–3½″	8′–3½″	9′–0½″	9′–0½″
Dead load steel p. lin. ft. trk. (effective)	910	950	1,040	1,060
Dead load moment in.-lb.	9,004,000	10,291,000	12,076,000	13,455,000
Live load moment in.-lb.	25,110,000	27,572,000	30,192,000	33,175,000
Impact load moment in.-lb.	19,209,000	20,845,000	22,523,000	24,485,000
Total moment in.-lb.	53,323,000	58,708,000	64,791,000	71,115,000
Effective depth, inches	98.5	98.4	107.7	108.1
Flange stress	541,000	587,000	602,000	658,000
Flange area required	31.8	35.1	35.4	38.7
Flange section	⅛″ web = 5.4	⅛″ web = 5.4	⅛″ web = 6.8	⅛″ web = 6.8
	2 Ls 6×6×⅝″ = 11.7	2 Ls 6×6×⅝″ = 11.7	2 Ls 6×6×⅝″ = 11.7	2 Ls 6×6×⅝″ = 11.7
	1 Pl. 16×$\frac{9}{16}$×67′ = 7.9	1 Pl. 16×⅝×72′ = 8.8	1 Pl. 16×⅝×75′ = 8.8	1 Pl. 16×½×82′ = 7.0
	1 Pl. 16×½×49′ = 7.0	1 Pl. 16×⅝×54′ = 8.8	1 Pl. 16×$\frac{9}{16}$×55′ = 7.9	1 Pl. 16×½×69′ = 7.0
				1 Pl. 16×½×52′ = 7.0
Total net area sq. in.	32.0	34.7	35.2	39.5
Dead load shear	32,500	35,300	39,500	41,700
Live load shear	105,000	110,000	114,500	119,000
Impact load shear	80,300	83,200	85,500	88,000
Total shear	217,800	228,500	239,500	248,700
Web area required sq. in.	21.8	22.9	23.9	24.9
Web section	99×$\frac{7}{16}$″	99×$\frac{7}{16}$″	108×½″	108×½″
End pitch through flange	3″	3″	3″	3″
End stiffeners with fillers	4 Ls 5×3½×$\frac{7}{16}$″	4 Ls 5×3½×$\frac{7}{16}$″	4 Ls 5×3½×½″	4 Ls 5×3½×½″
Masonry plate	29×30¼″	29×30¼″	29×30¼″	29×30¼″
Base of rail to masonry	10′–9″	10′–9″	11′–8″	11′–6″
Weight of steel per span (actual)	90,500	99,200	115,700	124,500
Dead load track 500 # p. lin. ft.				

TABLE XXIX—(Continue

Assumed Live Loading	E 40			
Length o. to o. of girders	15′	20′	25′	
Length c. to c bearings	13′–9″	18′–9″	23′–6″	
Depth b. to b of Ls	2′–2¼″	2′–6½″	3′–2½″	
Dead load steel # p. l. ft (effective)	220	260	330	
Dead load moment, in.-lb	88,000	174,000	302,000	
Live load moment, in -lb	1,276,000	2,186,000	3,304,000	
Impact load moment in -lb.	1,219,000	2,060,000	3,063,000	
Total moment, in -lb	2,583,000	4,420,000	6,669,000	
Effective depth in inches	24 8	28.6	35 2	
Flange stress	104,000	155,000	190,000	
Flange area required	6 1	9 1	11 2	
Flange section	⅛″ web =1 2 2 Ls 5×3½×⅜″=5 3	⅛″ web =1 6 2 Ls 6×4×7/16″=7 5	⅛″ web =2 1 2 Ls 6×6×7/16″=9 3	2
Total net area, sq. in	6 5	9 1	11.4	
Dead load shear	2,100	3,100	4,200	
Live load shear	38,100	47,900	54,500	
Impact load shear	36,400	45,100	50,500	
Total shear	76,600	96,100	109,200	
Web area required, sq in	7 7	9 6	10 9	
Web section	26×⅜″	30×7/16″	38×7/16″	
End pitch through flange	2¾″	3″	3″	
End stiffeners (with fillers)	4 Ls 3½×3½×⅜″	4 Ls 5×3½×⅜″	4 Ls 5×3½×⅜″	
Base of end casting	12×21″	12×21″	16×24″	
Base of rail to masonry	3′–5″	3′–9″	4′–6″	
Wt. of steel per span (actual)	4,800	7,000	10,400	
Dead load track 400 lb p. l ft.				

TABLE XXIX—(Continued)

Assumed Live Loading	E 40		
Length o to o of girders	45′–0″	50′–0″	55′–0″
Length c to c bearings	43′–3″	48′–3″	53′–0″
Depth back to back Ls	5′–6½″	5′–9½″	6′–0½″
Dead load steel # p l. ft. (effective)	530	550	610
Dead load moment in.-lb.	1,306,000	1,660,000	2,128,000
Live load moment in -lb	8,999,000	10,745,000	12,569,000
Impact load moment in -lb.	7,865,000	9,263,000	10,683,000
Total moment in -lb	18,170,000	21,668,000	25,380,000
Effective depth in inches	63 0	67 3	70 7
Flange stress	288,000	322,000	359,000
Flange area required	16.9	18 9	21.1
Flange section	$\frac{1}{8}$″ web = 3 1 2 Ls 6 × 6 × $\frac{11}{16}$″ = 14 2	$\frac{1}{8}$″ web = 3 2 2 Ls 6 × 6 × $\frac{9}{16}$″ = 10 6 Pl 14 × $\frac{7}{16}$ × 29′ = 5 3	$\frac{1}{8}$″ web = 3.4 2 Ls 6 × 6 × $\frac{5}{8}$″ = 11.7 Pl. 14 × $\frac{1}{2}$ × 33′ = 6 0
Total net area sq. inches	17 3	19 1	21 1
Dead load shear	10,000	11,500	13,400
Live load shear	79,700	85,100	90,200
Impact load shear	69,700	73,300	76,700
Total shear	159,400	169,900	180,300
Web area required sq in.	15 9	17 0	18 0
Web section	66 × $\frac{3}{8}$″	69 × $\frac{3}{8}$″	72 × $\frac{3}{8}$″
End pitch through flange	3″	3″	3″
End stiffeners (with fillers)	4 Ls 5 × 3½ × $\frac{3}{8}$″	4 Ls 5 × 3½ × $\frac{3}{8}$″	4 Ls 5 × 3½ × $\frac{3}{8}$″
Base of end casting	21 × 25″	21 × 25″	21 × 25″
Base of rail to masonry	6′–11″	7′–7″	7′–5″
Actual weight of span (steel only)	25,800	31,200	35,000
Dead load track 400 # p l ft.			

TABLE XXIX—(Continued)

Assumed Live Loading		E 40	
Length o. to o. of girders	70′–0″	75′–0″	80′–0″
Length c. to c. bearings	68′–0″	72′–9″	77′–9″
Depth b. to b of Ls	6′–6½″	6′–6½″	7′–0½″
Dead load steel # p. l ft (effective)	730	780	840
Dead load mom. in -lb.	4,268,000	5,081,000	6,076,000
Live load mom. in -lb.	19,356,000	21,757,000	24,501,000
Impact load mom in -lb.	15,775,000	17,515,000	19,478,000
Total mom. in -lb.	39,399,000	44,353,000	50,055,000
Effective depth in inches	77 5	78 0	84 2
Flange stress	508,000	569,000	595,000
Flange area required	29 9	33 5	35 0
Flange section	⅛″ web = 3 7 2 Ls 6×6×⅝″ = 11 7 Pl. 14×⅝×51′ = 7.5 Pl. 14×$\frac{9}{16}$×37′ = 6 8	⅛″ web = 3 7 2 Ls 6×6×⅝″ = 11 7 Pl. 14×$\frac{9}{16}$×57′ = 6 8 Pl 14×½×48′ = 6 0 Pl 14×½×36′ = 6 0	⅛″ web = 4 6 2 Ls 6×6×⅝″ = 11 7 Pl 16×½×61′ = 7 0 Pl 16×$\frac{7}{16}$×50′ = 6 1 Pl 16×$\frac{7}{16}$×38′ = 6 1
Total net area sq. in.	29 7	34 2	35 5
Dead load shear	20,900	23,300	26,100
Live load shear	107,500	114,100	121,100
Impact load shear	87,700	92,000	96,300
Total shear	216,100	229,400	243,500
Web area required sq in.	21 7	23 0	24 4
Web section	78×⅜″	78×⅜″	84×$\frac{7}{16}$″
End stiffeners (with fillers)	4 Ls 5×3½×$\frac{7}{16}$″	4 Ls 5×3½×$\frac{7}{16}$″	4 Ls 5×3½×½″
Masonry plate	23×29″	23×29″	23×29″
End pitch through flange	2¾″	2½″	3″
Base of rail to masonry	8′–2″	8′–2″	8′–8″
Wt. of steel per span (actual)	53,600	60,800	69,100
Dead load trk 500 # p. lin ft			

TABLE XXIX—(Continued)

E 40			
95′–0″	100′–0″	105′–0″	110′–0″
93′–3″	97′–3″	102′–3″	107′–3″
8 –3$\frac{1}{2}$″	8′–3$\frac{1}{2}$″	9′–0$\frac{1}{2}$″	9′–0$\frac{1}{2}$″
1,020	1,070	1,140	1,160
9,701,000	11,135,000	12,860,000	14,320,000
33,478,000	36,761,000	40,255,000	44,235,000
25,612,000	27,792,000	30,030,000	32,646,000
68,791,000	75,688,000	83,745,000	91,202,000
99 5	99 3	108 5	108 5
691,000	762,000	768,000	841,000
40 6	44 8	45 2	49 5
$\frac{1}{8}$″ web = 5 4	$\frac{1}{8}$″ web = 5 4	$\frac{1}{8}$″ web = 6 8	$\frac{1}{8}$″ web = 6 8
2 Ls 6 × 6 × $\frac{5}{8}$″ = 11 7	2 Ls 6 × 6 × $\frac{3}{4}$″ = 13 9	2 Ls 6 × 6 × $\frac{3}{4}$″ = 13 9	2 Ls 6 × 6 × $\frac{3}{4}$″ = 13 9
1 Pl 16 × $\frac{9}{16}$ × 74′ = 7 9	1 Pl 16 × $\frac{5}{8}$ × 77′ = 8 8	1 Pl 16 × $\frac{5}{8}$ × 79′ = 8 8	1 Pl 16 × $\frac{9}{16}$ × 85′ = 7 9
1 Pl. 16 × $\frac{9}{16}$ × 62′ = 7 9	1 Pl. 16 × $\frac{5}{8}$ × 64′ = 8 8	1 Pl 16 × $\frac{9}{16}$ × 66′ = 7 9	1 Pl 16 × $\frac{1}{2}$ × 74′ = 7 0
1 Pl. 16 × $\frac{9}{16}$ × 46′ = 7 9	1 Pl 16 × $\frac{9}{16}$ × 47′ = 7 9	1 Pl 16 × $\frac{9}{16}$ × 49′ = 7 9	1 Pl 16 × $\frac{1}{2}$ × 63′ = 7 0
			1 Pl 16 × $\frac{1}{2}$ × 47′ = 7 0
40 8	44 8	45 3	49 6
35,100	38,200	41,800	44,400
140,100	146,500	152,700	158,700
107,200	110,800	114,000	117,400
282,400	295,500	308,500	320,500
28.2	29 6	30 8	32 0
99 × $\frac{7}{16}$″	99 × $\frac{7}{16}$″	108 × $\frac{1}{2}$″	108 × $\frac{1}{2}$″
3″	3″	3″	3″
4 Ls 5 × 3$\frac{1}{2}$ × $\frac{9}{16}$″	4 Ls 5 × 3$\frac{1}{2}$ × $\frac{9}{16}$″	4 Ls 5 × 3$\frac{1}{2}$ × $\frac{9}{16}$″	4 Ls 5 × 3$\frac{1}{2}$ × $\frac{9}{16}$″
36 × 36″	36 × 36″	36 × 36″	36 × 36″
10′–3″	10′–9″	11′–6″	11′–6″
103,700	113,200	129,800	139,000

TABLE XXIX—(Continued)

E 50				
20′	25′	30′	35′	40′
18′–9″	23′–6″	28′–6″	33′–6″	38′–6″
2′–10$\frac{1}{2}$″	3′–5$\frac{1}{2}$″	4′–0$\frac{1}{2}$″	4′–3$\frac{1}{2}$″	4′–9$\frac{1}{2}$″
290	390	400	470	520
182,000	327,000	487,000	732,000	1,022,000
2,733,000	4,129,000	5,652,000	7,337,000	9,197,000
2,575,000	3,829,000	5,162,000	6,600,000	8,153,000
5,490,000	8,285,000	11,301,000	14,669,000	18,372,000
32 6	38 2	45 0	47 9	55 6
169,000	217,000	251,000	206,000	330,000
10 0	12 8	14 8	18 0	19 4
$\frac{3}{8}$″ web = 1 9	$\frac{3}{8}$″ web = 2 6	$\frac{3}{8}$″ web = 2 3	$\frac{3}{8}$″ web = 2 4	$\frac{3}{8}$″ web = 2 7
2 Ls 6×4×$\frac{1}{2}$″ = 8 5	2 Ls 6×6×$\frac{1}{2}$″ = 10 5	2 Ls 6×6×$\frac{5}{8}$″ = 13 0	2 Ls 6×6×$\frac{3}{4}$″ = 15 4	2 Ls 6×6×$\frac{9}{16}$″ = 10 6
10 4	13 1	15 3	17 8	20 1
3 200	4,600	5,700	7,300	8,900
59,900	68,100	67,300	84,400	92,100
56,400	63,100	69,600	75,900	81,600
119,500	135,800	151,600	167,600	182,600
11 9	13 6	15 2	16 8	18 3
34×$\frac{7}{16}$″	41×$\frac{1}{2}$″	48×$\frac{3}{8}$″	51×$\frac{3}{8}$″	57×$\frac{3}{8}$″
2$\frac{3}{4}$″	3″	2$\frac{1}{2}$″	2$\frac{1}{4}$″	2$\frac{1}{2}$″
4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$″	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$″	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$″	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$″	4 Ls 5×3$\frac{1}{2}$×$\frac{3}{8}$″
16×24″	16×24″	16×24″	21×25″	21×25″
4′–2″	4′–9″	5′–4″	5′–8″	6′–2″
7,800	11,800	14,500	18,800	23,300

TABLE XXIX—(Continued)

E 50				
45′–0″	50′–0″	55′–0″	60′–0″	65′–0″
43′–3″	48′–3″	53′–0″	58′–0″	63′–0″
5′–6½″	5′–9½″	6′–0½″	6′–0½″	6′–6½″
570	610	650	730	770
1,361,000	1,763,000	2,214,000	2,851,000	3,480,000
11,249,000	13,432,000	15,712,000	18,352,000	21,235,000
9,832,000	11,579,000	13,353,000	15,380,000	17,541,000
22,442,000	26,774,000	31,279,000	36,583,000	42,256,000
64.6	68 0	71 0	71 6	77 8
347,000	394,000	441,000	511,000	543,000
20 5	23 2	25 9	30 0	32 0
$\frac{1}{8}$″ web = 3 1 2 Ls 6 × 6 × $\frac{9}{16}$″ = 10 6 1 Pl. 14 × $\frac{9}{16}$ × 28′ = 6 8	$\frac{1}{8}$″ web = 3 2 2 Ls 6 × 6 × $\frac{9}{16}$″ = 10 6 1 Pl. 14 × $\frac{7}{16}$ × 34′ = 5 3 1 Pl. 14 × $\frac{3}{8}$ × 25′ = 4 5	$\frac{1}{8}$″ web = 3 4 2 Ls 6 × 6 × $\frac{9}{16}$″ = 11 7 Pl. 14 × $\frac{1}{2}$ × 38′ = 6 0 Pl. 14 × $\frac{7}{16}$ × 28′ = 5 3	$\frac{1}{8}$″ web = 3 4 2 Ls 6 × 6 × $\frac{5}{8}$″ = 11 7 Pl. 14 × $\frac{5}{8}$ × 44′ = 7 5 Pl. 14 × $\frac{5}{8}$ × 33′ = 7 5	$\frac{1}{8}$″ web = 3 7 2 Ls 6 × 6 × $\frac{5}{8}$″ = 11 7 1 Pl. 14 × $\frac{1}{2}$ × 48′ = 6 0 1 Pl. 14 × $\frac{7}{16}$ × 40′ = 5 3 1 Pl. 14 × $\frac{7}{8}$ × 31′ = 5 3
20.5	23 6	26 4	30 1	32 0
10,500	12,100	14,000	16,400	18,500
99,500	106,300	112,700	119,200	126,700
87,200	91,700	95,000	100,000	104,500
197,000	210,100	221,700	235,600	249,700
19 7	21 0	22 2	23 6	26 0
66 × $\frac{3}{8}$″	69 × $\frac{3}{8}$″	72 × $\frac{3}{8}$″	72 × $\frac{3}{8}$″	78 × $\frac{3}{8}$″
$2\frac{3}{4}$″	$2\frac{1}{2}$″	$2\frac{1}{2}$″	$2\frac{1}{4}$″	$2\frac{1}{2}$″
4 Ls 5 × $3\frac{1}{2}$ × $\frac{3}{8}$	4 Ls 5 × $3\frac{1}{2}$ × $\frac{3}{8}$	4 Ls 5 × $3\frac{1}{2}$ × $\frac{7}{16}$	4 Ls 5 × $3\frac{1}{2}$ × $\frac{7}{16}$	4 Ls 5 × $3\frac{1}{2}$ × $\frac{7}{16}$
21 × 25″	21 × 25″	23 × 29″	23 × 29″	23 × 29″
6′–11″	7′–2″	7′–6″	7′–6″	8′–0″
27,400	32,400	38,800	45,500	51,500

TABLE XXIX—(Continued)

E 50				
70′–0″	75′–0″	80′–0″	85′–0″	90′–0″
68′–0″	72′–9″	77′–9″	82′–9″	87′–9″
6′–6$\frac{1}{2}$″	6′–6$\frac{1}{2}$″	7′–0$\frac{1}{2}$″	7′–6$\frac{1}{2}$″	7′–6$\frac{1}{2}$″
810	860	910	1,010	1,080
4,546,000	5,399,000	6,396,000	7,754,000	9,125,000
24,198,000	27,200,000	30,629,000	34,414,000	38,265,000
19,721,000	21,895,000	24,350,000	26,947,000	29,580,000
48,465,000	54,494,000	61,375,000	69,115,000	76,970,000
78 3	78 5	84 5	90 5	90 5
619,000	694,000	726,000	764,000	850,000
36 4	40 8	42.7	44 9	50 0
$\frac{1}{8}$″ web = 3 7	$\frac{1}{8}$″ web = 3.7	$\frac{1}{8}$″ web = 4 6	$\frac{1}{8}$″ web = 4 9	$\frac{1}{8}$″ web = 4 9
2 Ls 6 × 6 × $\frac{5}{8}$″ = 11.7	2 Ls 6 × 6 × $\frac{5}{8}$″ = 11.7	2 Ls 6 × 6 × $\frac{5}{8}$″ = 11 7	2 Ls 6 × 6 × $\frac{5}{8}$″ = 11 7	2 Ls 6 × 6 × $\frac{3}{4}$″ = 12 9
Pl. 14 × $\frac{5}{8}$ × 55′ = 7.5	Pl 14 × $\frac{9}{16}$ × 60′ = 6.8	Pl 16 × $\frac{5}{8}$ × 64′ = 8 8	Pl 16 × $\frac{9}{16}$ × 69′ = 7 9	Pl 16 × $\frac{9}{16}$ × 73′ = 7 9
Pl. 14 × $\frac{9}{16}$ × 45′ = 6 8	Pl 14 × $\frac{9}{16}$ × 53′ = 6 8	Pl 16 × $\frac{5}{8}$ × 54′ = 8 8	Pl 16 × $\frac{1}{2}$ × 60′ = 7 0	Pl 16 × $\frac{9}{16}$ × 64′ = 7 9
Pl. 14 × $\frac{9}{16}$ × 34′ = 6 8	Pl 14 × $\frac{1}{2}$ × 44′ = 6 0	Pl 16 × $\frac{5}{8}$ × 40′ = 8 8	Pl. 16 × $\frac{1}{2}$ × 51′ = 7 0	Pl 16 × $\frac{9}{16}$ × 54′ = 7 9
	Pl 14 × $\frac{1}{2}$ × 34′ = 6 0		Pl 16 × $\frac{1}{2}$ × 39′ = 7 0	Pl 16 × $\frac{9}{16}$ × 41′ = 7 9
36 5	41 0	42 7	45 5	50 4
23,300	24,700	27,400	31,300	34,700
134,500	142,800	151,500	160,000	168,000
109,500	115,000	120,500	125,200	130,200
266,300	282,500	299,400	316,500	332,900
26 7	28 3	30 0	31 7	33 3
78 × $\frac{3}{8}$″	78 × $\frac{3}{8}$″	84 × $\frac{7}{16}$″	90 × $\frac{7}{16}$″	90 × $\frac{7}{16}$″
4 Ls 5 × 3$\frac{1}{2}$ × $\frac{1}{2}$″	4 Ls 5 × 3$\frac{1}{2}$ × $\frac{9}{16}$″	4 Ls 5 × 3$\frac{1}{2}$ × $\frac{9}{16}$″	4 Ls 5 × 3$\frac{1}{2}$ × $\frac{5}{8}$″	4 Ls 5 × 3$\frac{1}{2}$ × $\frac{5}{8}$″
25 × 32″	25 × 32″	25 × 32″	36 × 36″	36 × 36″
2$\frac{1}{4}$″	2″	2$\frac{1}{2}$″	2$\frac{1}{2}$″	2$\frac{1}{4}$″
8′–3″	8′–3″	8′–9″	10′–0″	10′–0″
59,500	67,300	76,300	94,200	105,500

TABLE XXIX—(Continued)

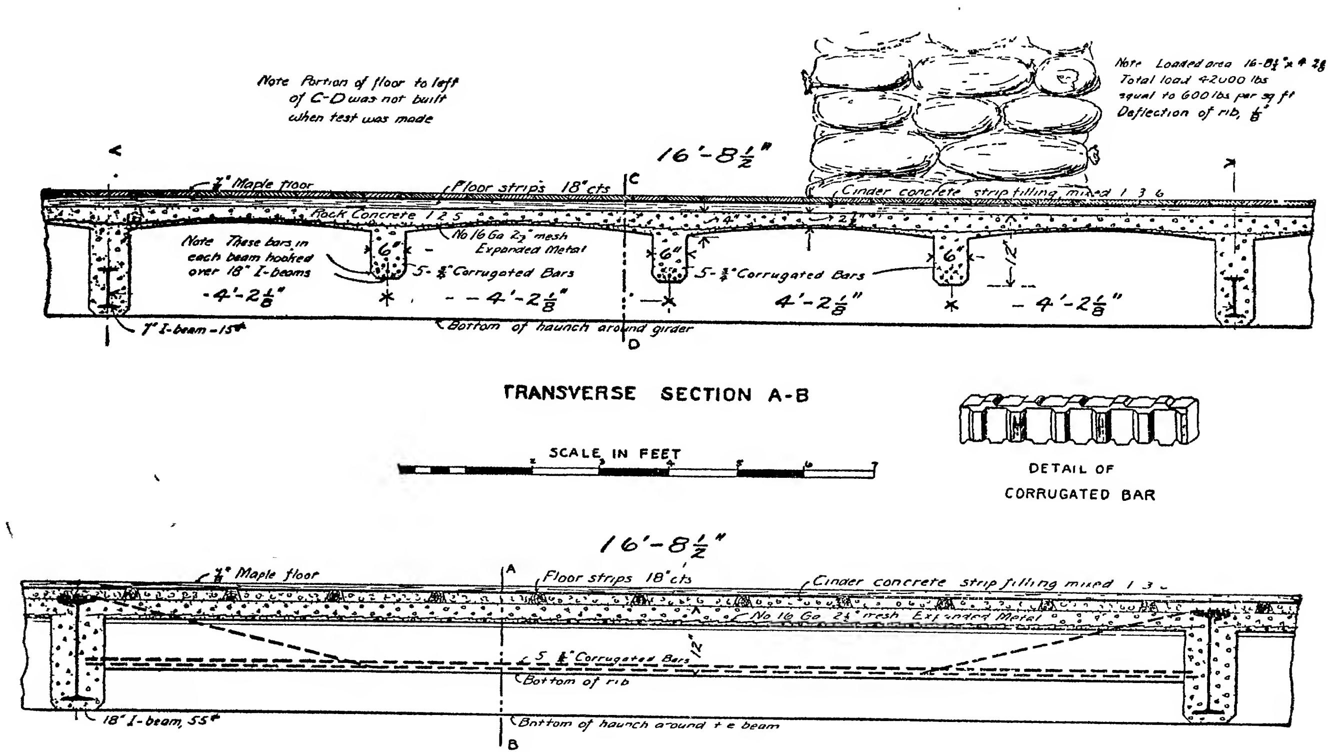
Assumed Live Loading	E 50			
Length o to o of girders	95′–0″	100′–0″	105′–0″	110′–0″
Length c to c of bearings	92′–3″	97′–3″	102′–3″	107′–3″
Depth b to b. of Ls	8′–3$\frac{1}{2}$″	8′–3$\frac{1}{2}$″	9′–0$\frac{1}{2}$″	9′–0$\frac{1}{2}$″
Dead load steel p lin ft (effective)	1,120	1,190	1,280	1,340
Dead load moment in -lb.	10,340,000	11,985,000	13,958,000	15,872,000
Live load moment in -lb.	41,852,000	45,958,000	50,320,000	55,295,000
Impact load moment in -lb.	32,018,000	34,242,000	37,538,000	40,801,000
Total moment in.-lb.	84,210,000	92,685,000	101,813,000	111,977,000
Effective depth, inches	99 5	98 7	107 6	108 0
Flange stress	846,000	939,000	946,000	1,037,000
Flange area required	49 8	55 3	55 7	61 0
Flange section	$\frac{1}{8}$″ web = 5 4 2 Ls 6×6×$\frac{3}{4}$″ = 13 9 1 Pl 16×$\frac{9}{16}$×76′ = 7 9 1 Pl 16×$\frac{9}{16}$×66′ = 7 9 1 Pl 16×$\frac{9}{16}$×55′ = 7 9 1 Pl 16×$\frac{1}{2}$×41′ = 7 0	$\frac{1}{8}$″ web = 5 4 2 Ls 8×8×$\frac{3}{4}$″ = 19 9 1 Pl. 20×$\frac{9}{16}$×76′ = 10 1 1 Pl 20×$\frac{9}{16}$×64′ = 10 1 1 Pl 20×$\frac{9}{16}$×48′ = 10 1	$\frac{1}{8}$″ web = 6 8 2 Ls 8×8×$\frac{3}{4}$″ = 19 9 1 Pl. 20×$\frac{9}{16}$×78′ = 10 1 1 Pl. 20×$\frac{9}{16}$×65′ = 10 1 1 Pl 20×$\frac{1}{2}$×48′ = 9 0	$\frac{1}{8}$″ web = 6 8 2 Ls 8×8×$\frac{3}{4}$″ = 19 9 1 Pl. 20×$\frac{5}{8}$×84′ = 11 3 1 Pl 20×$\frac{5}{8}$×70′ = 11 3 1 Pl 20×$\frac{5}{8}$×53′ = 11 3
Total net area sq. in.	50 0	55 6	55 9	60 6
Dead load shear	37,300	41,100	44,900	49,300
Live load shear	175,100	183,200	190,900	198,400
Impact load shear	134,000	138,500	142,500	146,800
Total shear	346,400	362,800	378,300	394,500
Web area required sq in.	34 6	36 3	37 8	39 4
Web section	99×$\frac{7}{16}$″	99×$\frac{7}{16}$″	108×$\frac{1}{2}$″	108×$\frac{1}{2}$″
End pitch through flange	2$\frac{1}{2}$″	2$\frac{1}{2}$″	3″	2$\frac{3}{4}$″
End stiffeners with fillers	4 Ls 5×3$\frac{1}{2}$×$\frac{5}{8}$″	4 Ls 6×4×$\frac{5}{8}$″	4 Ls 6×4×$\frac{5}{8}$″	4 Ls 6×4×$\frac{5}{8}$″
Masonry plate	36×36″	36×36″	36×36″	36×36″
Base of rail to masonry	10′–9″	10′–9″	11′–6″	11′–6″
Weight of steel per span (actual)	114,200	123,600	146,000	161,700
Dead load track 500 # p lin. ft				

TABLE XXX

Unit Costs of Bridge Construction

Item	Cost in Dollars	Unit
Foundation piling, in place	$0 50	Lin ft.
Earth, dry excavation	0 50	C Y
Earth, wet excavation	3 00	C Y.
Rock, dry loose excavation	2 00	C. Y
Cofferdam lumber in place	50 00	MBM
Concrete	10 00	C Y.
Reinforcing—bars, in place	0 05	Per lb.
Old rails in place	24 00	Ton
Masonry—stone for culverts	8 00	C Y
Bolts, etc , in place	0 06	Per lb.
Rip-rap in place (piers)	2 00	C Y.
Waterproofing—painted on	0 10	Sq. ft.
Waterproofing—mastic	0 30	Sq ft
Trestle timber in place	45 00	MBM
Guard rails in place	40 00	MBM
Structural steel erected:		
Pin Conn. Tr	0 045	Lb
Riveted trusses	0 046	Lb
Deck plate girders	0 040	Lb.
Through plate girders	0 042	Lb.
Draw spans—truss	0 053	Lb
Draw spans—girder	0 050	Lb
Trough floor through plate girder	0 043	Lb.
Beam spans braced	0 030	Lb.
Wooden boxes per MBM in place, incl. iron	40 00	MBM
Wooden piles, foundation	0 50	Lin. ft.
Wooden piles, for trestles	0 40	Lin. ft.
Driving wooden piles for trestles	0 16	Lin. ft.
Bridge ties	40 00	MBM

NOTE: The prices for structural steel are for the middle west with Pittsburgh freight and a base price of 1 6 cents per pound for raw material at Pittsburgh. To obtain the price for any other location and base, add or subtract the difference between 19 cents per hundred pounds and the freight rate to the place under consideration and also add or subtract the difference between 1 6 cents and the present price per pound of raw metal.



LONGITUDINAL SECTION C-D

SECTIONS THROUGH REINFORCED CONCRETE FLOOR, SHOWING TEST OF FLOOR

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STRENGTH OF MATERIALS.

PART I

SIMPLE STRESS.

1. Stress. When forces are applied to a body they tend in a greater or less degree to break it. Preventing or tending to prevent the rupture, there arise, generally, forces between every two adjacent parts of the body. Thus, when a load is suspended by means of an iron rod, the rod is subjected to a downward pull at its lower end and to an upward pull at its upper end, and these two forces tend to pull it apart. At any cross-section of the rod the iron on either side "holds fast" to that on the other, and these forces which the parts of the rod exert upon each other prevent the tearing of the rod. For example, in Fig. 1, let *a* represent the rod and its suspended load, 1,000 pounds; then the pull on the lower end equals 1,000 pounds. If we neglect the weight of the rod, the pull on the upper end is also 1,000 pounds, as shown in Fig. 1 (*b*); and the upper part A exerts on the lower part B an upward pull Q equal to 1,000 pounds, while the lower part exerts on the upper a force P also equal to 1,000 pounds. These two forces, P and Q, prevent rupture of the rod at the "section" C; at any other section there are two forces like P and Q preventing rupture at that section.

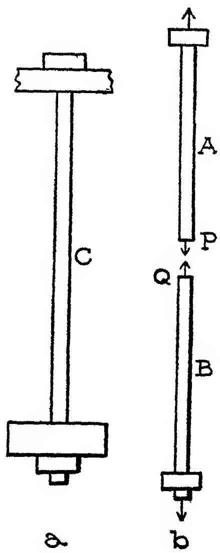


Fig. 1.

By *stress at a section* of a body is meant the force which the part of the body on either side of the section exerts on the other. Thus, the stress at the section C (Fig. 1) is P (or Q), and it equals 1,000 pounds.

2. Stresses are usually expressed (in America) in pounds, sometimes in tons. Thus the stress **P** in the preceding article is

1,000 pounds, or $\frac{1}{2}$ ton. Notice that this value has nothing to do with the size of the cross-section on which the stress acts.

3. Kinds of Stress. (a) When the forces acting on a body (as a rope or rod) are such that they tend to tear it, the stress at any cross-section is called a *tension* or a *tensile stress.* The stresses P and Q, of Fig. 1, are tensile stresses. Stretched ropes, loaded "tie rods" of roofs and bridges, etc., are under tensile stress.

(b.) When the forces acting on a body (as a short post, brick, etc.) are such that they tend to crush it, the stress at any section at right angles to the direction of the crushing forces is called a *pressure* or a *compressive stress.* Fig. 2 (*a*) represents a loaded post, and Fig. 2 (*b*) the upper and lower parts. The upper part presses down on B, and the lower part presses up on A, as shown. P or Q is the compressive stress in the post at section C. Loaded posts, or struts, piers, etc., are under compressive stress.

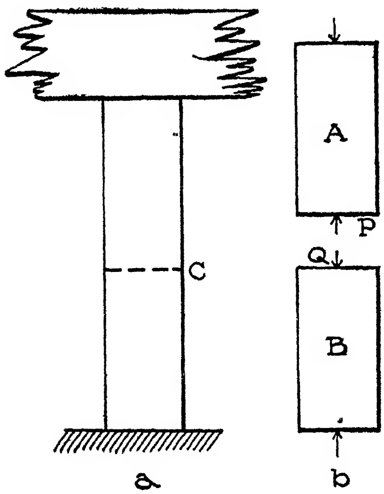


Fig. 2.

(c.) When the forces acting on a body (as a rivet in a bridge joint) are such that they tend to cut or "shear" it across, the stress at a section along which there is a tendency to cut is called a *shear* or a *shearing stress.* This kind of stress takes its name from the act of cutting with a pair of shears. In a material which is being cut in this way, the stresses that are being "overcome" are shearing stresses. Fig. 3 (*a*) represents a riveted joint, and Fig. 3 (*b*) two parts of the rivet. The forces applied to the joint are such that A tends to slide to the left, and B to the right; then B exerts on A a force P toward the right, and A on B a force Q toward the left as shown. P or Q is the shearing stress in the rivet.

Tensions, Compressions and Shears are called *simple stresses.* Forces may act upon a body so as to produce a combination of simple stresses on some section; such a combination is called a *complex*

stress. The stresses in beams are usually complex. There are other terms used to describe stress; they will be defined farther on.

4. **Unit-Stress.** It is often necessary to specify not merely the amount of the entire stress which acts on an area, but also the amount which acts on each unit of area (square inch for example). By unit-stress is meant stress per unit area.

To find the value of a unit-stress: *Divide the whole stress by the whole area of the section on which it acts, or over which it is distributed.* Thus, let

P denote the value of the whole stress,
A the area on which it acts, and
S the value of the unit-stress; then

$$S = \frac{P}{A}, \text{ also } P = AS. \qquad (1)$$

Strictly these formulas apply only when the stress P is uniform,

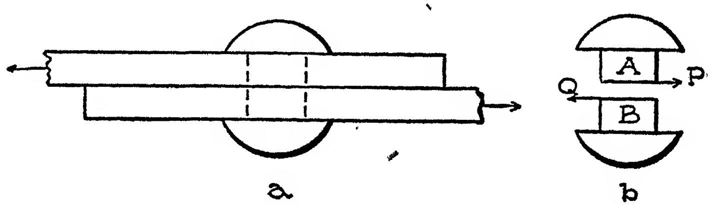


Fig. 3.

that is, when it is uniformly distributed over the area, each square inch for example sustaining the same amount of stress. When the stress is not uniform, that is, when the stresses on different square inches are not equal, then $P \div A$ equals the *average value* of the unit-stress.

5. Unit-stresses are usually expressed (in America) in pounds per square inch, sometimes in tons per square inch. If P and A in equation 1 are expressed in pounds and square inches respectively, then S will be in pounds per square inch; and if P and A are expressed in tons and square inches, S will be in tons per square inch.

Examples. **1.** Suppose that the rod sustaining the load in Fig. 1 is 2 square inches in cross-section, and that the load weighs 1,000 pounds. What is the value of the unit-stress?

Here $P = 1,000$ pounds, $A = 2$ square inches; hence.

$$S = \frac{1,000}{2} = 500 \text{ pounds per square inch.}$$

2. Suppose that the rod is one-half square inch in cross-section. What is the value of the unit-stress?

$$A = \frac{1}{2} \text{ square inch, and, as before, } P = 1,000 \text{ pounds; hence}$$

$$S = 1,000 \div \frac{1}{2} = 2,000 \text{ pounds per square inch.}$$

Notice that one must always divide the whole stress by the area to get the unit-stress, whether the area is greater or less than one.

6. Deformation. Whenever forces are applied to a body it changes in size, and usually in shape also. This change of size and shape is called deformation. Deformations are usually measured in inches; thus, if a rod is stretched 2 inches, the "elongation" = 2 inches.

7. Unit-Deformation. It is sometimes necessary to specify not merely the value of a total deformation but its amount per unit length of the deformed body. Deformation per unit length of the deformed body is called unit-deformation.

To find the value of a unit-deformation: *Divide the whole deformation by the length over which it is distributed.* Thus, if

D denotes the value of a deformation,
l the length,
s the unit-deformation, then

$$s = \frac{D}{l}, \text{ also } D = ls. \qquad (2)$$

Both D and l should always be expressed in the same unit.

Example. Suppose that a 4-foot rod is elongated $\frac{1}{2}$ inch. What is the value of the unit-deformation?

Here $D = \frac{1}{2}$ inch, and $l = 4$ feet $= 48$ inches;
hence $s = \frac{1}{2} \div 48 = \frac{1}{96}$ inch per inch.

That is, each inch is elongated $\frac{1}{96}$ inch.

Unit-elongations are sometimes expressed in per cent. To express an elongation in per cent: *Divide the elongation in inches by the original length in inches, and multiply by 100.*

8. Elasticity. Most solid bodies when deformed will regain more or less completely their natural size and shape when the de-

forming forces cease to act. This property of regaining size and shape is called elasticity.

We may classify bodies into kinds depending on the degree of elasticity which they have, thus:

1. *Perfectly elastic* bodies; these will regain their original form and size no matter how large the applied forces are if less than breaking values. Strictly there are no such materials, but rubber, practically, is perfectly elastic.

2. *Imperfectly elastic* bodies; these will fully regain their original form and size if the applied forces are not too large, and practically even if the loads are large but less than the breaking value. Most of the constructive materials belong to this class.

3. *Inelastic* or *plastic* bodies; these will not regain in the least their original form when the applied forces cease to act. Clay and putty are good examples of this class.

9. Hooke's Law, and Elastic Limit. If a gradually increasing force is applied to a perfectly elastic material, the deformation increases proportionally to the force; that is, if P and P′ denote two values of the force (or stress), and D and D′ the values of the deformation produced by the force,

$$\text{then } P:P' :: D:D'.$$

This relation is also true for imperfectly elastic materials, provided that the loads P and P′ do not exceed a certain limit depending on the material. Beyond this limit, the deformation increases much faster than the load; that is, if within the limit an addition of 1,000 pounds to the load produces a stretch of 0.01 inch, beyond the limit an equal addition produces a stretch larger and usually much larger than 0.01 inch.

Beyond this limit of proportionality a part of the deformation is permanent; that is, if the load is removed the body only partially recovers its form and size. The permanent part of a deformation is called *set.*

The fact that for most materials the deformation is proportional to the load within certain limits, is known as Hooke's Law. The unit-stress within which Hooke's law holds, or above which the deformation is not proportional to the load or stress, is called *elastic limit.*

10. Ultimate Strength. By ultimate tensile, compressive, or shearing strength of a material is meant the greatest tensile, compressive, or shearing unit-stress which it can withstand.

As before mentioned, when a material is subjected to an increasing load the deformation increases faster than the load beyond the elastic limit, and much faster near the stage of rupture. Not only do tension bars and compression blocks elongate and shorten respectively, but their cross-sectional areas change also; tension bars thin down and compression blocks "swell out" more or less. The value of the ultimate strength for any material is ascertained by subjecting a specimen to a gradually increasing tensile, compressive, or shearing stress, as the case may be, until rupture occurs, and measuring the greatest load. *The breaking load divided by the area of the original cross-section sustaining the stress, is the value of the ultimate strength.*

Example. Suppose that in a tension test of a wrought-iron rod $\frac{1}{2}$ inch in diameter the greatest load was 12,540 pounds. What is the value of the ultimate strength of that grade of wrought iron?

The original area of the cross-section of the rod was

$0.7854\ (\text{diameter})^2 = 0.7854 \times \frac{1}{4} = 0.1964$ square inches; hence the ultimate strength equals

$$12{,}540 \div 0.1964 = 63{,}850 \text{ pounds per square inch. (nearly).}$$

11. Stress-Deformation Diagram. A "test" to determine the elastic limit, ultimate strength, and other information in regard to a material is conducted by applying a gradually increasing load until the specimen is broken, and noting the deformation corresponding to many values of the load. The first and second columns of the following table are a record of a tension test on a steel rod one inch in diameter. The numbers in the first column are the values of the pull, or the loads, at which the elongation of the specimen was measured. The elongations are given in the second column. The numbers in the third and fourth columns are the values of the unit-stress and unit-elongation corresponding to the values of the load opposite to them. The numbers in the third column were obtained from those in the first by dividing the latter by the area of the cross-section of the rod, 0.7854 square inches. Thus,

$$3{,}930 \div 0.7854 = 5{,}000$$
$$7{,}850 \div 0.7854 = 10{,}000, \text{ etc.}$$

Total Pull in pounds, P	Deformation in inches, D	Unit-Stress in pounds per square inch, S	Unit-Deformation, s
3930	0.00136	5000	0.00017
7850	.00280	10000	.00035
11780	.00404	15000	.00050
15710	.00538	20000	.00067
19635	.00672	25000	.00084
23560	.00805	30000	.00101
27490	.00942	35000	.00118
31415	.01080	40000	.00135
35345	.01221	45000	.00153
39270	.0144	50000	.00180
43200	.0800	55000	.0100
47125	.1622	60000	.0202
51050	.201	65000	.0251
54980	.281	70000	.0351
58910	.384	75000	.048
62832	.560	80000	.070
65200	1.600	83000	.200

The numbers in the fourth column were obtained by dividing those in the second by the length of the specimen (or rather the length of that part whose elongation was measured), 8 inches Thus,

$$0.00136 \div 8 = 0.00017,$$
$$.00280 \div 8 = .00035, \text{ etc.}$$

Looking at the first two columns it will be seen that the elongations are practically proportional to the loads up to the ninth load, the increase of stretch for each increase in load being about 0.00135 inch; but beyond the ninth load the increases of stretch are much greater. Hence the elastic limit was reached at about the ninth load, and its value is about 45,000 pounds per square inch. The greatest load was 65,200 pounds, and the corresponding unit-stress, 83,000 pounds per square inch, is the ultimate strength.

Nearly all the information revealed by such a test can be well represented in a diagram called a *stress-deformation diagram.* It is made as follows: Lay off the values of the unit-deformation (fourth column) along a horizontal line, according to some convenient scale, from some fixed point in the line. At the points on the horizontal line representing the various unit-elongations, lay off perpendicular distances equal to the corresponding unit-stresses. Then connect by a smooth curve the upper ends of all those distances, last distances laid off. Thus, for instance, the highest unit-

elongation (0.20) laid off from *o* (Fig. 4) fixes the point *a*, and a perpendicular distance to represent the highest unit-stress (83,000) fixes the point *b*. All the points so laid off give the curve *ocb*. The part *oc*, within the elastic limit, is straight and nearly vertical while the remainder is curved and more or less horizontal, especially toward the point of rupture *b*. Fig. 5 is a typical stress-deformation diagram for timber, cast iron, wrought iron, soft and hard steel, in tension and compression.

12. Working Stress and Strength, and Factor of Safety. The greatest unit-stress in any part of a structure when it is sustaining its loads is called the *working stress* of that part. If it is under tension, compression and shearing stresses, then the corresponding highest unit-stresses in it are called its working stress in tension, in compression, and in shear respectively; that is, we speak of as many working stresses as it has kinds of stress.

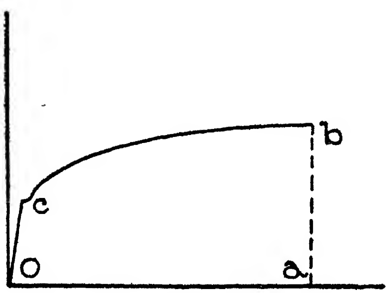


Fig. 4.

By *working strength* of a material to be used for a certain purpose is meant the highest unit-stress to which the material ought to be subjected when so used. Each material has a working strength for tension, for compression, and for shear, and they are in general different.

By *factor of safety* is meant the ratio of the ultimate strength of a material to its working stress or strength. Thus, if

S_u denotes ultimate strength,
S_w denotes working stress or strength, and
f denotes factor of safety, then

$$f = \frac{S_u}{S_w}; \text{ also } S_w = \frac{S_u}{f}. \qquad (3)$$

When a structure which has to stand certain loads is about to be designed, it is necessary to select working strengths or factors of safety for the materials to be used. Often the selection is a matter of great importance, and can be wisely performed only by an experienced engineer, for this is a matter where hard-and-

fast rules should not govern but rather the judgment of the expert. But there are certain principles to be used as guides in making a selection, chief among which are:

1. The working strength should be considerably below the elastic limit. (Then the deformations will be small and not permanent.)

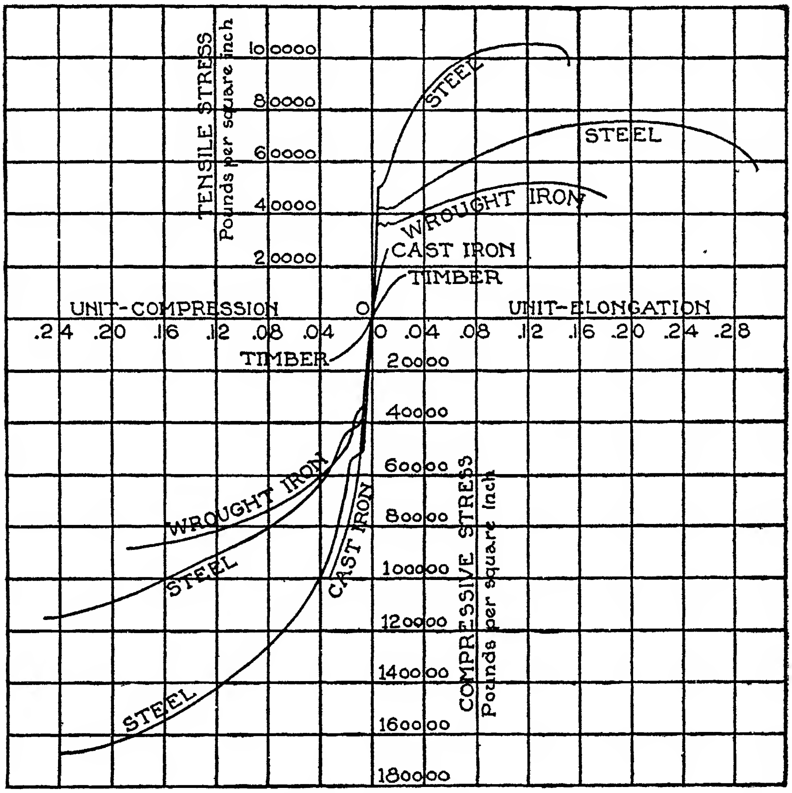


Fig. 5. (After Johnson.)

2. The working strength should be smaller for parts of a structure sustaining varying loads than for those whose loads are steady. (Actual experiments have disclosed the fact that the strength of a specimen depends on the kind of load put upon it, and that in a general way it is less the less steady the load is.)

3. The working strength must be taken low for non-uniform material, where poor workmanship may be expected, when the

loads are uncertain, etc. Principles 1 and 2 have been reduced to figures or formulas for many particular cases, but the third must remain a subject for display of judgment, and even good guessing in many cases.

The following is a table of factors of safety* which will be used in the problems:

Factors of Safety.

Materials.	For steady stress. (Buildings.)	For varying stress. (Bridges.)	For shocks. (Machines.)
Timber	8	10	15
Brick and stone	15	25	30
Cast iron	6	15	20
Wrought iron	4	6	10
Steel	5	7	15

They must be regarded as average values and are not to be adopted in every case in practice.

Examples. 1. A wrought-iron rod 1 inch in diameter sustains a load of 30,000 pounds. What is its working stress? If its ultimate strength is 50,000 pounds per square inch, what is its factor of safety?

The area of the cross-section of the rod equals $0.7854\times$ (diameter)$^2 = 0.7854\times 1^2 = 0.7854$ square inches. Since the whole stress on the cross-section is 30,000 pounds, equation 1 gives for the unit working stress

$$S = \frac{30{,}000}{0.7854} = 38{,}197 \text{ pounds per square inch.}$$

Equation 3 gives for factor of safety

$$f = \frac{50{,}000}{38{,}197} = 1.3$$

2. How large a steel bar or rod is needed to sustain a steady pull of 100,000 pounds if the ultimate strength of the material is 65,000 pounds?

The load being steady, we use a factor of safety of 5 (see table above); hence the working strength to be used (see equation 3) is

$$S = \frac{65{,}000}{5} = 13{,}000 \text{ pounds per square inch.}$$

The proper area of the cross-section of the rod can now be computed from equation 1 thus:

*Taken from Merriman's "Mechanics of Materials."

$$A = \frac{P}{S} = \frac{100,000}{13,000} = 7.692 \text{ square inches.}$$

A bar 2×4 inches in cross-section would be a little stronger than necessary. To find the diameter (d) of a round rod of sufficient strength, we write $0.7854\ d^2 = 7.692$, and solve the equation for d; thus:

$$d^2 = \frac{7.692}{0.7854} = 9.794, \text{ or } d = 3.129 \text{ inches.}$$

3. How large a steady load can a short timber post safely sustain if it is 10×10 inches in cross-section and its ultimate compressive strength is 10,000 pounds per square inch?

According to the table (page 10) the proper factor of safety is 8, and hence the working strength according to equation 3 is

$$S = \frac{10,000}{8} = 1,250 \text{ pounds per square inch.}$$

The area of the cross-section is 100 square inches; hence the safe load (see equation 1) is

$$P = 100 \times 1,250 = 125,000 \text{ pounds.}$$

4. When a hole is punched through a plate the shearing strength of the material has to be overcome. If the ultimate shearing strength is 50,000 pounds per square inch, the thickness of the plate $\frac{1}{2}$ inch, and the diameter of the hole $\frac{3}{4}$ inch, what is the value of the force to be overcome?

The area shorn is that of the cylindrical surface of the hole or the metal punched out; that is

$$3.1416 \times \text{diameter} \times \text{thickness} = 3.1416 \times \tfrac{3}{4} \times \tfrac{1}{2} = 1.178 \text{ sq. in.}$$

Hence, by equation 1, the total shearing strength or resistance to punching is

$$P = 1.178 \times 50,000 = 58,900 \text{ pounds.}$$

STRENGTH OF MATERIALS UNDER SIMPLE STRESS.

13. **Materials in Tension.** Practically the only materials used extensively under tension are timber, wrought iron and steel, and to some extent cast iron.

14. **Timber.** A successful tension test of wood is difficult, as the specimen usually crushes at the ends when held in the testing machine, splits, or fails otherwise than as desired. Hence the

tensile strengths of woods are not well known, but the following may be taken as approximate average values of the ultimate strengths of the woods named, when "dry out of doors."

Hemlock,	7,000	pounds per square inch.
White pine,	8,000	" "
Yellow pine, long leaf,	12,000	" "
" " , short leaf,	10,000	" "
Douglas spruce,	10,000	" "
White oak,	12,000	" "
Red oak,	9,000	" "

15. Wrought Iron. The process of the manufacture of wrought iron gives it a "grain," and its tensile strengths along and across the grain are unequal, the latter being about three-fourths of the former. The ultimate tensile strength of wrought iron along the grain varies from 45,000 to 55,000 pounds per square inch. Strength along the grain is meant when not otherwise stated.

The strength depends on the size of the piece, it being greater for small than for large rods or bars, and also for thin than for thick plates. The elastic limit varies from 25,000 to 40,000 pounds per square inch, depending on the size of the bar or plate even more than the ultimate strength. Wrought iron is very ductile, a specimen tested in tension to destruction elongating from 5 to 25 per cent of its length.

16. Steel. Steel has more or less of a grain but is practically of the same strength in all directions. To suit different purposes, steel is made of various grades, chief among which may be mentioned rivet steel, sheet steel (for boilers), medium steel (for bridges and buildings), rail steel, tool and spring steel. In general, these grades of steel are hard and strong in the order named, the ultimate tensile strength ranging from about 50,000 to 160,000 pounds per square inch.

There are several grades of structural steel, which may be described as follows:*

1. Rivet steel:

   Ultimate tensile strength, 48,000 to 58,000 pounds per square inch.
   Elastic limit, not less than one-half the ultimate strength.
   Elongation, 26 per cent.
   Bends 180 degrees flat on itself without fracture.

*Taken from "Manufacturer's Standard Specifications."

2. Soft steel:

Ultimate tensile strength, 52,000 to 62,000 pounds per square inch.
Elastic limit, not less than one-half the ultimate strength.
Elongation, 25 per cent.
Bends 180 degrees flat on itself.

3. Medium steel:

Ultimate tensile strength, 60,000 to 70,000 pounds per square inch.
Elastic limit, not less than one-half the ultimate strength.
Elongation, 22 per cent.
Bends 180 degrees to a diameter equal to the thickness of the specimen without fracture.

17. Cast Iron. As in the case of steel, there are many grades of cast iron. The grades are not the same for all localities or districts, but they are based on the appearance of the fractures, which vary from coarse dark grey to fine silvery white.

The ultimate tensile strength does not vary uniformly with the grades but depends for the most part on the percentage of "combined carbon" present in the iron. This strength varies from 15,000 to 35,000 pounds per square inch, 20,000 being a fair average.

Cast iron has no well-defined elastic limit (see curve for cast iron, Fig. 5). Its ultimate elongation is about one per cent.

EXAMPLES FOR PRACTICE.

1. A steel wire is one-eighth inch in diameter, and the ultimate tensile strength of the material is 150,000 pounds per square inch. How large is its breaking load? Ans. 1,845 pounds.

2. A wrought-iron rod (ultimate tensile strength 50,000 pounds per square inch) is 2 inches in diameter. How large a steady pull can it safely bear? Ans. 39,270 pounds.

18. Materials in Compression. Unlike the tensile, the compressive strength of a specimen or structural part depends on its dimension in the direction in which the load is applied, for, in compression, a long bar or rod is weaker than a short one. At present we refer only to the strength of short pieces such as do not bend under the load, the longer ones (columns) being discussed farther on.

Different materials break or fail under compression, in two very different ways:

1. Ductile materials (structural steel, wrought iron, etc.),

and wood compressed across the grain, do not fail by breaking into two distinct parts as in tension, but the former bulge out and flatten under great loads, while wood splits and mashes down. There is no particular point or instant of failure under increasing loads, and such materials have no definite ultimate strength in compression.

2. Brittle materials (brick, stone, hard steel, cast iron, etc.), and wood compressed along the grain, do not mash gradually, but fail suddenly and have a definite ultimate strength in compression. Although the surfaces of fracture are always much inclined to the direction in which the load is applied (about 45 degrees), the ultimate strength is computed by dividing the total breaking load by the cross-sectional area of the specimen.

The principal materials used under compression in structural work are timber, wrought iron, steel, cast iron, brick and stone.

19. Timber. As before noted, timber has no definite ultimate compressive strength across the grain. The U. S. Forestry Division has adopted certain amounts of compressive *deformation* as marking stages of failure. Three per cent compression is regarded as "a working limit allowable," and fifteen per cent as "an extreme limit, or as failure." The following (except the first) are values for compressive strength from the Forestry Division Reports, all in pounds per square inch:

	Ultimate strength along the grain.	3% Compression across the grain
Hemlock	6,000	
White pine	5,400	700
Long-leaf yellow pine	8,000	1,260
Short-leaf yellow pine	6,500	1,050
Douglas spruce	5,700	800*
White oak	8,500	2,200
Red oak	7,200	2,300

20. Wrought Iron. The elastic limit of wrought iron, as before noted, depends very much upon the size of the bars or plate, it being greater for small bars and thin plates. Its value for compression is practically the same as for tension, 25,000 to 40,000 pounds per square inch.

21. Steel. The hard steels have the highest compressive strength; there is a recorded value of nearly 400,000 pounds per square inch, but 150,000 is probably a fair average.

The elastic limit in compression is practically the same as in tension, which is about 60 per cent of the ultimate tensile strength, or, for structural steel, about 25,000 to 42,000 pounds per square inch.

22. Cast Iron. This is a very strong material in compression, in which way, principally, it is used structurally. Its ultimate strength depends much on the proportion of "combined carbon" and silicon present, and varies from 50,000 to 200,000 pounds per square inch, 90,000 being a fair average. As in tension, there is no well-defined elastic limit in compression (see curve for cast iron, Fig. 5).

23. Brick. The ultimate strengths are as various as the kinds and makes of brick. For soft brick, the ultimate strength is as low as 500 pounds per square inch, and for pressed brick it varies from 4,000 to 20,000 pounds per square inch, 8,000 to 10,000 being a fair average. The ultimate strength of good paving brick is still higher, its average value being from 12,000 to 15,000 pounds per square inch.

24. Stone. Sandstone, limestone and granite are the principal building stones. Their ultimate strengths in pounds per square inch are about as follows:

Sandstone,*	5,000	to	16,000,	average	8,000.
Limestone,*	8,000	"	16,000,	"	10,000.
Granite,	14,000	"	24,000,	"	16,000.

*Compression at right angles to the "bed" of the stone.

EXAMPLES FOR PRACTICE.

1. A limestone 12×12 inches on its bed is used as a pier cap, and bears a load of 120,000 pounds. What is its factor of safety?
Ans. 12.

2. How large a post (short) is needed to sustain a steady load of 100,000 pounds if the ultimate compressive strength of the wood is 10,000 pounds per square inch? Ans. 8×10 inches.

25. Materials in Shear. The principal materials used under shearing stress are timber, wrought iron, steel and cast iron. Partly on account of the difficulty of determining shearing strengths, these are not well known.

26. Timber. The ultimate shearing strengths of the more important woods *along the grain* are about as follows:

Hemlock,	300	pounds per square inch.
White pine,	400	" "
Long-leaf yellow pine,	850	" "
Short-leaf " "	775	" "
Douglas spruce,	500	" "
White oak,	1,000	" "
Red oak,	1,100	" "

Wood rarely fails by shearing across the grain. Its ultimate

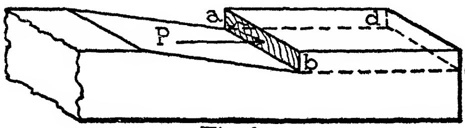


Fig. 6 *a*.

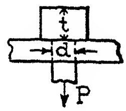


Fig. 6 *b*.

shearing strength in that direction is probably four or five times the values above given.

27. Metals. The ultimate shearing strength of wrought iron, steel, and cast iron is about 80 per cent of their respective ultimate tensile strengths.

EXAMPLES FOR PRACTICE.

1. How large a pressure P (Fig. 6 *a*) exerted on the shaded area can the timber stand before it will shear off on the surface *abcd*, if $ab = 6$ inches and $bc = 10$ inches, and the ultimate shearing strength of the timber is 400 pounds per square inch?

Ans. 24,000 pounds.

2. When a bolt is under tension, there is a tendency to tear the bolt and to "strip" or shear off the head. The shorn area would be the surface of the cylindrical hole left in the head. Compute the tensile and shearing unit-stresses when P (Fig. 6 *b*) equals 30,000 pounds, $d = 2$ inches, and $t = 3$ inches.

Ans. { Tensile unit-stress, 9,550 pounds per square inch.
Shearing unit-stress, 1,591 pounds per square inch.

REACTIONS OF SUPPORTS.

28. Moment of a Force. By moment of a force with respect to a point is meant its tendency to produce rotation about that point. Evidently the tendency depends on the magnitude of the force and on the perpendicular distance of the line of action of the force from the point: the greater the force and the perpendicular distance, the greater the tendency; hence *the moment*

of a force with respect to a point equals the product of the force and the perpendicular distance from the force to the point.

The point with respect to which the moment of one or more forces is taken is called an *origin* or *center of moments*, and the perpendicular distance from an origin of moments to the line of action of a force is called the *arm* of the force with respect to that origin. Thus, if F_1 and F_2 (Fig. 7) are forces, their arms with respect to O′ are a_1' and a_2' respectively, and their moments are $F_1a'_1$ and $F_2a'_2$. With respect to O″ their arms are a_1'' and a_2'' respectively, and their moments are F_1a_1'' and F_2a_2''.

If the force is expressed in pounds and its arm in feet, the moment is in foot-pounds; if the force is in pounds and the arm in inches, the moment is in inch-pounds.

29. A *sign* is given to the moment of a force for convenience; the rule used herein is as follows: *The moment of a force about a point is positive or negative according as it tends to turn the body about that point in the clockwise or counter-clockwise direction**.

Thus the moment (Fig. 7)

of F_1 about O′ is negative, about O″ positive;
" F_2 " O′ " " , about O″ negative.

30. Principle of Moments. In general, a single force of proper magnitude and line of action can balance any number of forces. That single force is called the *equilibrant* of the forces, and the single force that would balance the equilibrant is called the *resultant* of the forces. Or, otherwise stated, the resultant of any number of forces is a force which produces the same effect. It can be proved that—*The algebraic sum of the moments of any number of forces with respect to a point, equals the moment of their resultant about that point.*

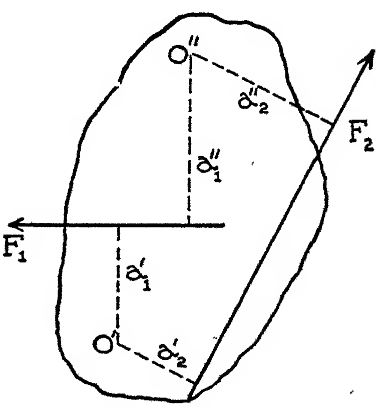


Fig. 7.

*By clockwise direction is meant that in which the hands of a clock rotate; and by counter-clockwise, the opposite direction.

This is a useful principle and is called "principle of moments."

31. All the forces acting upon a body which is at rest are said to be *balanced* or *in equilibrium.* No force is required to balance such forces and hence their equilibrant and resultant are zero.

Since their resultant is zero, *the algebraic sum of the mom-*

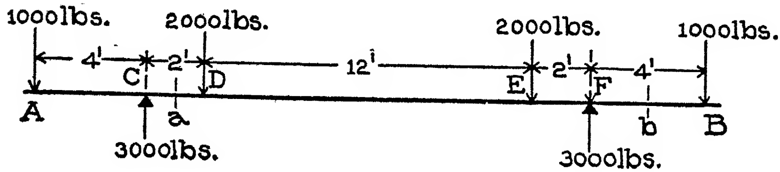


Fig. 8.

ents of any number of forces which are balanced or in equilibrium equals zero.

This is known as the principle of moments for forces in equilibrium; for brevity we shall call it also "the principle of moments."

The principle is easily verified in a simple case. Thus, let AB (Fig. 8) be a beam resting on supports at C and F. It is evident from the symmetry of the loading that each reaction equals one-half of the whole load, that is, $\frac{1}{2}$ of 6,000=3,000 pounds. (We neglect the weight of the beam for simplicity.)

With respect to C, for example, the moments of the forces are, taking them in order from the left:

$-1{,}000 \times 4 =$	$-4{,}000$	foot-pounds
$3{,}000 \times 0 =$	0	"
$2{,}000 \times 2 =$	$4{,}000$	"
$2{,}000 \times 14 =$	$28{,}000$	"
$-3{,}000 \times 16 =$	$-48{,}000$	"
$1{,}000 \times 20 =$	$20{,}000$	"

The algebraic sum of these moments is seen to equal zero.

Again, with respect to B the moments are:

$-1{,}000 \times 24 =$	$-24{,}000$	foot-pounds
$3{,}000 \times 20 =$	$60{,}000$	"
$-2{,}000 \times 18 =$	$-36{,}000$	"
$-2{,}000 \times 6 =$	$-12{,}000$	"
$3{,}000 \times 4 =$	$12{,}000$	"
$1{,}000 \times 0 =$	0	"

The sum of these moments also equals zero. In fact, no matter

where the center of moments is taken, it will be found in this and any other balanced system of forces that the algebraic sum of their moments equals zero. The chief use that we shall make of this principle is in finding the supporting forces of loaded beams.

32. Kinds of Beams. A *cantilever beam* is one resting on one support or fixed at one end, as in a wall, the other end being free.

A *simple beam* is one resting on two supports.

A *restrained beam* is one fixed at both ends; a beam fixed at one end and resting on a support at the other is said to be restrained at the fixed end and simply supported at the other.

A *continuous beam* is one resting on more than two supports.

33. Determination of Reactions on Beams. The forces which the supports exert on a beam, that is, the "supporting forces," are called *reactions*. We shall deal chiefly with simple beams. The reaction on a cantilever beam supported at one point evidently equals the total load on the beam.

When the loads on a horizontal beam are all vertical (and

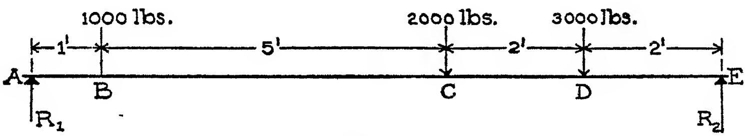


Fig. 9.

this is the usual case), the supporting forces are also vertical and *the sum of the reactions equals the sum of the loads.* This principle is sometimes useful in determining reactions, but in the case of simple beams the principle of moments is sufficient. The general method of determining reactions is as follows:

1. Write out two equations of moments for all the forces (loads and reactions) acting on the beam with origins of moments at the supports.

2. Solve the equations for the reactions.

3. As a check, try if the sum of the reactions equals the sum of the loads.

Examples. 1. Fig. 9 represents a beam supported at its ends and sustaining three loads. We wish to find the reactions due to these loads.

Let the reactions be denoted by R_1 and R_2 as shown; then the moment equations are:
For origin at A,

$$1,000\times1+2,000\times6+3,000\times8-R_2\times10=0.$$

For origin at E,

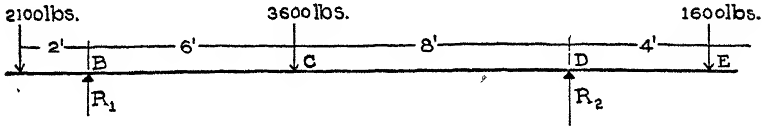


Fig. 10.

$$R_1\times10-1,000\times9-2,000\times4-3,000\times2=0.$$

The first equation reduces to

$$10\ R_2=1,000+12,000+24,000=37,000;\ \text{or}$$

$R_2=3,700$ pounds.
The second equation reduces to

$$10\ R_1=9,000+8,000+6,000=23,000;\ \text{or}$$

$R_1=2,300$ pounds.
The sum of the loads is 6,000 pounds and the sum of the reactions is the same; hence the computation is correct.

2. Fig. 10 represents a beam supported at B and D (that is, it has overhanging ends) and sustaining three loads as shown. We wish to determine the reactions due to the loads.

Let R_1 and R_2 denote the reactions as shown; then the moment equations are:
For origin at B,

$$-2,100\times2+0+3,600\times6-R_2\times14+1,600\times18=0.$$

For origin at D,

$$-2,100\times16+R_1\times14-3,600\times8+0+1,600\times4=0.$$

The first equation reduces to

$$14\ R_2=-4,200+21,600+28,800=46,200;\ \text{or}$$

$R_2=3,300$ pounds.
The second equation reduces to

$$14\ R_1=33,600+28,800-6,400=56,000;\ \text{or}$$

$R_1=4,000$ pounds.
The sum of the loads equals **7,300** pounds and the sum of the reactions is the same; hence the computation checks.

3. What are the total reactions in example 1 if the beam weighs 400 pounds?

(1.) Since we already know the reactions due to the loads (2,300 and 3,700 pounds at the left and right ends respectively (see illustration 1 above), we need only to compute the reactions due to the weight of the beam and add. Evidently the reactions due to the weight equal 200 pounds each: hence the

left reaction $= 2{,}300 + 200 = 2{,}500$ pounds, and the

right " $= 3{,}700 + 200 = 3{,}900$ " .

(2.) Or, we might compute the reactions due to the loads and weight of the beam together and directly. In figuring the moment due to the weight of the beam, we imagine the weight as concentrated at the middle of the beam; then its moments with respect to the left and right supports are (400×5) and $-(400 \times 5)$ respectively. The moment equations for origins at A and E are like those of illustration 1 except that they contain one more term, the moment due to the weight; thus they are respectively:

$$1{,}000 \times 1 + 2{,}000 \times 6 + 3{,}000 \times 8 - R_2 \times 10 + 400 \times 5 = 0,$$

$$R_1 \times 10 - 1{,}000 \times 9 - 2{,}000 \times 4 - 3{,}000 \times 2 - 400 \times 5 = 0.$$

The first one reduces to

$$10\ R_2 = 39{,}000, \text{ or } R = 3{,}900 \text{ pounds;}$$

and the second to

$$10\ R_1 = 25{,}000, \text{ or } R_1 = 2{,}500 \text{ pounds.}$$

4. What are the total reactions in example 2 if the beam weighs 42 pounds per foot ?

As in example 3, we might compute the reactions due to the weight and then add them to the corresponding reactions due to the loads (already found in example 2), but we shall determine the total reactions due to load and weight directly.

The beam being 20 feet long, its weight is 42×20, or 840 pounds. Since the middle of the beam is 8 feet from the left and 6 feet from the right support, the moments of the weight with to the left and right supports are respectively:

$$840 \times 8 = 6{,}720, \text{ and } -840 \times 6 = -5{,}040 \text{ foot-pounds.}$$

The moment equations for all the forces applied to the beam for origins at B and D are like those in example 2, with an additional term, the moment of the weight; they are respectively:

$$-2{,}100 \times 2 + 0 + 3{,}600 \times 6 - R_2 \times 14 + 1{,}600 \times 18 + 6{,}720 = 0,$$

$$-2{,}100 \times 16 + R_1 \times 14 - 3{,}600 \times 8 + 0 + 1{,}600 \times 4 - 5{,}040 = 0.$$

The first equation reduces to

$$14\ R_2 = 52{,}920, \text{ or } R_2 = 3{,}780 \text{ pounds,}$$

and the second to

$$14\ R_1 = 61{,}040, \text{ or } R_1 = 4{,}360 \text{ pounds.}$$

The sum of the loads and weight of beam is 8,140 pounds; and since the sum of the reactions is the same, the computation checks.

EXAMPLES FOR PRACTICE.

1. AB (Fig. 11) represents a simple beam supported at its ends. Compute the reactions, neglecting the weight of the beam.

Ans. { Right reaction = 1,443.75 pounds. Left reaction = 1,556.25 pounds.

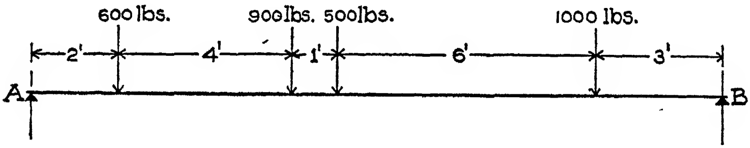


Fig. 11.

2. Solve example 1 taking into account the weight of the beam, which suppose to be 400 pounds.

Ans. { Right reaction = 1,643.75 pounds. Left reaction = 1,756.25 pounds.

3. Fig. 12 represents a simple beam weighing 800 pounds supported at A and B, and sustaining three loads as shown. What are the reactions ?

Ans. { Right reaction = 2,014.28 pounds. Left reaction = 4,785.72 pounds.

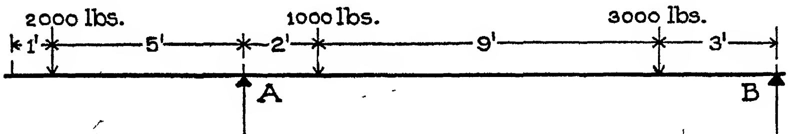


Fig. 12.

4. Suppose that in example 3 the beam also sustains a uniformly distributed load (as a floor) over its entire length, of 500 pounds per foot. Compute the reactions due to all the loads and the weight of the beam.

Ans. { Right reaction = 4,871.43 pounds. Left reaction = 11,928.57 pounds.

EXTERNAL SHEAR AND BENDING MOMENT.

On almost every cross-section of a loaded beam there are three kinds of stress, namely tension, compression and shear. The first two are often called *fibre stresses* because they act along the real fibres of a wooden beam or the imaginary ones of which we may suppose iron and steel beams composed. Before taking up the subject of these stresses in beams it is desirable to study certain quantities relating to the loads, and on which the stresses in a beam depend. These quantities are called *external shear* and *bending moment*, and will now be discussed.

34. External Shear. By external shear at (or for) any section of a loaded beam is meant the algebraic sum of all the loads (including weight of beam) and reactions on *either side* of the section. This sum is called external shear because, as is shown later, it equals the shearing stress (internal) at the section. For brevity, we shall often say simply "shear" when external shear is meant.

35. Rule of Signs. In computing external shears, it is customary to give the plus sign to the reactions and the minus sign to the loads. But in order to get the same sign for the external shear whether computed from the right or left, we *change the sign* of the sum when computed from the loads and reactions *to the right*. Thus for section a of the beam in Fig. 8 the algebraic sum is, when computed from the left,

$$-1{,}000+3{,}000 = +2{,}000 \text{ pounds;}$$

and when computed from the right,

$$-1{,}000+3{,}000-2{,}000-2{,}000 = -2{,}000 \text{ pounds.}$$

The external shear at section a is $+2{,}000$ pounds.

Again, for section b the algebraic sum is, when computed from the left,

$$-1{,}000+3{,}000-2{,}000-2{,}000+3{,}000 = +1{,}000 \text{ pounds;}$$

and when computed from the right, $-1{,}000$ pounds.

The external shear at the section is $+1{,}000$ pounds.

It is usually convenient to compute the shear at a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left sides of the section.

36. Units for Shears. It is customary to express external shears in pounds, but any other unit for expressing force and weight (as the ton) may be used.

37. Notation. We shall use V to stand for external shear at any section, and the shear at a particular section will be denoted by that letter subscripted; thus V_1, V_2, etc., stand for the shears at sections one, two, etc., feet from the left end of a beam.

The shear has different values just to the left and right of a support or concentrated load. We shall denote such values by V' and V''; thus V_5' and V_5'' denote the values of the shear at sections a little less and a little more than 5 feet from the left end respectively.

Examples. 1. Compute the shears for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively; see example 1, Art. 33.)

All the following values of the shear are computed from the left. The shear just to the right of the left support is denoted by V_0'', and $V_0'' = 2{,}300$ pounds. The shear just to the left of B is denoted by V_1', and since the only force to the left of the section is the left reaction, $V_1' = 2{,}300$ pounds. The shear just to the right of B is denoted by V_1'', and since the only forces to the left of this section are the left reaction and the 1,000-pound load, $V_1'' = 2{,}300 - 1{,}000 = 1{,}300$ pounds. To the left of all sections between B and C, there are but two forces, the left reaction and the 1,000-pound load; hence the shear at any of those sections equals $2{,}300 - 1{,}000 = 1{,}300$ pounds, or

$$V_2 = V_3 = V_4 = V_5 = V_6' = 1{,}300 \text{ pounds.}$$

The shear just to the right of C is denoted by V_6''; and since the forces to the left of that section are the left reaction and the 1,000- and 2,000-pound loads,

$$V_6'' = 2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds.}$$

Without further explanation, the student should understand that,

$$V_7 = +2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds,}$$
$$V_8' = -700,$$
$$V_8'' = +2{,}300 - 1{,}000 - 2{,}000 - 3{,}000 = -3{,}700,$$
$$V_9 = V_{10}' = -3{,}700,$$
$$V_{10}'' = +2{,}300 - 1{,}000 - 2{,}000 - 3{,}000 + 3{,}700 = 0$$

2. A simple beam 10 feet long, and supported at each end, weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the shears for sections two feet apart.

Evidently each reaction equals one-half the sum of the load and weight of the beam, that is, $\frac{1}{2}$ (1,600+400) =1,000 pounds. To the left of a section 2 feet from the left end, the forces acting on the beam consist of the left reaction, the load on that part of the beam, and the weight of that part; then since the load and weight of the beam *per foot* equal 200 pounds,

$$V_2 = 1{,}000 - 200 \times 2 = 600 \text{ pounds.}$$

To the left of a section four feet from the left end, the forces are the left reaction, the load on that part of the beam, and the weight; hence

$$V_4 = 1{,}000 - 200 \times 4 = 200 \text{ pounds.}$$

Without further explanation, the student should see that

$$V_6 = 1{,}000 - 200 \times 6 = -200 \text{ pounds,}$$
$$V_8 = 1{,}000 - 200 \times 8 = -600 \text{ pounds,}$$
$$V_{10}' = 1{,}000 - 200 \times 10 = -1{,}000 \text{ pounds,}$$
$$V_{10}'' = 1{,}000 - 200 \times 10 + 1{,}000 = 0.$$

3. Compute the values of the shear in example 1, taking into account the weight of the beam (400 pounds). (The right and left reactions are then 3,900 and 2,500 pounds respectively; see example 3, Art. 33.)

We proceed just as in example 1, except that in each computation we include the weight of the beam to the left of the section (or to the right when computing from forces to the right). The weight of the beam being 40 pounds per foot, then (computing from the left)

$$V_0'' = +2{,}500 \text{ pounds,}$$
$$V_1' = +2{,}500 - 40 = +2{,}460,$$
$$V_1'' = +2{,}500 - 40 - 1{,}000 = +1{,}460,$$
$$V_2 = +2{,}500 - 1{,}000 - 40 \times 2 = +1{,}420,$$
$$V_3 = +2{,}500 - 1{,}000 - 40 \times 3 = +1{,}380,$$
$$V_4 = +2{,}500 - 1{,}000 - 40 \times 4 = +1{,}340,$$
$$V_5 = +2{,}500 - 1{,}000 - 40 \times 5 = +1{,}300,$$
$$V_6' = +2{,}500 - 1{,}000 - 40 \times 6 = +1{,}260,$$
$$V_6'' = +2{,}500 - 1{,}000 - 40 \times 6 - 2{,}000 = -740,$$
$$V_7 = +2{,}500 - 1{,}000 - 2{,}000 - 40 \times 7 = -780,$$

$$V_8' = +2{,}500-1{,}000-2{,}000-40\times 8 = -820,$$
$$V_8'' = +2{,}500-1{,}000-2{,}000-40\times 8-3{,}000 = -3{,}820,$$
$$V_9 = +2{,}500-1{,}000-2{,}000-3{,}000-40\times 9 = -3{,}860,$$
$$V'_{10} = +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 = -3{,}900,$$
$$V''_{10} = +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 + 3{,}900 = 0.$$

Computing from the right, we find, as before, that

$$V_7 = -(3{,}900-3{,}000-40\times 3) = -780 \text{ pounds},$$
$$V_8' = -(3{,}900-3{,}000-40\times 2) = -820,$$
$$V_8'' = -(3{,}900-40\times 2) = -3{,}820,$$

etc., etc.

EXAMPLES FOR PRACTICE.

1. Compute the values of the shear for sections of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

$$\text{Ans.}\begin{cases} V_1 = V_2' = -2{,}100 \text{ pounds}, \\ V_2'' = V_3 = V_4 = V_5 = V_6 = V_7 = V'_8 = +1{,}900, \\ V_8'' = V_9 = V_{10} = V_{11} = V_{12} = V_{13} = V_{14} = V_{15} = V'_{16} = -1{,}700, \\ V''_{16} = V_{17} = V_{18} = V_{19} = V'_{20} = +1{,}600. \end{cases}$$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

$$\text{Ans.}\begin{cases} V_0'' = -2{,}100 \text{ lbs.} & V_7 = +1{,}966 \text{ lbs.} & V_{14} = -1{,}928 \text{ lbs.} \\ V_1 = -2{,}142 & V_8' = +1{,}924 & V_{15} = -1{,}970 \\ V_2' = -2{,}184 & V_8'' = -1{,}676 & V_{16}' = -2{,}012 \\ V_2'' = +2{,}176 & V_9 = -1{,}718 & V_{16}'' = +1{,}768 \\ V_3 = +2{,}134 & V_{10} = -1{,}760 & V_{17} = +1{,}726 \\ V_4 = +2{,}092 & V_{11} = -1{,}802 & V_{18} = +1{,}684 \\ V_5 = +2{,}050 & V_{12} = -1{,}844 & V_{19} = +1{,}642 \\ V_6 = +2{,}008 & V_{13} = -1{,}886 & V_{20}' = +1{,}600 \end{cases}$$

3. Compute the values of the shear at sections one foot apart in the beam of Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

$$\text{Ans.}\begin{cases} V_0'' = V_1 = V_2' = +1{,}556 \text{ pounds}, \\ V_2'' = V_3 = V_4 = V_5 = V_6' = +956, \\ V_6'' = V_7' = +56, \\ V_7'' = V_8 = V_9 = V_{10} = V_{11} = V_{12} = V_{13}' = -444, \\ V_{13}'' = V_{14} = V_{15} = V_{16}' = -1{,}444. \end{cases}$$

4. Compute the vertical shear at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam, 800 pounds, and a distributed load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively; see examples 3 and 4, Art. 33.)

$$\text{Ans.}\begin{cases} \begin{array}{lll} V_0 = 0 & V_7 = +6{,}150 \text{ lbs.} & V_{15} = +830 \text{ lbs} \\ V_1' = -540 \text{ lbs.} & V_8' = +5{,}610 & V_{16} = +290 \\ V_1'' = -2{,}540 & V_8'' = +4{,}610 & V_{17}' = -250 \\ V_2 = -3{,}080 & V_9 = +4{,}070 & V_{17}'' = -3{,}250 \\ V_3 = -3{,}620 & V_{10} = +3{,}530 & V_{18} = -3{,}790 \\ V_4 = -4{,}160 & V_{11} = +2{,}990 & V_{19} = -4{,}330 \\ V_5 = -4{,}700 & V_{12} = +2{,}450 & V_{20}' = -4{,}870 \\ V_6' = -5{,}240 & V_{13} = +1{,}910 & V_{20}'' = 0 \\ V_6'' = +6{,}690 & V_{14} = +1{,}370 & \end{array} \end{cases}$$

38. Shear Diagrams. The way in which the external shear varies from section to section in a beam can be well represented by means of a diagram called a *shear diagram.* To construct such a diagram for any loaded beam,

1. Lay off a line equal (by some scale) to the length of the beam, and mark the positions of the supports and the loads. (This is called a "base-line.")

2. Draw a line such that the distance of any point of it from the base equals (by some scale) the shear at the corresponding section of the beam, and so that the line is above the base where the shear is positive, and below it where negative. (This is called a *shear line*, and the distance from a point of it to the base is called the "ordinate" from the base to the shear line at that point.)

We shall explain these diagrams further by means of illustrative examples.

Examples. 1. It is required to construct the shear diagram for the beam represented in Fig. 13, *a* (a copy of Fig. 9).

Lay off A′E′ (Fig. 13, *b*) to represent the beam, and mark the positions of the loads B′, C′ and D′. In example 1, Art. 37, we computed the values of the shear at sections one foot apart; hence we lay off ordinates at points on A′E′ one foot apart, to represent those shears.

Use a scale of 4,000 pounds to one inch. Since the shear for any section in AB is 2,300 pounds, we draw a line *ab* parallel to the base 0.575 inch (2,300÷4,000) therefrom; this is the shear line for the portion AB. Since the shear for any section in BC equals 1,300 pounds, we draw a line *b′c* parallel to the base and

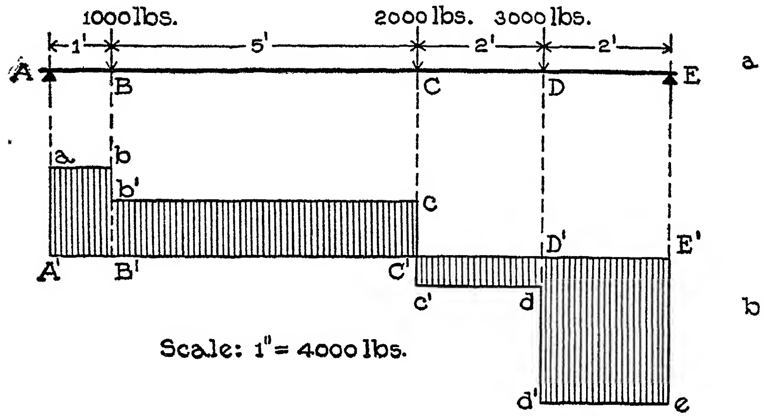


Fig. 13.

0.325 inch (1,300÷4,000) therefrom; this is the shear line for the portion BC. Since the shear for any section in CD is –700 pounds, we draw a line *c′d* below the base and 0.175 inch (700÷4,000) therefrom; this is the shear line for the portion CD. Since the shear for any section in DE equals –3,700 lbs., we draw a line *d′e* below the base and 0.925 inch (3,700÷4,000) therefrom; this is the shear line for the portion DE. Fig. 13, *b*, is the required shear diagram.

2.– It is required to construct the shear diagram for the beam of Fig. 14, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the shear for sections one foot apart were computed in example 3, Art. 37, so we have only to erect ordinates at the various points on a base line A′E′ (Fig. 14, *b*), equal to those

values. We shall use the same scale as in the preceding illustration, 4,000 pounds to an inch. Then the lengths of the ordinates corresponding to the values of the shear (see example 3, Art. 37) are respectively:

$$2{,}500 \div 4{,}000 = 0.625 \text{ inch}$$
$$2{,}460 \div 4{,}000 = 0.615 \quad \text{"}$$
$$1{,}460 \div 4{,}000 = 0.365 \quad \text{"}$$
etc. etc.

Laying these ordinates off from the base (upwards or downwards according as they correspond to positive or negative shears), we get ab, $b'c$, $c'd$, and $d'e$ as the shear lines.

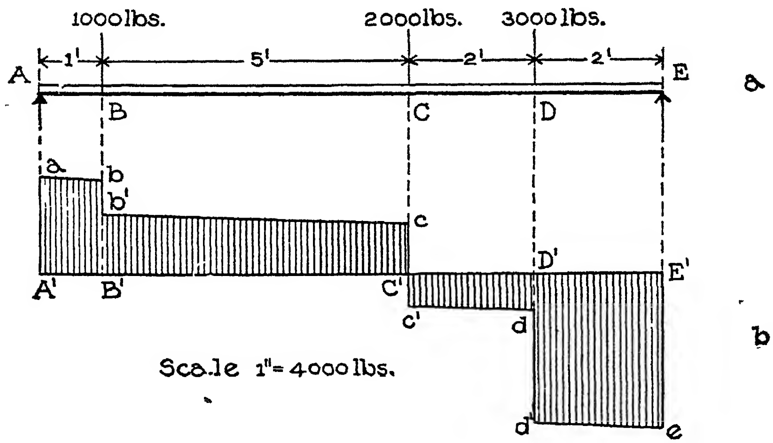


Fig. 14.

3. It is required to construct the shear diagram for the cantilever beam represented in Fig. 15, a, neglecting the weight of the beam.

The value of the shear for any section in AB is -500 pounds; for any section in BC, $-1{,}500$ pounds; and for any section in CD, $-3{,}500$ pounds. Hence the shear lines are ab, $b'c$, $c'd$. The scale being 5,000 pounds to an inch,

$$A'a = 500 \div 5{,}000 = 0.1 \text{ inch,}$$
$$B'b' = 1{,}500 \div 5{,}000 = 0.3 \quad \text{"}$$
$$C'c' = 3{,}500 \div 5{,}000 = 0.7 \quad \text{"}$$

The shear lines are all below the base because all the values of the shear are negative.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of 200 pounds per foot (see Fig. 16, a). Construct a shear diagram.

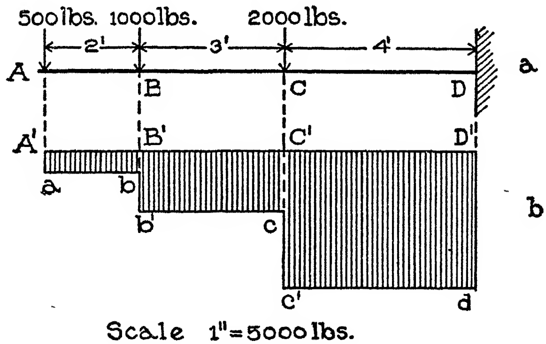


Fig. 15.

First, we compute the values of the shear at several sections. Thus

$$
\begin{aligned}
V_0'' &= -500 \text{ pounds,}\\
V_1 &= -500 - 200 = -700,\\
V_2' &= -500 - 200 \times 2 = -900,\\
V_2'' &= -500 - 200 \times 2 - 1{,}000 = -1{,}900,\\
V_3 &= -500 - 1{,}000 - 200 \times 3 = -2{,}100,\\
V_4 &= -500 - 1{,}000 - 200 \times 4 = -2{,}300,\\
V_5' &= -500 - 1{,}000 - 200 \times 5 = -2{,}500,\\
V_5'' &= -500 - 1{,}000 - 200 \times 5 - 2{,}000 = -4{,}500,\\
V_6 &= -500 - 1{,}000 - 2{,}000 - 200 \times 6 = -4{,}700,\\
V_7 &= -500 - 1{,}000 - 2{,}000 - 200 \times 7 = -4{,}900,\\
V_8 &= -500 - 1{,}000 - 2{,}000 - 200 \times 8 = -5{,}100,\\
V_9 &= -500 - 1{,}000 - 2{,}000 - 200 \times 9 = -5{,}300.
\end{aligned}
$$

The values, being negative, should be plotted downward. To a scale of 5,000 pounds to the inch they give the shear lines ab, $b'c$, $c'd$ (Fig. 16, b).

EXAMPLES FOR PRACTICE.

1. Construct a shear diagram for the beam represented in Fig. 10, neglecting the weight of the beam (see example 1, Art. 37).

2. Construct the shear diagram for the beam represented in Fig. 11, neglecting the weight of the beam (see example 3, Art. 37).

3. Construct the shear diagram for the beam of Fig. 12 when it sustains, in addition to the loads represented, its own weight, 800 pounds, and a uniform load of 500 pounds per foot (see example 4, Art. 37).

4. Figs. *a*, cases 1 and 2, Table B, represent two cantilever beams, the first bearing a concentrated load P at the free end, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that the diagrams are correct.
and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, same table, represent simple beams supported at their ends, the first bearing a concentrated

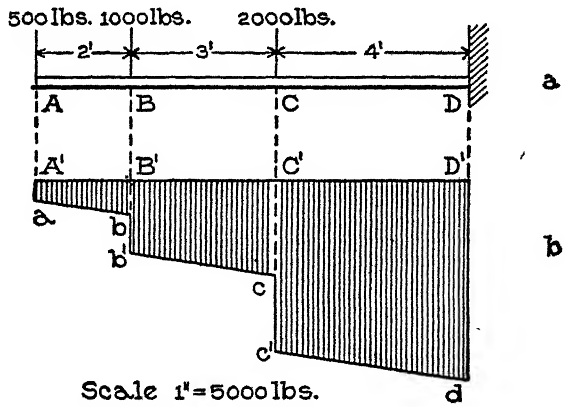


Fig. 16.

load P at the middle, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that they are correct.

39. Maximum Shear. It is sometimes desirable to know the greatest or maximum value of the shear in a given case. This value can always be found with certainty by constructing the shear diagram, from which the maximum value of the shear is evident at a glance. In any case it can most readily be computed if one knows the section for which the shear is a maximum. The student should examine all the shear diagrams in the preceding articles and those that he has drawn, and see that

1. *In cantilevers fixed in a wall, the maximum shear occurs at the wall.*

2. *In simple beams, the maximum shear occurs at a section next to one of the supports.*

By the use of these propositions one can determine the value of the maximum shear without constructing the whole shear diagram. Thus, it is easily seen (referring to the diagrams, page 53) that for a

Cantilever, end load P,	maximum	shear	$=P$
" , uniform load W,	"	"	$=W$
Simple beam, middle load P,	"	"	$=\frac{1}{2}P$
" " , uniform " W,	"	"	$=\frac{1}{2}W$

40. Bending Moment. By bending moment at (or for) a section of a loaded beam, is meant the algebraic sum of the moments of all the loads (including weight of beam) and reactions to the left or right of the section with respect to any point in the section.

41. Rule of Signs. We follow the rule of signs previously stated (Art. 29) that the moment of a force which tends to produce clockwise rotation is plus, and that of a force which tends to produce counter-clockwise rotation is minus; but in order to get the same sign for the bending moment whether computed from the right or left, we *change the sign* of the sum of the moments when computed from the loads and reactions *on the right*. Thus for section *a*, Fig. 8, the algebraic sums of the moments of the forces are:

when computed from the left,

$$-1{,}000\times5+3{,}000\times1=-2{,}000 \text{ foot-pounds;}$$

and when computed from the right,

$$1{,}000\times19-3{,}000\times15+2{,}000\times13+2{,}000\times1=+2{,}000 \text{ foot-pounds.}$$

The bending moment at section *a* is $-2{,}000$ foot-pounds.

Again, for section *b*, the algebraic sums of the moments of the forces are:

when computed from the left,

$$-1{,}000\times22+3{,}000\times18-2{,}000\times16-2{,}000\times4+3{,}000\times2=-2{,}000 \text{ foot-pounds;}$$

and when computed from the right,

$$1{,}000\times2=+2{,}000 \text{ foot-pounds.}$$

The bending moment at the section is $-2{,}000$ foot-pounds.

It is usually convenient to compute the bending moment for a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left side of the section.

42. Units. It is customary to express bending moments in inch-pounds, but often the foot-pound unit is more convenient. *To reduce foot-pounds to inch-pounds, multiply by twelve.*

43. Notation. We shall use M to denote bending moment at any section, and the bending moment at a particular section will be denoted by that letter subscripted; thus M_1, M_2, etc., denote values of the bending moment for sections one, two, etc., feet from the left end of the beam.

Examples. 1. Compute the bending moments for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively. See example 1, Art. 33.)

Since there are no forces acting on the beam to the left of the left support, $M_0=0$. To the left of the section one foot from the left end there is but one force, the left reaction, and its arm is one foot; hence $M_1=+2{,}300\times1=2{,}300$ foot-pounds. To the left of a section two feet from the left end there are two forces, 2,300 and 1,000 pounds, and their arms are 2 feet and 1 foot respectively; hence $M_2=+2{,}300\times2-1{,}000\times1=3{,}600$ foot-pounds. At the left of all sections between B and C there are only two forces, 2,300 and 1,000 pounds; hence

$$
\begin{aligned}
M_3&=+2{,}300\times3-1{,}000\times2=+4{,}900 \text{ foot-pounds,}\\
M_4&=+2{,}300\times4-1{,}000\times3=+6{,}200 \quad \text{"}\\
M_5&=+2{,}300\times5-1{,}000\times4=+7{,}500 \quad \text{"}\\
M_6&=+2{,}300\times6-1{,}000\times5=+8{,}800 \quad \text{"}
\end{aligned}
$$

To the right of a section seven feet from the left end there are two forces, the 3,000-pound load and the right reaction (3,700 pounds), and their arms with respect to an origin in that section are respectively one foot and three feet; hence

$$M_7=-(-3{,}700\times3+3{,}000\times1)=+8{,}100 \text{ foot-pounds.}$$

To the right of any section between E and D there is only one force, the right reaction; hence

$$M_8 = -(-3{,}700\times 2) = 7{,}400 \text{ foot-pounds},$$
$$M_9 = -(-3{,}700\times 1) = 3{,}700 \quad \text{"}$$

Clearly $M_{10}=0$.

2. A simple beam 10 feet long and supported at its ends weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the bending moments for sections two feet apart.

Each reaction equals one-half the whole load, that is, $\frac{1}{2}$ of $(1{,}600+400)=1{,}000$ pounds, and the load per foot including weight of the beam is 200 pounds. The forces acting on the beam to the left of the first section, two feet from the left end, are the left reaction (1,000 pounds) and the load (including weight) on the part of the beam to the left of the section (400 pounds). The arm of the reaction is 2 feet and that of the 400-pound force is 1 foot (the distance from the middle of the 400-pound load to the section). Hence

$$M_2 = +1{,}000\times 2 - 400\times 1 = +1{,}600 \text{ foot-pounds}.$$

The forces to the left of the next section, 4 feet from the left end, are the left reaction and all the load (including weight of beam) to the left (800 pounds). The arm of the reaction is 4 feet, and that of the 800-pound force is 2 feet; hence

$$M_4 = +1{,}000\times 4 - 800\times 2 = +2{,}400 \text{ foot-pounds}.$$

Without further explanation the student should see that

$$M_6 = +1{,}000\times 6 - 1{,}200\times 3 = +2{,}400 \text{ foot-pounds},$$
$$M_8 = +1{,}000\times 8 - 1{,}600\times 4 = +1{,}600 \quad \text{"}$$

Evidently $M_0 = M_{10} = 0$.

3. Compute the values of the bending moment in example 1, taking into account the weight of the beam, 400 pounds. (The right and left reactions are respectively 3,900 and 2,500 pounds; see example 3, Art. 33.)

We proceed as in example 1, except that the moment of the weight of the beam to the left of each section (or to the right when computing from forces to the right) must be included in the respective moment equations. Thus, computing from the left,

$$\begin{aligned}
M_0 &= 0\\
M_1 &= +2{,}500\times1-40\times\tfrac{1}{2}=+2{,}480 \text{ foot-pounds},\\
M_2 &= +2{,}500\times2-1{,}000\times1-80\times1=+3{,}920,\\
M_3 &= +2{,}500\times3-1{,}000\times2-120\times1\tfrac{1}{2}=+5{,}320,\\
M_4 &= +2{,}500\times4-1{,}000\times3-160\times2=+6{,}680,\\
M_5 &= +2{,}500\times5-1{,}000\times4-200\times2\tfrac{1}{2}=+8{,}000,\\
M_6 &= +2{,}500\times6-1{,}000\times5-240\times3=+9{,}280.
\end{aligned}$$

Computing from the right,

$$\begin{aligned}
M_7 &= -(-3{,}900\times3+3{,}000\times1+120\times1\tfrac{1}{2})=+8{,}520,\\
M_8 &= -(-3{,}900\times2+80\times1)=+7{,}720,\\
M_9 &= -(-3{,}900\times1+40\times\tfrac{1}{2})=+3{,}880,\\
M_{10} &= 0.
\end{aligned}$$

EXAMPLES FOR PRACTICE.

1. Compute the values of the bending moment for sections one foot apart, beginning one foot from the left end of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

Ans. (in foot-pounds)

$M_1=-2{,}100$	$M_6=+3{,}400$	$M_{11}=+2{,}100$	$M_{16}=-6{,}400$
$M_2=-4{,}200$	$M_7=+5{,}300$	$M_{12}=+\ 400$	$M_{17}=-4{,}800$
$M_3=-2{,}300$	$M_8=+7{,}200$	$M_{13}=-1{,}300$	$M_{18}=-3{,}200$
$M_4=-\ 400$	$M_9=+5{,}500$	$M_{14}=-3{,}000$	$M_{19}=-1{,}600$
$M_5=+1{,}500$	$M_{10}=+3{,}800$	$M_{15}=-4{,}700$	$M_{20}=\ 0$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

Ans. (in foot-pounds)

$M_1=-2{,}121$	$M_6=+4{,}084$	$M_{11}=+2{,}799$	$M_{16}=-6{,}736$
$M_2=-4{,}284$	$M_7=+6{,}071$	$M_{12}=+\ 976$	$M_{17}=-4{,}989$
$M_3=-2{,}129$	$M_8=+8{,}016$	$M_{13}=-\ 889$	$M_{18}=-3{,}284$
$M_4=-\ 16$	$M_9=+6{,}319$	$M_{14}=-2{,}796$	$M_{19}=-1{,}621$
$M_5=+2{,}055$	$M_{10}=+4{,}580$	$M_{15}=-4{,}745$	$M_{20}=\ 0$

3. Compute the bending moments for sections one foot apart, of the beam represented in Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

Ans. (in foot-pounds)

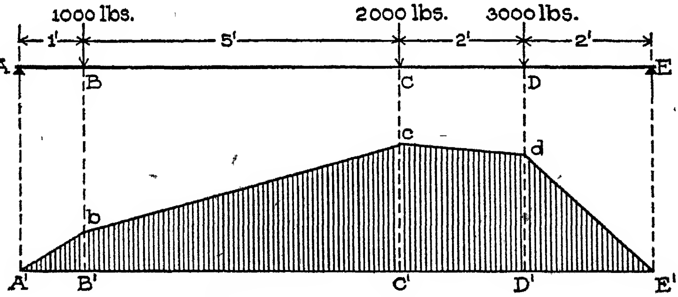
$M_1=+1{,}556$	$M_5=+5{,}980$	$M_9=+6{,}104$	$M_{13}=+4{,}328$
$M_2=+3{,}112$	$M_6=+6{,}936$	$M_{10}=+5{,}660$	$M_{14}=+2{,}884$
$M_3=+4{,}068$	$M_7=+6{,}992$	$M_{11}=+5{,}216$	$M_{15}=+1{,}440$
$M_4=+5{,}024$	$M_8=+6{,}548$	$M_{12}=+4{,}772$	$M_{16}=0$

4 Compute the bending moments at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam, 800 pounds, and a uniform load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively; see Exs. 3 and 4, Art. 33.)

Ans. (in foot-pounds)

$M_1=-270$	$M_6=-19{,}720$	$M_{11}=+3{,}980$	$M_{16}=12{,}180$
$M_2=-3{,}080$	$M_7=-13{,}300$	$M_{12}=+6{,}700$	$M_{17}=12{,}200$
$M_3=-6{,}430$	$M_8=-7{,}420$	$M_{13}=+8{,}880$	$M_{18}=8{,}680$
$M_4=-10{,}320$	$M_9=-3{,}080$	$M_{14}=+10{,}520$	$M_{19}=4{,}620$
$M_5=-14{,}750$	$M_{10}=+720$	$M_{15}=+11{,}620$	$M_{20}=0$

44. Moment Diagrams. The way in which the bending moment varies from section to section in a loaded beam can be well represented by means of a diagram called a *moment diagram*. To construct such a diagram for any loaded beam,



Scale. 1 = 10000 ft-lbs

Fig. 17.

1. Lay off a base-line just as for a shear diagram (see Art. 38).

2. Draw a line such that the distance from any point of it to the base-line equals (by some scale) the value of the bending moment at the corresponding section of the beam, and so that the line is above the base where the bending moment is positive and below it where it is negative. (This line is called a "moment line.")

Examples. 1. It is required to construct a moment diagram for the beam of Fig. 17, *a* (a copy of Fig. 9), loaded as there shown.

Lay off A′E′ (Fig. 17, *b*) as a base. In example 1, Art. 43, we computed the values of the bending moment for sections one foot apart, so we erect ordinates at points of A′E′ one foot apart, to represent the bending moments.

We shall use a scale of 10,000 foot-pounds to the inch; then the ordinates (see example 1, Art. 43, for values of M) will be:

One foot	from	left	end,	$2,300 \div 10,000 = 0.23$ inch,
Two feet	"	"	"	$3,600 \div 10,000 = 0.36$ "
Three "	"	"	"	$4,900 \div 10,000 = 0.49$ "
Four "	"	"	"	$6,200 \div 10,000 = 0.62$ "

etc., etc.

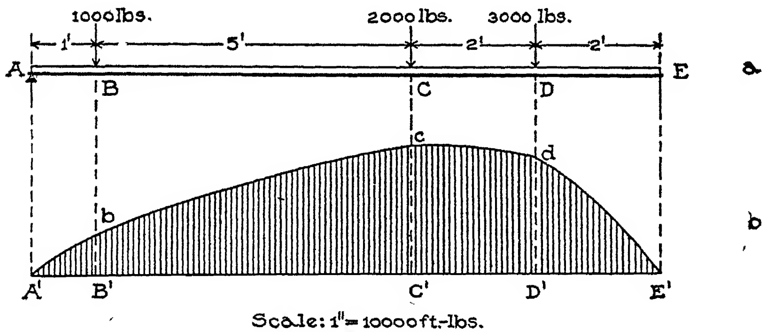


Fig. 18.

Laying these ordinates off, and joining their ends in succession, we get the line A′*bcd*E′, which is the bending moment line. Fig. 17, *b*, is the moment diagram.

2. It is required to construct the moment diagram for the beam, Fig. 18, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the bending moment for sections one foot apart were computed in example 3, Art. 43. So we have only to lay off ordinates equal to those values, one foot apart, on the base A′E′ (Fig. 18, *b*).

To a scale of 10,000 foot-pounds to the inch the ordinates (see example 3, Art. 43, for values of M) are:

At left end, 0
One foot from left end, 2,480÷10,000=0.248 inch
Two feet " " " 3,920÷10,000=0.392 "
Three " " " " 5,320÷10,000=0.532 "
Four " " " " 6,680÷10,000=0.668 "

Laying these ordinates off at the proper points, we get A′*bcd*E as the moment line.

3. It is required to construct the moment diagram for the cantilever beam represented in Fig. 19, *a*, neglecting the weight of the beam. The bending moment at B equals

$$-500\times 2=-1,000 \text{ foot-pounds};$$

at C,

$$-500\times 5-1,000\times 3=-5,500;$$

and at D,

$$-500\times 9-1,000\times 7-2,000\times 4=-19,500.$$

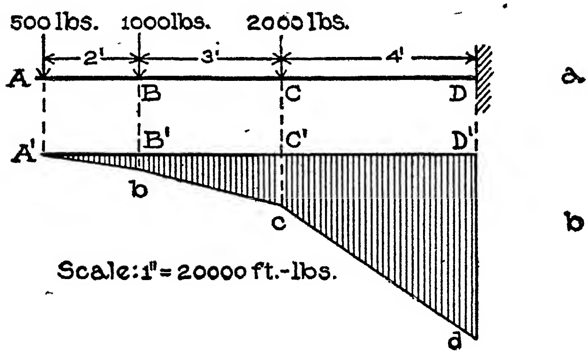


Fig. 19.

Using a scale of 20,000 foot-pounds to one inch, the ordinates in the bending moment diagram are:

At B, 1,000÷20,000=0.05 inch,
" C, 5,500÷20,000=0.275 "
" D, 19,500÷20,000=0.975 "

Hence we lay these ordinates off, and downward because the bending moments are negative, thus fixing the points *b*, *c* and *d*. The bending moment at A is zero; hence the moment line connects A *b*, *c* and *d*. Further, the portions A*b*, *bc* and *cd* are straight, as can be shown by computing values of the bending moment for sections in AB, BC and CD, and laying off the corresponding ordinates in the moment diagram.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of **100** pounds per foot (see Fig. 20, *a*). Construct a moment diagram.

First, we compute the values of the bending moment at several sections; thus,

$$\begin{aligned}
M_1 &= -500\times 1 - 100\times \tfrac{1}{2} = -550 \text{ foot-pounds,}\\
M_2 &= -500\times 2 - 200\times 1 = -1{,}200,\\
M_3 &= -500\times 3 - 1{,}000\times 1 - 300\times 1\tfrac{1}{2} = -2{,}950,\\
M_4 &= -500\times 4 - 1{,}000\times 2 - 400\times 2 = -4{,}800,\\
M_5 &= -500\times 5 - 1{,}000\times 3 - 500\times 2\tfrac{1}{2} = -6{,}750,\\
M_6 &= -500\times 6 - 1{,}000\times 4 - 2{,}000\times 1 - 600\times 3 = -10{,}800,\\
M_7 &= -500\times 7 - 1{,}000\times 5 - 2{,}000\times 2 - 700\times 3\tfrac{1}{2} = -14{,}950,\\
M_8 &= -500\times 8 - 1{,}000\times 6 - 2{,}000\times 3 - 800\times 4 = -19{,}200,\\
M_9 &= -500\times 9 - 1{,}000\times 7 - 2{,}000\times 4 - 900\times 4\tfrac{1}{2} = -23{,}550.
\end{aligned}$$

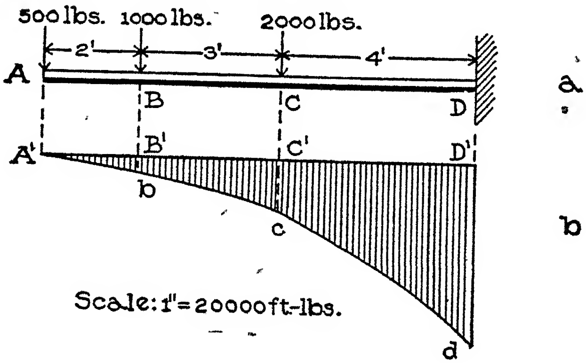


Fig. 20.

These values all being negative, the ordinates are all laid off downwards. To a scale of 20,000 foot-pounds to one inch, they fix the moment line A'*bcd*.

EXAMPLES FOR PRACTICE.

1. Construct a moment diagram for the beam represented in Fig. 10, neglecting the weight of the beam. (See example 1, Art. 43).

2. Construct a moment diagram for the beam represented in Fig. 11, neglecting the weight of the beam. (See example 3, Art. 43).

3. Construct the moment diagram for the beam of Fig. 12

when it sustains, in addition to the loads represented and its own weight (800 pounds), a uniform load of 500 pounds per foot. (See example 4, Art. 43.)

4. Figs. *a*, cases 1 and 2, page 53, represent two cantilever beams, the first bearing a load P at the free end, and the second a uniform load W. Figs. *c* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and l equal to 10 feet, and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, page 53, represent simple beams on end supports, the first bearing a middle load P, and the other a uniform load W. Figs. *c* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and l equal to 10 feet, and satisfy yourself that the diagrams are correct.

45. Maximum Bending Moment. It is sometimes desirable to know the greatest or maximum value of the bending moment in a given case. This value can always be found with certainty by constructing the moment diagram, from which the maximum value of the bending moment is evident at a glance. But in any case, it can be most readily computed if one knows the section for which the bending moment is greatest. If the student will compare the corresponding shear and moment diagrams which have been constructed in foregoing articles (Figs. 13 and 17, 14 and 18, 15 and 19, 16 and 20), and those which he has drawn, he will see that—*The maximum bending moment in a beam occurs where the shear changes sign.*

By the help of the foregoing principle we can readily compute the maximum moment in a given case. We have only to construct the shear line, and observe from it where the shear changes sign; then compute the bending moment for that section. If a simple beam has one or more overhanging ends, then the shear changes sign more than once—twice if there is one overhanging end, and three times if two. In such cases we compute the bending moment for each section where the shear changes sign; the largest of the values of these bending moments is the maximum for the beam.

The section of maximum bending moment in a cantilever fixed at one end (as when built into a wall) is always at the wall.

Thus, without reference to the moment diagrams, it is readily seen that,

for a cantilever whose length is l,

with an end load P, the maximum moment is Pl,

" a uniform " W, " " " " $\frac{1}{2}Wl$.

Also by the principle, it is seen that,

for a beam whose length is l, on end supports,

with a middle load P, the maximum moment is $\frac{1}{4}Pl$,

" uniform " W, " " " " $\frac{1}{8}Wl$.

46. Table of Maximum Shears, Moments, etc. Table B on page 53 shows the shear and moment diagrams for eight simple cases of beams. The first two cases are built-in cantilevers: the next four, simple beams on end supports; and the last two, restrained beams built in walls at each end. In each case l denotes the length.

CENTER OF GRAVITY AND MOMENT OF INERTIA.

It will be shown later that the strength of a beam depends partly on the form of its cross-section. The following discussion relates principally to cross-sections of beams, and the results reached (like shear and bending moment) will be made use of later in the subject of strength of beams.

47. Center of Gravity of an Area. The student probably knows what is meant by, and how to find, the center of gravity of any flat disk, as a piece of tin. Probably his way is to balance the piece of tin on a pencil point, the point of the tin at which it so balances being the center of gravity. (Really it is midway between the surfaces of the tin and over the balancing point.) The center of gravity of the piece of tin, is also that point of it through which the resultant force of gravity on the tin (that is, the weight of the piece) acts.

By "center of gravity" of a plane area of any shape we mean that point of it which corresponds to the center of gravity of a piece of tin when the latter is cut out in the shape of the area. The center of gravity of a quite irregular area can be found most readily by balancing a piece of tin or stiff paper cut in the shape of the area. But when an area is simple in shape, or consists of parts which are simple, the center of gravity of the whole can be

found readily by computation, and such a method will now be described.

48. **Principle of Moments Applied to Areas.** Let Fig. 21 represent a piece of tin which has been divided off into any number of parts in any way, the weight of the whole being W, and that of the parts W_1, W_2, W_3, etc. Let C_1, C_2, C_3, etc., be the centers of gravity of the parts, C that of the whole, and c_1, c_2, c_3, etc., and c the distances from those centers of gravity respectively to some line (L L) in the plane of the sheet of tin. When the tin is lying in a horizontal position, the moment of the weight of the entire piece about L L is Wc, and the moments of the parts are W_1c_1, W_2c_2, etc. Since the weight of the whole is the resultant of the weights of the parts, the moment of the weight of the whole equals the sum of the moments of the weights of the parts; that is,

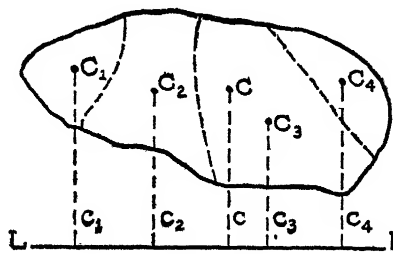


Fig. 21.

$$Wc = W_1c_1 + W_2c_2 + \text{etc.} \ldots$$

Now let A_1, A_2, etc. denote the areas of the parts of the pieces of tin, and A the area of the whole; then since the weights are proportional to the areas, we can replace the W's in the preceding equation by corresponding A's, thus:

$$Ac = A_1c_1 + A_2c_2 + \text{etc.} \ldots \qquad (4)$$

If we call the product of an area and the distance of its center of gravity from some line in its plane, the "moment" of the area with respect to that line, then the preceding equation may be stated in words thus:

The moment of an area with respect to any line equals the algebraic sum of the moments of the parts of the area.

If all the centers of gravity are on one side of the line with respect to which moments are taken, then all the moments should be given the plus sign; but if some centers of gravity are on one side and some on the other side of the line, then the moments of the areas whose centers of gravity are on one side should be given the

same sign, and the moments of the others the opposite sign. The foregoing is the principle of moments for areas, and it is the basis of all rules for finding the center of gravity of an area.

To find the center of gravity of an area which can be divided up into simple parts, we write the principle in forms of equations for two different lines as "axes of moments," and then solve the equations for the unknown distances of the center of gravity of the whole from the two lines. We explain further by means of specific examples.

Examples. 1. It is required to find the center of gravity of Fig. 22, *a*, the width being uniformly one inch.

The area can be divided into two rectangles. Let C_1 and

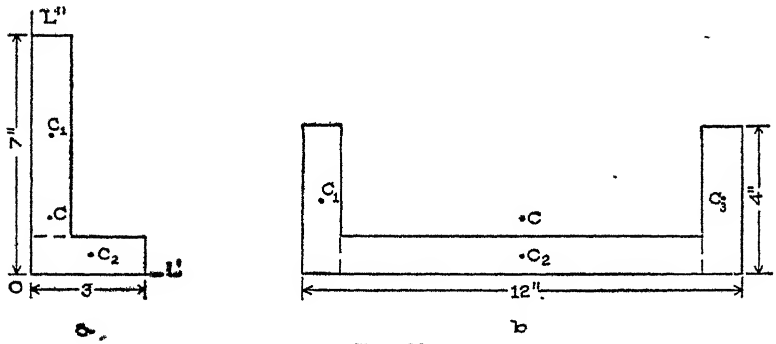


Fig. 22.

C_2 be the centers of gravity of two such parts, and C the center of gravity of the whole. Also let a and b denote the distances of C from the two lines OL′ and OL″ respectively.

The areas of the parts are 6 and 3 square inches, and their arms with respect to OL′ are 4 inches and $\frac{1}{2}$ inch respectively, and with respect to OL″ $\frac{1}{2}$ inch and $1\frac{1}{2}$ inches. Hence the equations of moments with respect to OL′ and OL″ (the whole area being 9 square inches) are:

$$9\times a = 6\times 4 + 3\times\tfrac{1}{2} = 25.5,$$
$$9\times b = 6\times\tfrac{1}{2} + 3\times 1\tfrac{1}{2} = 7.5.$$

Hence,
$$a = 25.5\div 9 = 2.83 \text{ inches},$$
$$b = 7.5\div 9 = 0.83 \quad \text{"} \quad .$$

2. It is required to locate the center of gravity of Fig. 22, *b*, the width being uniformly one inch.

The figure can be divided up into three rectangles. Let C_1, C_2 and C_3 be the centers of gravity of such parts, C the center of gravity of the whole; and let a denote the (unknown) distance of C from the base. The areas of the parts are 4, 10 and 4 square inches, and their "arms" with respect to the base are 2, $\frac{1}{2}$ and 2 inches respectively; hence the equation of moments with respect to the base (the entire area being 18 square inches) is:

$$18 \times a = 4 \times 2 + 10 \times \tfrac{1}{2} + 4 \times 2 = 21.$$

Hence, $a = 21 \div 18 = 1.17$ inches.

From the symmetry of the area it is plain that the center of gravity is midway between the sides.

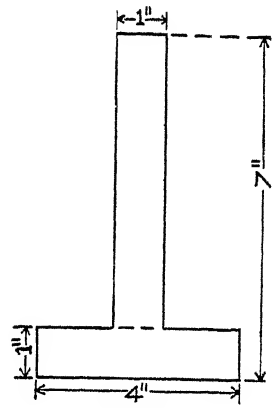


Fig. 23.

EXAMPLE FOR PRACTICE.

1. Locate the center of gravity of Fig. 23.

Ans. 2.3 inches above the base.

49. Center of Gravity of Built-up Sections. In Fig. 24 there are represented cross-sections of various kinds of rolled steel, called "shape steel," which is used extensively in steel construction. Manufacturers of this material publish "handbooks" giving full information in regard thereto, among other things, the position of the center of gravity of each cross section. With such a handbook

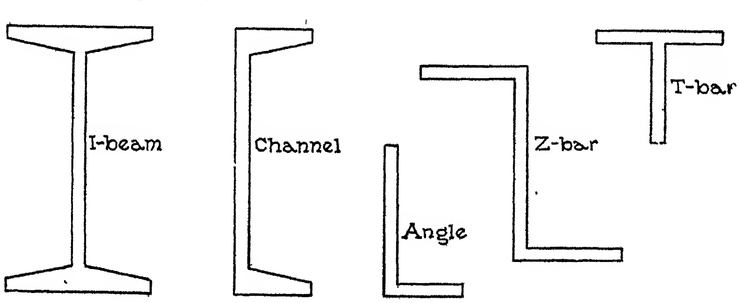


Fig. 24.

available, it is therefore not necessary actually to compute the position of the center of gravity of any section, as we did in the preceding article; but sometimes several shapes are riveted together to

make a "built-up" section (see Fig. 25), and then it may be necessary to compute the position of the center of gravity of the section.

Example. It is desired to locate the center of gravity of the section of a built-up beam represented in Fig. 25. The beam con-

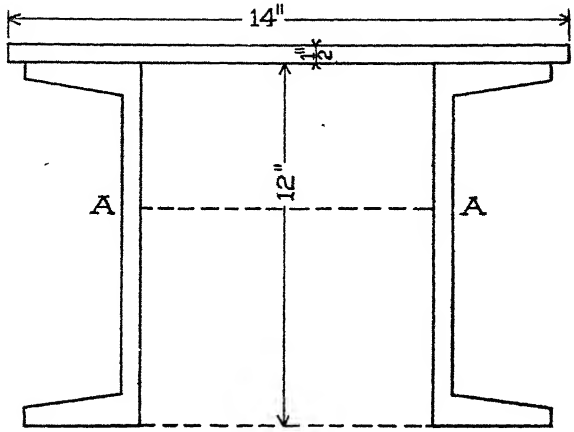


Fig. 25.

sists of two channels and a plate, the area of the cross-section of a channel being 6.03 square inches.

Evidently the center of gravity of each channel section is 6 inches, and that of the plate section is $12\frac{1}{4}$ inches, from the bottom.

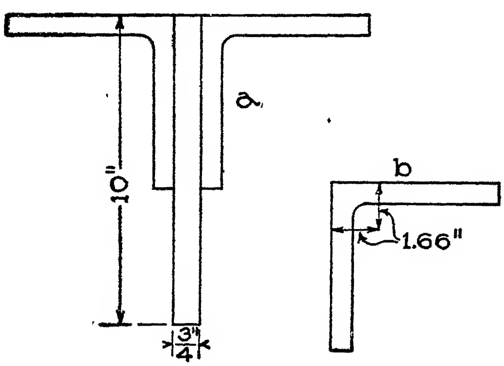


Fig. 26.

Let c denote the distance of the center of gravity of the whole section from the bottom; then since the area of the plate section is 7 square inches, and that of the whole section is 19.06,

$$19.06 \times c = 6.03 \times 6 + 6.03 \times 6 + 7 \times 12\tfrac{1}{4} = 158.11.$$

Hence, $c = 158.11 \div 19.06 = 8.30$ inches. (about).

EXAMPLES FOR PRACTICE.

1. Locate the center of gravity of the built-up section of

Fig. 26, a, the area of each "angle" being 5.06 square inches, and the center of gravity of each being as shown in Fig. 26, b.

Ans. Distance from top, 3.08 inches.

2. Omit the left-hand angle in Fig. 26, a, and locate the center of gravity of the remainder.

Ans. { Distance from top, 3.65 inches,
" " left side, 1.19 inches.

50. Moment of Inertia. If a plane area be divided into an infinite number of infinitesimal parts, then the sum of the products obtained by multiplying the area of each part by the square of its distance from some line is called the *moment of inertia* of the area with respect to the line. The line to which the distances are measured is called the *inertia-axis;* it may be taken anywhere in the plane of the area. In the subject of beams (where we have sometimes to compute the moment of inertia of the cross-section of a beam), the inertia-axis is taken through the center of gravity of the section and horizontal.

An approximate value of the moment of inertia of an area can be obtained by dividing the area into small parts (not infinitesimal), and adding the products obtained by multiplying the area of each part by the square of the distance from its center to the inertia-axis.

Example. If the rectangle of Fig. 27, a, is divided into 8 parts as shown, the area of each is one square inch, and the distances from the axis to the centers of gravity of the parts are $\frac{1}{2}$ and $1\frac{1}{2}$ inches. For the four parts lying nearest the axis the product (area times distance squared) is:

$1\times(\frac{1}{2})^2=\frac{1}{4}$; and for the other parts it is
$1\times(1\frac{1}{2})^2=\frac{9}{4}$.

Hence the approximate value of the moment of inertia of the area with respect to the axis, is

$$4(\tfrac{1}{4})+4(\tfrac{9}{4})=10.$$

If the area is divided into 32 parts, as shown in Fig. 27, b, the area of each part is $\frac{1}{4}$ square inch. For the eight of the little squares farthest away from the axis, the distance from their centers of gravity to the axis is $1\frac{3}{4}$ inches; for the next eight it is $1\frac{1}{4}$; for the next eight $\frac{3}{4}$; and for the remainder $\frac{1}{4}$ inch. Hence an

approximate value of the moment of inertia of the rectangle with respect to the axis is:

$$8\times\tfrac{1}{4}\times(1\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(1\tfrac{1}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{1}{4})^2=10\tfrac{1}{2}.$$

If we divide the rectangle into still smaller parts and form the products

$$(\text{small area})\times(\text{distance})^2,$$

and add the products just as we have done, we shall get a larger answer than $10\frac{1}{2}$. The smaller the parts into which the rectangle is divided, the larger will be the answer, but it will never be larger than $10\frac{2}{3}$. This $10\frac{2}{3}$ is the sum corresponding to a division of the rectangle into an infinitely large number of parts (infinitely small) and it is the exact value of the moment of inertia of the rectangle with respect to the axis selected.

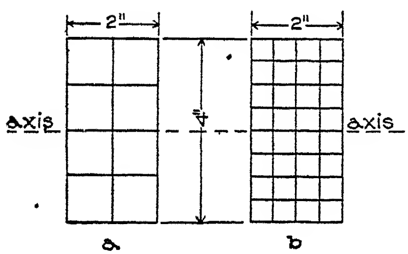


Fig. 27.

There are short methods of computing the exact values of the moments of inertia of simple figures (rectangles, circles, etc.,), but they cannot be given here since they involve the use of difficult mathematics. The foregoing method to obtain approximate values of moments of inertia is used especially when the area is quite irregular in shape, but it is given here to explain to the student the *meaning* of the moment of inertia of an area. He should understand now that the moment of inertia of an area is simply a name for such sums as we have just computed. The name is not a fitting one, since the sum has nothing whatever to do with inertia. It was first used in this connection because the sum is very similar to certain other sums which had previously been called moments of inertia.

51. Unit of Moment of Inertia. The product (area $\times$ distance^2) is really the product of four lengths, two in each factor; and since a moment of inertia is the sum of such products, a moment of inertia is also the product of four lengths. Now the product of two lengths is an area, the product of three is a volume, and the product of four is moment of inertia—unthinkable in

the way in which we can think of an area or volume, and therefore the source of much difficulty to the student. The units of these quantities (area, volume, and moment of inertia) are respectively:

the square inch, square foot, etc.,
" cubic " , cubic " ",
" biquadratic inch, biquadratic foot, etc.;

but the biquadratic inch is almost exclusively used in this connection; that is, the inch is used to compute values of moments of inertia, as in the preceding illustration. It is often written thus: Inches^4.

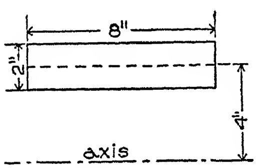


Fig. 23.

52. Moment of Inertia of a Rectangle. Let b denote the base of a rectangle, and a its altitude; then by higher mathematics it can be shown that the moment of inertia of the rectangle with respect to a line through its center of gravity and parallel to its base, is $\frac{1}{12}\ ba^3$.

Example. Compute the value of the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the long side.

Here $b=12$, and $a=4$ inches ; hence the moment of inertia desired equals

$$\tfrac{1}{12}(12\times4^3)=64 \text{ inches}^4.$$

EXAMPLE FOR PRACTICE.

1. Compute the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the short side. Ans. 576 inches^4.

53. Reduction Formula. In the previously mentioned "handbooks" there can be found tables of moments of inertia of all the cross-sections of the kinds and sizes of rolled shapes made. The inertia-axes in those tables are always taken through the center of gravity of the section, and usually parallel to some edge of the section. Sometimes it is necessary to compute the moment of inertia of a "rolled section" with respect to some other axis, and if the two axes (that is, the one given in the tables, and the other) are parallel, then the desired moment of inertia can be easily computed from the one given in the tables by the following rule:

The moment of inertia of an area with respect to any axis equals the moment of inertia with respect to a parallel axis through the center of gravity, plus the product of the area and the square of the distance between the axes.

Or, if I denotes the moment of inertia with respect to any axis; I_o the moment of inertia with respect to a parallel axis through the center of gravity; A the area; and d the distance between the axes, then

$$I = I_o + Ad^2 \ldots. \qquad \textbf{(5)}$$

Example. It is required to compute the moment of inertia of a rectangle 2×8 inches with respect to a line parallel to the long side and 4 inches from the center of gravity.

Let I denote the moment of inertia sought, and I_o the moment of inertia of the rectangle with respect to a line parallel to the long side and through the center of gravity (see Fig. 28). Then

$I_o = \frac{1}{12}ba^3$ (see Art. 52); and,

since $b=8$ inches and $a=2$ inches,

$I_o = \frac{1}{12}(8\times 2^3) = 5\frac{1}{3}$ biquadratic inches.

The distance between the two inertia-axes is 4 inches, and the area of the rectangle is 16 square inches, hence equation 5 becomes

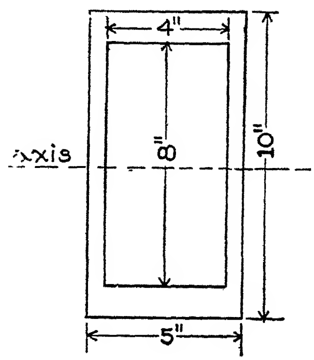


Fig. 29.

$$I = 5\tfrac{1}{3} + 16\times 4^2 = 261\tfrac{1}{3} \text{ biquadratic inches.}$$

EXAMPLE FOR PRACTICE.

1. The moment of inertia of an "angle" $2\frac{1}{2}\times 2\times \frac{1}{2}$ inches (lengths of sides and width respectively) with respect to a line through the center of gravity and parallel to the long side, is 0.64 inches^4. The area of the section is 2 square inches, and the distance from the center of gravity to the long side is 0.63 inches. (These values are taken from a "handbook".) It is required to compute the moment of inertia of the section with respect to a line parallel to the long side and 4 inches from the center of gravity. Ans. 32.64 inches^4.

54. Moment of Inertia of Built-up Sections. As before stated, beams are sometimes "built up" of rolled shapes (angles,

channels, etc.). The moment of inertia of such a section with respect to a definite axis is computed by adding the moments of inertia of the parts, *all with respect to that same axis.* This is the method for computing the moment of any area which can be divided into simple parts.

The moment of inertia of an area which may be regarded as consisting of a larger area *minus* other areas, is computed by subtracting from the moment of inertia of the large area those of the "minus areas."

Examples. 1. Compute the moment of inertia of the built-up section represented in Fig. 30 (in part same as Fig. 25) with respect to a horizontal axis passing through the center of gravity, it being given that the moment of inertia of each channel section with respect to a horizontal axis through its center of gravity is 128.1 inches^4, and its area 6.03 square inches.

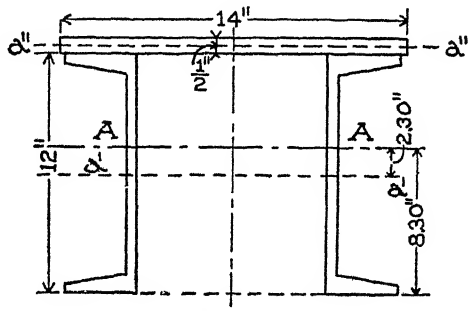


Fig. 30.

The center of gravity of the whole section was found in the example of Art. 49 to be 8.30 inches from the bottom of the section; hence the distances from the inertia-axis to the centers of gravity of the channel section and the plate are 2.30 and 3.95 inches respectively (see Fig. 30).

The moment of inertia of one channel section with respect to the axis AA (see equation 5, Art. 53) is:

$$128.1 + 6.03 \times 2.30^2 = 160.00 \text{ inches}^4.$$

The moment of inertia of the plate section (rectangle) with respect to the line $a'a''$ (see Art. 52) is:

$$\tfrac{1}{12}\,ba^3 = \tfrac{1}{12}[14 \times (\tfrac{1}{2})^3] = 0.15 \text{ inches}^4;$$

and with respect to the axis AA (the area being 7 square inches) it is:

$$0.15 + 7 \times 3.95^2 = 109.37 \text{ inches}^4.$$

Therefore the moment of inertia of the whole section with respect to AA is:

$$2 \times 160.00 + 109.37 = 429.37 \text{ inches}^4.$$

2. It is required to compute the moment of inertia of the "hollow rectangle" of Fig. 29 with respect to a line through the center of gravity and parallel to the short side.

The amount of inertia of the large rectangle with respect to the named axis (see Art. 52) is:

$$\tfrac{1}{12}(5\times 10^3) = 416\tfrac{2}{3};$$

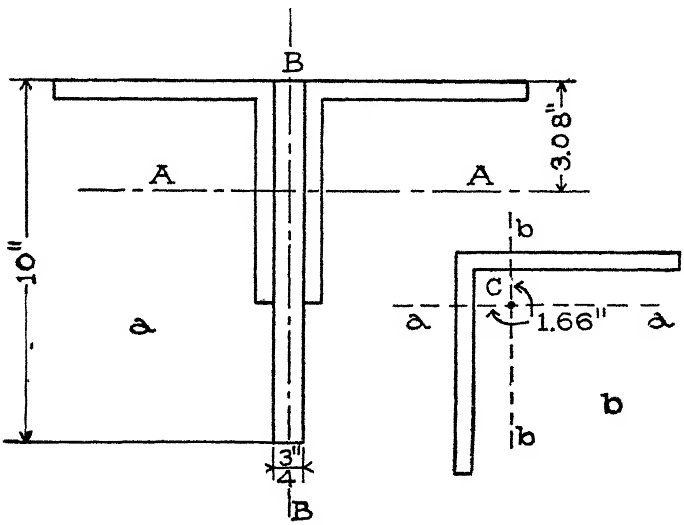


Fig. 31.

and the moment of inertia of the smaller one with respect to the same axis is:

$$\tfrac{1}{12}(4\times 8^3) = 170\tfrac{2}{3};$$

hence the moment of inertia of the hollow section with respect to the axis is:

$$416\tfrac{2}{3} - 170\tfrac{2}{3} = 246 \text{ inches}^4.$$

EXAMPLES FOR PRACTICE.

1. Compute the moment of inertia of the section represented in Fig. 31, *a*, about the axis AA, it being 3.08 inches from the top. Given also that the area of one angle section is 5.06 square inches, its center of gravity C (Fig. 31, *b*) 1.66 inches from the top, and its moment of inertia with respect to the axis *aa* 17.68 inches⁴. Ans. 145.8 inches⁴.

2. Compute the moment of inertia of the section of Fig. 31, *a*,

with respect to the axis BB. Given that distance of the center of gravity of one angle from one side is 1.66 inches (see Fig. 31, *b*), and its moment of inertia with respect to *bb* 17.68 inches.

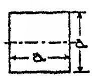
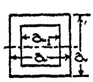
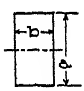
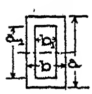
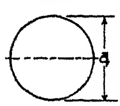
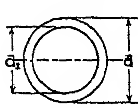
Ans. 77.618 inches^4.

55. Table of Centers of Gravity and Moments of Inertia. Column 2 in Table A below gives the formula for moment of inertia with respect to the horizontal line through the center of gravity. The numbers in the third column are explained in Art. 62; and those in the fourth, in Art. 80.

TABLE A.

Moments of Inertia, Section Moduli, and Radii of Gyration.

In each case the axis is horizontal and passes through the center of gravity

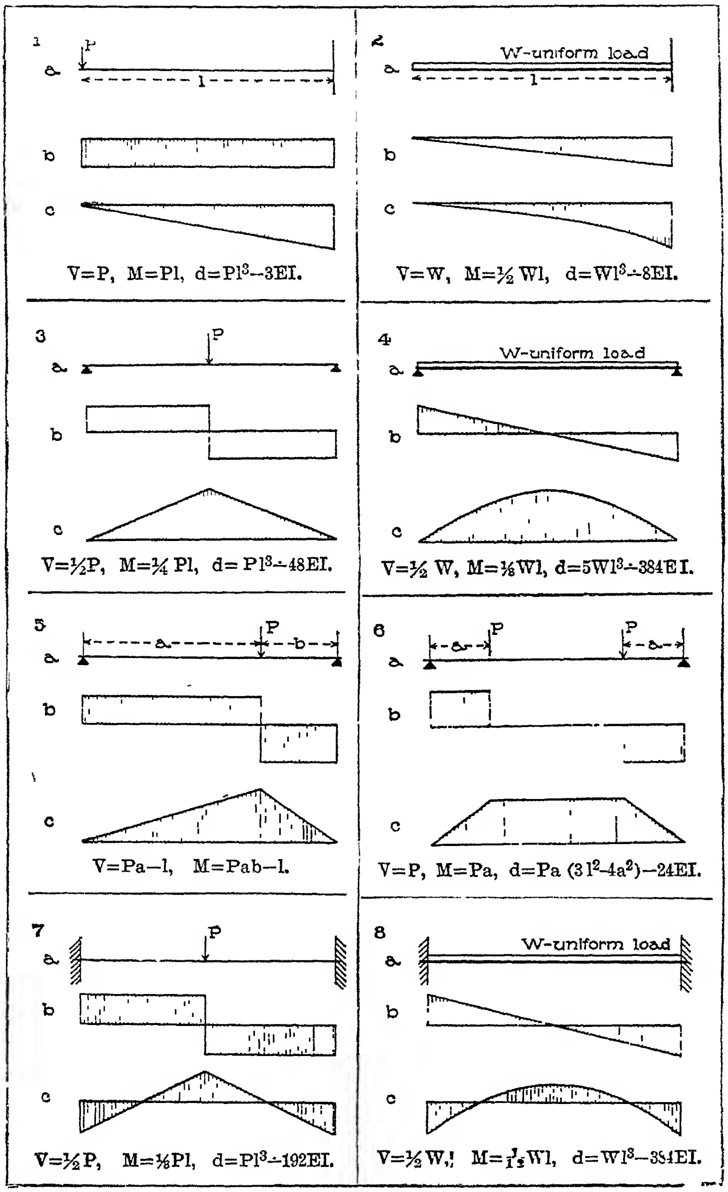
Section.	Moment of Inertia.	Section Modulus.	Radius of Gyration.
	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{a^4 - a_1^4}{12}$	$\frac{a^4 - a_1^4}{6a}$	$\sqrt{\frac{a^2 + a_1^2}{12}}$
	$\frac{ba^3}{12}$	$\frac{ba^2}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{ba^3 - b_1a_1^3}{12}$	$\frac{ba^3 - b_1a_1^3}{6a}$	$\sqrt{\frac{ba^3 - b_1a_1^3}{12(ba - b_1a_1)}}$
	$0.049d^4$	$0.098d^3$	$\frac{d}{4}$
	$0.049\,(d^4 - d_1^4)$	$0.098\frac{d^4 - d_1^4}{d}$	$\frac{\sqrt{d^2 + d_1^2}}{4}$

STRENGTH OF BEAMS.

56. Kinds of Loads Considered. The loads that are applied to a horizontal beam are usually vertical, but sometimes forces are applied otherwise than at right angles to beams. Forces acting on beams at right angles are called **transverse forces;** those applied

TABLE B.

Shear Diagrams (b) and Moment Diagrams (c) for Eight Different Cases (a). Also Values of Maximum Shear (V), Bending Moment (M), and Deflection (d)



parallel to a beam are called **longitudinal forces**; and others are called **inclined forces.** For the present we deal only with beams subjected to transverse forces (loads and reactions).

57. Neutral Surface, Neutral Line, and Neutral Axis. When a beam is loaded it may be wholly convex up (concave down), as a cantilever; wholly convex down (concave up), as a simple beam on end supports; or partly convex up and partly convex down, as a simple beam with overhanging ends, a restrained beam, or a con-

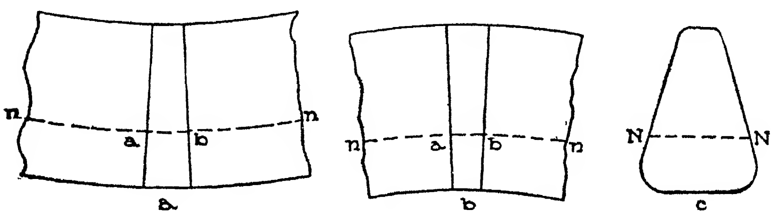


Fig. 32.

tinuous beam. Two vertical parallel lines drawn close together on the side of a beam before it is loaded will not be parallel after it is loaded and bent. If they are on a convex-down portion of a beam, they will be closer at the top and farther apart below than when drawn (Fig. 32*a*), and if they are on a convex-up portion, they will be closer below and farther apart above than when drawn (Fig. 32*b*).

The "fibres" on the convex side of a beam are stretched and therefore under tension, while those on the concave side are shortened and therefore under compression. Obviously there must be some intermediate fibres which are neither stretched nor shortened, *i. e.*, **under neither tension nor compression.** These make up a sheet of fibres and define a surface in the beam, which surface is called the **neutral surface** of the beam. The intersection of the neutral surface with either side of the beam is called the **neutral line,** and its intersection with any cross-section of the beam is called the **neutral axis** of that section. Thus, if *ab* is a fibre that has been neither lengthened nor shortened with the bending of the beam, then *nn* is a portion of the neutral line of the beam; and, if Fig. 32*c* be taken to represent a cross-section of the beam, NN is the neutral axis of the section.

It can be proved that *the neutral axis of any cross-section of*

a loaded beam passes through the center of gravity of that section, provided that all the forces applied to the beam are transverse, and that the tensile and compressive stresses at the cross-section are all within the elastic limit of the material of the beam.

58. Kinds of Stress at a Cross-section of a Beam. It has already been explained in the preceding article that there are tensile and compressive stresses in a beam, and that the tensions are on the convex side of the beam and the compressions on the concave (see Fig. 33). The forces T and C are exerted upon the portion of the beam represented by the adjoining portion to the

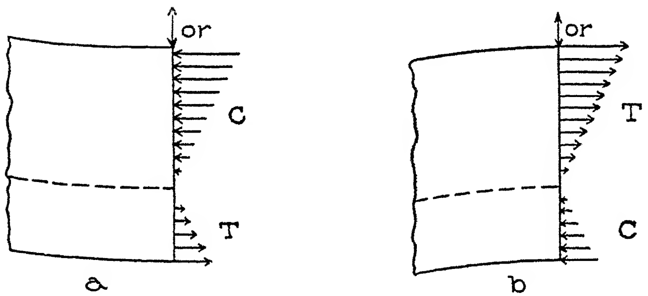


Fig. 33.

right (not shown). These, the student is reminded, are often called **fibre stresses.**

Besides the fibre stresses there is, in general, a shearing stress at every cross-section of a beam. This may be proved as follows: Fig. 34 represents a simple beam on end supports which has actually been cut into two parts as shown. The two parts can maintain loads when in a horizontal position, if forces are applied at the cut ends equivalent to the forces that would act there if the beam were not cut. Evidently in the solid beam there are at the section a compression above and a tension below, and such forces can be applied in the cut beam by means of a short block C and a chain or cord T, as shown. The block furnishes the compressive forces and the chain the tensile forces. At first sight it appears as if the beam would stand up under its load after the block and chain have been put into place. Except in certain cases*, however, it would not remain in a horizontal position, as would the

* When the external shear for the section is zero.

solid beam. This shows that the forces exerted by the block and chain (horizontal compression and tension) are not equivalent to the actual stresses in the solid beam. What is needed is a vertical force at each cut end.

Suppose that R_1 is less than $L_1 + L_2 +$ weight of A, *i. e.*, that the external shear for the section is negative; then, if vertical pulls be applied at the cut ends, upward on A and downward on B, the beam will stand under its load and in a horizontal position, just as a solid beam. These pulls can be supplied, in the model of the beam, by means of a cord S tied to two brackets fastened on A and

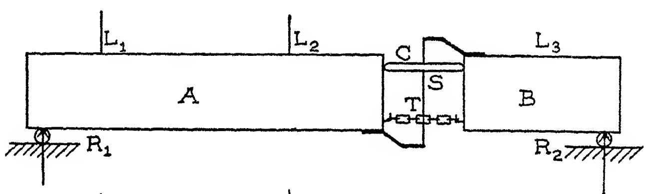


Fig. 34.

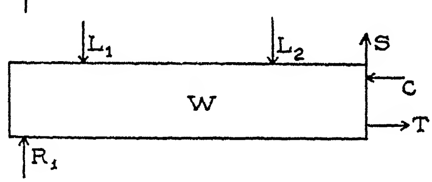


Fig. 35

B, as shown. In the solid beam the two parts act upon each other directly, and the vertical forces are shearing stresses, since they act in the plane of the surfaces to which they are applied.

59. Relation Between the Stress at a Section and the Loads and Reactions on Either Side of It. Let Fig. 35 represent the portion of a beam on the left of a section; and let R_1 denote the left reaction; L_1 and L_2 the loads; W the weight of the left part; C, T, and S the compression, tension, and shear respectively which the right part exerts upon the left.

Since the part of the beam here represented is at rest, all the forces exerted upon it are balanced; and when a number of horizontal and vertical forces are balanced, then

1. The algebraic sum of the horizontal forces equals zero.
2. " " " " " vertical " " "
3. " " " " " moments of all the forces with respect to any point equals zero.

To satisfy condition 1, since the tension and compression are the only horizontal forces, *the tension must equal the compression.*

To satisfy condition 2, S (the internal shear) must equal the

algebraic sum of all the other vertical forces on the portion, that is, must equal the external shear for the section; also, S must act up or down according as the external shear is negative or positive. In other words, briefly expressed, *the internal and external shears at a section are equal and opposite.*

To satisfy condition 3, the algebraic sum of the moments of the fibre stresses about the neutral axis must be equal to the sum of the moments of all the other forces acting on the portion about the same line, and the signs of those sums must be opposite. (The moment of the shear about the neutral axis is zero.) Now, the sum of the moments of the loads and reactions is called the bending moment at the section, and if we use the term **resisting moment** to signify the sum of the moments of the fibre stresses (tensions and compressions) about the neutral axis, then we may say briefly that *the resisting and the bending moments at a section are equal, and the two moments are opposite in sign.*

60. The Fibre Stress. As before stated, the fibre stress is not a uniform one, that is, it is not uniformly distributed over the section on which it acts. At any section, the compression is most "intense" (or the unit-compressive stress is greatest) on the concave side; the tension is most intense (or the unit-tensile stress is greatest) on the convex side; and the unit-compressive and unit-tensile stresses decrease toward the neutral axis, at which place the unit-fibre stress is zero.

If the fibre stresses are within the elastic limit, then the two straight lines on the side of a beam referred to in Art. 57 will still be straight after the beam is bent; hence the elongations and shortenings of the fibres vary directly as their distance from the neutral axis. Since the stresses (if within the elastic limit) and deformations in a given material are proportional, *the unit-fibre stress varies as the distance from the neutral axis.*

Let Fig. 36*a* represent a portion of a bent beam, 36*b* its cross-section, *nn* the neutral line, and NN the neutral axis. The way in which the unit-fibre stress on the section varies can be represented graphically as follows: Lay off *ac*, by some scale, to represent the unit-fibre stress in the top fibre, and join *c* and *n*, extending the line to the lower side of the beam; also make *bc′* equal to *bc″* and draw *nc′*. Then the arrows represent the unit-fibre stresses, for their lengths vary as their distances from the neutral axis.

61. Value of the Resisting Moment. If S denotes the unit-fibre stress in the fibre farthest from the neutral axis (the greatest unit-fibre stress on the cross-section), and c the distance from the neutral axis to the remotest fibre, while S_1, S_2, S_3, etc., denote the unit-fibre stresses at points whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. (see Fig. 36b), then

$$S : S_1 :: c : y_1; \text{ or } S_1 = \frac{S}{c} y_1.$$

Also,
$$S_2 = \frac{S}{c} y_2; \; S_3 = \frac{S}{c} y_3, \text{ etc.}$$

Let a_1, a_2, a_3, etc., be the areas of the cross-sections of the fibres

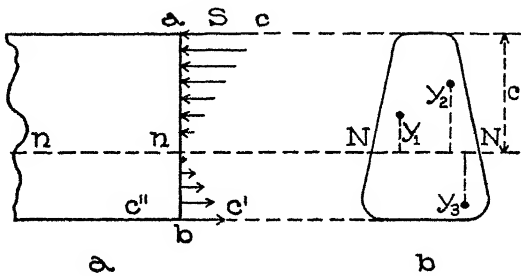


Fig. 36.

whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. Then the stresses on those fibres are, respectively,

$$S_1 a_1, \; S_2 a_2, \; S_3 a_3, \text{ etc.};$$

or,
$$\frac{S}{c} y_1 a_1, \; \frac{S}{c} y_2 a_2, \; \frac{S}{c} y_3 a_3, \text{ etc.}$$

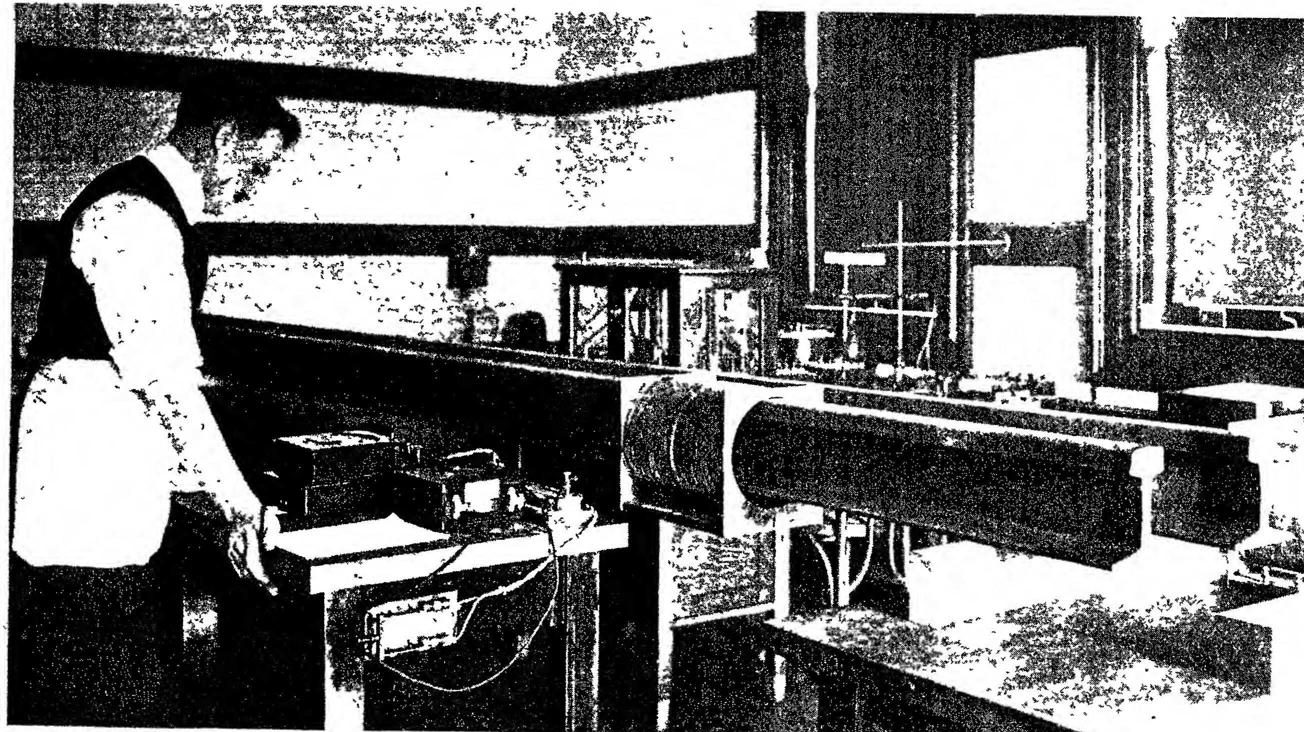
The arms of these forces or stresses with respect to the neutral axis are, respectively, y_1, y_2, y_3, etc.; hence their moments are

$$\frac{S}{c} a_1 y_1^2, \; \frac{S}{c} a_2 y_2^2, \; \frac{S}{c} a_3 y_3^2, \text{ etc.},$$

and the sum of the moments (that is, the resisting moment) is

$$\frac{S}{c} a_1 y_1^2 + \frac{S}{c} a_2 y_2^2 + \text{ etc.} = \frac{S}{c}(a_1 y_1^2 + a_2 y_2^2 + \text{ etc.})$$

Now $a_1 y_1^2 + a_2 y_2^2 +$ etc. is the sum of the products obtained by multiplying each infinitesimal part of the area of the cross-section by the square of its distance from the neutral axis; hence, it is the moment of inertia of the cross-section with respect to the neutral axis. If this moment is denoted by I, then the value of the resisting moment is $\frac{SI}{c}$.



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STRENGTH OF MATERIALS.

PART II.

STRENGTH OF BEAMS—(Concluded).

62. First Beam Formula. As shown in the preceding article, the resisting and bending moments for any section of a beam are equal; hence

$$\frac{SI}{c} = M, \qquad (6)$$

all the symbols referring to the same section. This is the most important formula relating to beams, and will be called the "first beam formula."

The ratio $I \div c$ is now quite generally called the **section modulus.** Observe that for a given beam it depends only on the dimensions of the cross-section, and not on the material or anything else. Since I is the product of four lengths (see article 51), $I \div c$ is the product of three; and hence a section modulus can be expressed in units of volume. The cubic inch is practically always used; and in this connection it is written thus, inches^3. See Table A, page 52, for values of the section moduli of a few simple sections.

63. Applications of the First Beam Formula. There are three principal applications of equation 6, which will now be explained and illustrated.

64. *First Application.* The dimensions of a beam and its manner of loading and support are given, and it is required to compute the greatest unit-tensile and compressive stresses in the beam.

This problem can be solved by means of equation 6, written in this form,

$$S = \frac{Mc}{I} \text{ or } \frac{M}{I \div c} \qquad (6')$$

Unless otherwise stated, we assume that the beams are uniform in cross-section, as they usually are; then the section modulus $(I \div c)$ is the same for all sections, and S (the unit-fibre stress on

the remotest fibre) varies just as M varies, and is therefore greatest where M is a maximum.* Hence, to compute the value of the greatest unit-fibre stress in a given case, *substitute the values of the section modulus and the maximum bending moment in the preceding equation, and reduce.*

If the neutral axis is equally distant from the highest and lowest fibres, then the greatest tensile and compressive unit-stresses are equal, and their value is S. If the neutral axis is unequally distant from the highest and lowest fibres, let c' denote its distance from the nearer of the two, and S′ the unit-fibre stress there. Then, since the unit-stresses in a cross-section are proportional to the distances from the neutral axis,

$$\frac{S'}{S} = \frac{c'}{c}, \text{ or } S' = \frac{c'}{c}S.$$

If the remotest fibre is on the convex side of the beam, S is tensile and S′ compressive; if the remotest fibre is on the concave side, S is compressive and S′ tensile.

Examples. 1. A beam 10 feet long is supported at its ends, and sustains a load of 4,000 pounds two feet from the left end (Fig. 37, a). If the beam is 4×12 inches in cross-section (the long side vertical as usual), compute the maximum tensile and compressive unit-stresses.

The section modulus of a rectangle whose base and altitude are b and a respectively (see Table A, page 52), is $\frac{1}{6}ba^2$; hence, for the beam under consideration, the modulus is

$$\frac{1}{6} \times 4 \times 12^2 = 96 \text{ inches}^3.$$

To compute the maximum bending moment, we have, first, to find the dangerous section. This section is where the shear changes sign (see article 45); hence, we have to construct the shear diagram, or as much thereof as is needed to find where the change of sign occurs. Therefore we need the values of the reaction. Neglecting the weight of the beam, the moment equation with origin at C (Fig. 37, a) is

$$R_1 \times 10 - 4{,}000 \times 8 = 0, \text{ or } R_1 = 3{,}200 \text{ pounds}$$

* NOTE. Because S is greatest in the section where M is maximum, this section is usually called the "dangerous section" of the beam.

Then, constructing the shear diagram, we see (Fig. 37, *b*) that the change of sign of the shear (also the dangerous section) is at the load. The value of the bending moment there is

$$3{,}200 \times 2 = 6{,}400 \text{ foot-pounds,}$$

or

$$6{,}400 \times 12 = 76{,}800 \text{ inch-pounds.}$$

Substituting in equation 6′, we find that

$$S = \frac{76{,}800}{96} = 800 \text{ pounds per square inch.}$$

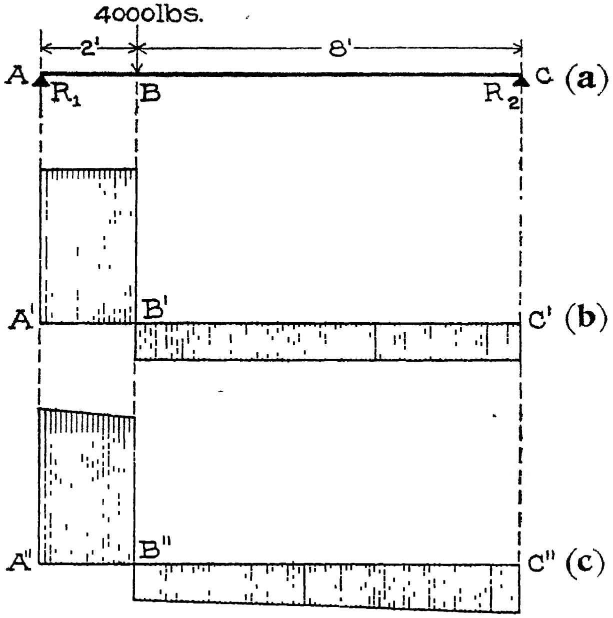


Fig. 37.

2. It is desired to take into account the weight of the beam in the preceding example, supposing the beam to be wooden.

The volume of the beam is

$$\frac{4 \times 12}{144} \times 10 = 3\tfrac{1}{3} \text{ cubic feet;}$$

and supposing the timber to weigh 45 pounds per cubic foot, the beam weighs 150 pounds (insignificant compared to the load). The left reaction, therefore, is

$$3{,}200 + (\frac{1}{2} \times 150) = 3{,}275;$$

and the shear diagram looks like Fig. 37, c, the shear changing sign at the load as before. The weight of the beam to the left of the dangerous section is 30 pounds; hence the maximum bending moment equals

$$3{,}275 \times 2 - 30 \times 1 = 6{,}520 \text{ foot-pounds,}$$

or

$$6{,}520 \times 12 = 78{,}240 \text{ inch-pounds.}$$

Substituting in equation 6′, we find that

$$S = \frac{78{,}240}{96} = 815 \text{ pounds per square inch.}$$

The weight of the beam therefore increases the unit-stress produced by the load at the dangerous section by 15 pounds per square inch.

3. A T-bar (see Fig. 38) 8 feet long and supported at each end, bears a uniform load of 1,200 pounds. The moment of inertia of its cross-section with respect to the neutral axis being 2.42 inches[4], compute the maximum tensile and compressive unit-stresses in the beam.

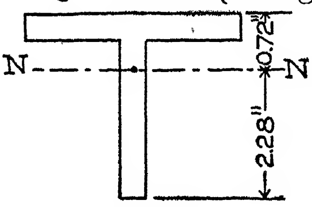


Fig. 38.

Evidently the dangerous section is in the middle, and the value of the maximum bending moment (see Table B, page 53, Part I) is $\frac{1}{8}$ Wl, W and l denoting the load and length respectively. Here

$$\frac{1}{8} Wl = \frac{1}{8} \times 1{,}200 \times 8 = 1{,}200 \text{ foot-pounds,}$$

or

$$1{,}200 \times 12 = 14{,}400 \text{ inch-pounds.}$$

The section modulus equals $2.42 \div 2.28 = 1.06$; hence

$$S = \frac{14{,}400}{1.06} = 13{,}585 \text{ pounds per square inch.}$$

This is the unit-fibre stress on the lowest fibre at the middle section, and hence is tensile. On the highest fibre at the middle section the unit-stress is compressive, and equals (see page 60):

$$S' = \frac{c'}{c} S = \frac{0.72}{2.28} \times 13{,}585 = 4{,}290 \text{ pounds per square inch.}$$

EXAMPLES FOR PRACTICE.

1. A beam 12 feet long and 6×12 inches in cross-section rests on end supports, and sustains a load of 3,000 pounds in the middle. Compute the greatest tensile and compressive unit-stresses in the beam, neglecting the weight of the beam.

Ans. 750 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 300 pounds

Ans. 787.5 pounds per square inch.

3. Suppose that a built-in cantilever projects 5 feet from the wall and sustains an end load of 250 pounds. The cross-section of the cantilever being represented in Fig. 38, compute the greatest tensile and compressive unit-stresses, and tell at what places they occur. (Neglect the weight.)

Ans. { Tensile, 4,471 pounds per square inch.
Compressive, 14,150 " " " "

4. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 18, *a*, due to the loads and the weight of beam (400 pounds). (A moment diagram is represented in Fig. 18, *b*; for description see example 2, Art. 44, p. 39.) The section of the beam is a rectangle 8×12 inches.

Ans. 580 pounds per square inch.

5. Compute the greatest tensile and compressive unit-stresses in the cantilever beam of Fig. 19, *a*, it being a steel I-beam whose section modulus is 20.4 inches^3. (A bending moment diagram for it is represented in Fig. 19, *b*; for description, see Ex. 3, Art. 44.)

Ans. 11,470 pounds per square inch.

6. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 10, neglecting its weight, the cross-sections being rectangular 6×12 inches. (See example for practice 1, Art. 43.)

Ans. 600 pounds per square inch.

65. *Second Application.* The dimensions and the working strengths of a beam are given, and it is required to determine its safe load (the manner of application being given).

This problem can be solved by means of equation 6 written in this form,

$$M = \frac{SI}{c} \qquad \textbf{(6'')}$$

We substitute for S the given working strength for the material of the beam, and for I and c their values as computed from the given dimensions of the cross-section; then reduce, thus obtaining the value of the safe resisting moment of the beam, which equals the greatest safe bending moment that the beam can stand. We next compute the value of the maximum bending moment in terms of the unknown load; equate this to the value of the resisting moment previously found; and solve for the unknown load.

In cast iron, the tensile and compressive strengths are very different; and the smaller (the tensile) should always be used if the neutral surface of the beam is midway between the top and bottom of the beam; but if it is unequally distant from the top and bottom, proceed as in example 4, following.

Examples. 1. A wooden beam 12 feet long and 6×12 inches in cross-section rests on end supports. If its working strength is 800 pounds per square inch, how large a load uniformly distributed can it sustain?

The section modulus is $\frac{1}{6}ba^2$, b and a denoting the base and altitude of the section (see Table A, page 52); and here

$$\frac{1}{6}ba^2 = \frac{1}{6} \times 6 \times 12^2 = 144 \text{ inches}^3.$$

Hence $$S\frac{I}{c} = 800 \times 144 = 115{,}200 \text{ inch-pounds.}$$

For a beam on end supports and sustaining a uniform load, the maximum bending moment equals $\frac{1}{8}Wl$ (see Table B, page 55), W denoting the sum of the load and weight of beam, and l the length. If W is expressed in pounds, then

$$\frac{1}{8}Wl = \frac{1}{8}W \times 12 \text{ foot-pounds} = \frac{1}{8}W \times 144 \text{ inch-pounds.}$$

Hence, equating the two values of maximum bending moment and the safe resisting moment, we get

$$\frac{1}{8}W \times 144 = 115{,}200;$$

or, $$W = \frac{115{,}200 \times 8}{144} = 6{,}400 \text{ pounds.}$$

The safe load for the beam is 6,400 pounds minus the weight of the beam.

2. A steel I-beam whose section modulus is 20.4 inches^3 rests on end supports 15 feet apart. Neglecting the weight of the beam, how large a load may be placed upon it 5 feet from one end, if the working strength is 16,000 pounds per square inch?

The safe resisting moment is

$$\frac{SI}{c} = 16{,}000 \times 20.4 = 326{,}400 \text{ inch-pounds;}$$

hence the bending moment must not exceed that value. The dangerous section is under the load; and if P denotes the unknown value of the load in pounds, the maximum moment (see Table B, page 53, Part I) equals $\frac{2}{3}$ P $\times$ 5 foot-pounds, or $\frac{2}{3}$ P $\times$ 60 inch-pounds. Equating values of bending and resisting moments, we get

$$\frac{2}{3} P \times 60 = 326{,}400;$$

or,

$$P = \frac{326{,}400 \times 3}{2 \times 60} = 8{,}160 \text{ pounds.}$$

3. In the preceding example, it is required to take into account the weight of the beam, 375 pounds.

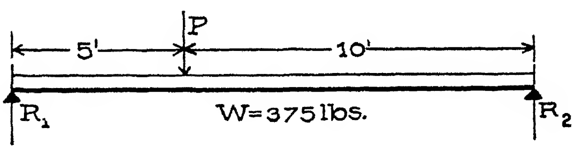


Fig. 39.

As we do not know the value of the safe load, we cannot construct the shear diagram and thus determine where the dangerous section is. But in cases like this, where the distributed load (the weight) is small compared with the concentrated load, the dangerous section is practically always where it is under the concentrated load alone; in this case, at the load. The reactions due to the weight equal $\frac{1}{2} \times 375 = 187.5$; and the reactions due to the load equal $\frac{1}{3}$ P and $\frac{2}{3}$ P, P denoting the value of the load. The larger reaction R_1 (Fig. 39) hence equals $\frac{2}{3}$ P + 187.5. Since

the weight of the beam per foot is 375 ÷ 15 = 25 pounds, the maximum bending moment (at the load) equals

$$\left(\frac{2}{3} P + 187.5\right) 5 - (25 \times 5) 2\tfrac{1}{2} =$$

$$\frac{10}{3} P + 937.5 - 312.5 = \frac{10}{3} P + 625.$$

This is in foot-pounds if P is in pounds.

The safe resisting moment is the same as in the preceding illustration, 326,400 inch-pounds; hence

$$\left(\frac{10}{3} P + 625\right) 12 = 326{,}400.$$

Solving for P, we have

$$\frac{10}{3} P + 625 = \frac{326{,}400}{12};$$

$$10 P + 625 \times 3 = \frac{326{,}400 \times 3}{12} = 81{,}600;$$

$$10 P = 79{,}725;$$

or, $$P = 7{,}972.5 \text{ pounds.}$$

It remains to test our assumption that the dangerous section is at the load. This can be done by computing R_1 (with P = 7,972.5), constructing the shear diagram, and noting where the shear changes sign. It will be found that the shear changes sign at the load, thus verifying the assumption.

4. A cast-iron built-in cantilever beam projects 8 feet from the wall. Its cross-section is represented in Fig. 40, and the moment of inertia with respect to the neutral axis is 50 inches⁴; the working strengths in tension and compression are 2,000 and 9,000 pounds per square inch respectively. Compute the safe uniform load which the beam can sustain, neglecting the weight of the beam.

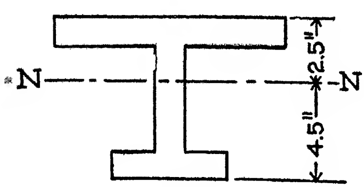


Fig. 40.

The beam being convex up, the upper fibres are in tension and the lower in compression. The resisting moment (SI ÷ c), as determined by the compressive strength, is

$$\frac{9{,}000 \times 50}{4.5} = 100{,}000 \text{ inch-pounds;}$$

and the resisting moment, as determined by the tensile strength, is

$$\frac{2{,}000 \times 50}{2.5} = 40{,}000 \text{ inch-pounds.}$$

Hence the safe resisting moment is the lesser of these two, or 40,000 inch-pounds. The dangerous section is at the wall (see Table B, page 53), and the value of the maximum bending moment is $\frac{1}{2}$ Wl, W denoting the load and l the length. If W is in pounds, then

$$M = \tfrac{1}{2} W \times 8 \text{ foot-pounds} = \tfrac{1}{2} W \times 96 \text{ inch-pounds.}$$

Equating bending and resisting moments, we have

$$\frac{1}{2} W \times 96 = 40{,}000;$$

or,
$$W = \frac{40{,}000 \times 2}{96} = 833 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. An 8 × 8-inch timber projects 8 feet from a wall. If its working strength is 1,000 pounds per square inch, how large an end load can it safely sustain?

Ans. 890 pounds.

2. A beam 12 feet long and 8 × 16 inches in cross-section, on end supports, sustains two loads P, each 3 feet from its ends respectively. The working strength being 1,000 pounds per square inch, compute P (see Table B, page 53).

Ans. 9,480 pounds.

3. An I-beam weighing 25 pounds per foot rests on end supports 20 feet apart. Its section modulus is 20.4 inches^3, and its working strength 16,000 pounds per square inch. Compute the safe uniform load which it can sustain.

Ans. 10,880 pounds-

66. *Third Application.* The loads, manner of support, and working strength of beam are given, and it is required to determine the size of cross-section necessary to sustain the load safely, that is, to "design the beam."

To solve this problem, we use the first beam formula (equation 6), written in this form,

$$\frac{\mathrm{I}}{c} = \frac{\mathrm{M}}{\mathrm{S}}. \qquad (6''')$$

We first determine the maximum bending moment, and then substitute its value for M, and the working strength for S. Then we have the value of the section modulus ($\mathrm{I} \div c$) of the required beam. Many cross-sections can be designed, all having a given section modulus. Which one is to be selected as most suitable will depend on the circumstances attending the use of the beam and on considerations of economy.

Examples. 1. A timber beam is to be used for sustaining a uniform load of 1,500 pounds, the distance between the supports being 20 feet. If the working strength of the timber is 1,000 pounds per square inch, what is the necessary size of cross-section ?

The dangerous section is at the middle of the beam; and the maximum bending moment (see Table B, page 53) is

$$\frac{1}{8}\mathrm{W}l = \frac{1}{8} \times 1{,}500 \times 20 = 3{,}750 \text{ foot-pounds,}$$

or $$3{,}750 \times 12 = 45{,}000 \text{ inch-pounds.}$$

Hence $$\frac{\mathrm{I}}{c} = \frac{45{,}000}{1{,}000} = 45 \text{ inches}^3.$$

Now the section modulus of a rectangle is $\frac{1}{6}ba^2$ (see Table A, page 54, Part I); therefore, $\frac{1}{6}ba^2 = 45$, or $ba^2 = 270$.

Any wooden beam (safe strength 1,000 pounds per square inch) whose breadth times its depth square equals or exceeds 270, is strong enough to sustain the load specified, 1,500 pounds.

To determine a size, we may choose any value for b or a, and solve the last equation for the unknown dimension. It is best, however, to select a value of the breadth, as 1, 2, 3, or 4 inches, and solve for a. Thus, if we try $b = 1$ inch, we have

$$a^2 = 270, \text{ or } a = 16.43 \text{ inches.}$$

This would mean a board 1×18 inches, which, if used, would have to be supported sidewise so as to prevent it from tipping or "buckling." Ordinarily, this would not be a good size.

Next try $b = 2$ inches; we have

$$2 \times a^2 = 270; \text{ or } a = \sqrt{270 \div 2} = 11.62 \text{ inches.}$$

This would require a plank 2×12, a better proportion than the first. Trying $b = 3$ inches, we have

$$3 \times a^2 = 270; \text{ or } a = \sqrt{270 \div 3} = 9.49 \text{ inches.}$$

This would require a plank 3 × 10 inches; and a choice between a 2 × 12 and a 3 × 10 plank would be governed by circumstances in the case of an actual construction.

It will be noticed that we have neglected the weight of the beam. Since the dimensions of wooden beams are not fractional, and we have to select a commercial size next larger than the one computed (12 inches instead of 11.62 inches, for example), the additional depth is usually sufficient to provide strength for the weight of the beam. If there is any doubt in the matter, we can settle it by computing the maximum bending moment including the weight of the beam, and then computing the greatest unit-fibre stress due to load and weight. If this is less than the safe strength, the section is large enough; if greater, the section is too small.

Thus, let us determine whether the 2 × 12-inch plank is strong enough to sustain the load and its own weight. The plank will weigh about 120 pounds, making a total load of

$$1{,}500 + 120 = 1{,}620 \text{ pounds.}$$

Hence the maximum bending moment is

$$\frac{1}{8}Wl = \frac{1}{8}1{,}620 \times 20 \times 12 = 48{,}600 \text{ inch-pounds.}$$

Since $\frac{I}{c} = \frac{1}{6}ba^2 = \frac{1}{6} \times 2 \times 12^2 = 48$, and $S = \frac{M}{I \div c}$,

$$S = \frac{48{,}600}{48} = 1{,}013 \text{ pounds per square inch.}$$

Strictly, therefore, the 2 × 12-inch plank is not large enough; but as the greatest unit-stress in it would be only 13 pounds per square inch too large, its use would be permissible.

2. What size of steel I–beam is needed to sustain safely the loading of Fig. 9 if the safe strength of the steel is 16,000 pounds per square inch?

The maximum bending moment due to the loads was found in example 1, Art. 43, to be 8,800 foot-pounds, or 8,800 × 12 = 105,600 inch-pounds.

Hence $$\frac{I}{c} = \frac{105{,}600}{16{,}000} = 6.6 \text{ inches}^3.$$

That is, an I-beam is needed whose section modulus is a little larger than 6.6, to provide strength for its own weight.

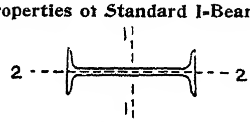
To select a size, we need a descriptive table of I-beams, such as is published in handbooks on structural steel.

Below is an abridged copy of such a table. (The last two columns contain information for use later.) The figure illustrates a cross-section of an I-beam, and shows the axes referred to in the table.

It will be noticed that two sizes are given for each depth; these are the lightest and heaviest of each size that are made, but intermediate sizes can be secured. In column 5 we find 7.3 as the next larger section modulus than the one required (6.6); and this corresponds to a $12\frac{1}{4}$-pound 6-inch I-beam, which is probably the proper size. To ascertain whether the excess $(7.3 - 6.6 = 0.70)$ in the section modulus is sufficient to provide for the weight of the beam, we might proceed as in example 1. In this case, however, the excess is quite large, and the beam selected is doubtless safe.

TABLE C.

Properties of Standard I-Beams



Section of beam, showing axes 1-1 and 2-2.

1	2	3	4	5	6
Depth of Beam, in inches.	Weight per foot, in pounds.	Area of cross-section, in square inches.	Moment of inertia, axis 1—1.	Section modulus, axis 1—1.	Moment of inertia, axis 2—2.
3	5.50	1.63	2.5	1.7	0.46
3	7.50	2.21	2.9	1.9	.60
4	7.50	2.21	6.0	3.0	.77
4	10.50	3.09	7.1	3.6	1.01
5	9.75	2.87	12.1	4.8	1.23
5	14.75	4.34	15.1	6.1	1.70
6	12.25	3.61	21.8	7.3	1.85
6	17.25	5.07	26.2	8.7	2.36
7	15.00	4.42	36.2	10.4	2.67
7	20.00	5.88	42.2	12.1	3.24
8	18.00	5.33	56.9	14.2	3.78
8	25.25	7.43	68.0	17.0	4.71
9	21.00	6.31	84.9	18.9	5.16
9	35.00	10.29	111.8	24.8	7.31
10	25.00	7.37	122.1	24.4	6.89
10	40.00	11.76	158.7	31.7	9.50
12	31.50	9.26	215.8	36.0	9.50
12	40.00	11.76	245.9	41.0	10.95
15	42.00	12.48	441.8	58.9	14.62
15	60.00	17.65	538.6	71.8	18.17
18	55.00	15.93	795.6	88.4	21.19
18	70.00	20.59	921.2	102.4	24.62
20	65.00	19.08	1,169.5	117.0	27.86
20	75.00	22.06	1,268.8	126.9	30.25
24	80.00	23.32	2,087.2	173.9	42.86
24	100.00	29.41	2,379.6	198.3	48.55

EXAMPLES FOR PRACTICE.

1. Determine the size of a wooden beam which can safely sustain a middle load of 2,000 pounds, if the beam rests on end supports 16 feet apart, and its working strength is 1,000 pounds per square inch. Assume width 6 inches.

Ans. 6×10 inches.

2. What sized steel I-beam is needed to sustain safely a uniform load of 200,000 pounds, if it rests on end supports 10 feet apart, and its working strength is 16,000 pounds per square inch?

Ans. 95-pound 24-inch.

3. What sized steel I-beam is needed to sustain safely the loading of Fig. 10, if its working strength is 16,000 pounds per square inch?

Ans. 14.75-pound 5-inch.

67. Laws of Strength of Beams. The strength of a beam is measured by the bending moment that it can safely withstand; or, since bending and resisting moments are equal, by its safe resisting moment ($SI \div c$). Hence the **safe strength** of a beam varies (1) directly as the working fibre strength of its material, and (2) directly as the section modulus of its cross-section. For beams rectangular in cross-section (as wooden beams), the section modulus is $\frac{1}{6}ba^2$, b and a denoting the breadth and altitude of the rectangle. Hence the strength of such beams varies also directly as the breadth, and as the square of the depth. Thus, doubling the breadth of the section for a rectangular beam doubles the strength, but doubling the depth quadruples the strength.

The **safe load** that a beam can sustain varies directly as its resisting moment, and depends on the way in which the load is distributed and how the beam is supported. Thus, in the first four and last two cases of the table on page 55,

$$
\begin{array}{llrl}
M = Pl, & \text{hence} & P = & SI \div lc, \\
M = \frac{1}{2}Wl, & \text{"} & W = & 2SI \div lc, \\
M = \frac{1}{4}Pl, & \text{"} & P = & 4SI \div lc, \\
M = \frac{1}{8}Wl, & \text{"} & W = & 8SI \div lc, \\
M = \frac{1}{8}Pl, & \text{"} & P = & 8SI \div lc, \\
M = \frac{1}{12}Wl, & \text{"} & W = & 12SI \div lc,
\end{array}
$$

Therefore the safe load in all cases varies inversely with the length; and for the different cases the safe loads are as 1, 2, 4, 8, 8, and 12 respectively.

Example. What is the ratio of the strengths of a plank 2×10 inches when placed edgewise and when placed flatwise on its supports?

When placed edgewise, the section modulus of the plank is $\frac{1}{6} \times 2 \times 10^2 = 33\frac{1}{3}$, and when placed flatwise it is $\frac{1}{6} \times 10 \times 2^2 = 6\frac{2}{3}$; hence its strengths in the two positions are as $33\frac{1}{3}$ to $6\frac{2}{3}$ respectively, or as 5 to 1.

EXAMPLE FOR PRACTICE.

What is the ratio of the safe loads for two beams of wood, one being 10 feet long, 3×12 inches in section, and having its load in the middle; and the other 8 feet long and 2×8 inches in section, with its load uniformly distributed.

Ans. As 28.8 to 21.3

68. Modulus of Rupture. If a beam is loaded to destruction, and the value of the bending moment for the rupture stage is computed and substituted for M in the formula $SI \div c = M$, then the value of S computed from the equation is the **modulus of rupture** for the material of the beam. Many experiments have been performed to ascertain the moduli of rupture for different materials and for different grades of the same material. The following are fair values, all in pounds per square inch:

TABLE D.

Moduli of Rupture.

Timber:	
Spruce	4,000— 7,000, average 5,000
Hemlock	3,500 7,000, " 4,500
White pine	5,500 10,500, " 8,000
Long-leaf pine	10,000 16,000, " 12,500
Short-leaf pine	8,000 14,000, " 10,000
Douglas spruce	4,000 12,000, " 8,000
White oak	7,500 18,500, " 13,000
Red oak	9,000 15,000, " 11,500
Stone:	
Sandstone	400— 1,200,
Limestone	400 1,000.
Granite	800 1,400.
Cast iron:	One and one-half to two and one-quarter times its ultimate tensile strength.
Hard steel:	Varies from 100,000 to 150,000

Wrought iron and structural steels have no modulus of rupture, as specimens of those materials will "bend double," but not break. The modulus of rupture of a material is used principally as a basis for determining its working strength. *The factor of safety of a loaded beam is computed by dividing the modulus of rupture of its material by the greatest unit-fibre stress in the beam.*

69. The Resisting Shear. The shearing stress on a cross-section of a loaded beam is not a uniform stress; that is, it is not uniformly distributed over the section. In fact the intensity or unit-stress is actually zero on the highest and lowest fibres of a cross-section, and is greatest, in such beams as are used in practice, on fibres at the neutral axis. In the following article we explain how to find the maximum value in two cases—cases which are practically important.

70. Second Beam Formula. Let S_s denote the average value of the unit-shearing stress on a cross-section of a loaded beam, and A the area of the cross-section. Then the value of the whole shearing stress on the section is:

$$\text{Resisting shear} = S_s\,A.$$

Since the resisting shear and the external shear at any section of a beam are equal (see Art. 59),

$$S_s\,A = V. \qquad (7)$$

This is called the "second beam formula" It is used to investigate and to design for shear in beams.

In beams uniform in cross-section, A is constant, and S_s is greatest in the section for which V is greatest. Hence the greatest unit-shearing stress in a loaded beam is at the neutral axis of the section at which the external shear is a maximum. There is a formula for computing this maximum value in any case, but it is not simple, and we give a simpler method for computing the value in the two practically important cases:

1. In wooden beams (rectangular or square in cross-section), the greatest unit-shearing stress in a section is 50 per cent larger than the average value S_s.

2. In I-beams, and in others with a thin vertical web, the greatest unit-shearing stress in a section practically equals S_s, as given by equation 7, if the area of the web is substituted for A.

Examples. 1. What is the greatest value of the unit-shearing stress in a wooden beam 12 feet long and 6 × 12 inches in cross-section when resting on end supports and sustaining a uniform load of 6,400 pounds? (This is the safe load as determined by working fibre stress; see example 1, Art. 65.)

The maximum external shear equals one-half the load (see Table B, page 53), and comes on the sections near the supports. Since

$$A = 6 \times 12 = 72 \text{ square inches};$$

$$S_s = \frac{3{,}200}{72} = 44 \text{ pounds per square inch},$$

and the greatest unit-shearing stress equals

$$\frac{3}{2} S_s = \frac{3}{2} 44 = 66 \text{ pounds per square inch}.$$

Apparently this is very insignificant; but it is not negligible, as is explained in the next article.

2. A steel I-beam resting on end supports 15 feet apart sustains a load of 8,000 pounds 5 feet from one end. The weight of the beam is 375 pounds, and the area of its web section is 3.2 square inches. (This is the beam and load described in examples 2 and 3, Art. 65.) What is the greatest unit-shearing stress?

The maximum external shear occurs near the support where the reaction is the greater, and its value equals that reaction. Calling that reaction R, and taking moments about the other end of the beam, we have

$$R \times 15 - 375 \times 7\frac{1}{2} - 8{,}000 \times 10 = 0;$$

therefore $15\ R = 80{,}000 + 2{,}812.5 = 82{,}812.5;$

or, $R = 5{,}520.8$ pounds.

Hence $S_s = \dfrac{5{,}520.8}{3.2} = 1{,}725$ pounds per square inch.

EXAMPLES FOR PRACTICE.

1. A wooden beam 10 feet long and 2 × 10 inches in cross-section sustains a middle load of 1,000 pounds. Neglecting the weight of the beam, compute the value of the greatest unit-shearing stress.

Ans. 37.5 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 60 pounds.

Ans. 40 pounds per square inch.

3. A wooden beam 12 feet long and 4×12 inches in cross-section sustains a load of 3,000 pounds 4 feet from one end. Neglecting the weight of the beam, compute the value of the greatest shearing unit-stress.

Ans. 62.5 pounds per square inch.

71. Horizontal Shear. It can be proved that there is a shearing stress on every horizontal section of a loaded beam. An experimental explanation will have to suffice here. Imagine a pile of six boards of equal length supported so that they do not bend. If the intermediate supports are removed, they will bend and their ends will not be flush but somewhat as represented in Fig. 41. This indicates that the boards slid over each other during the bending, and hence there was a rubbing and a frictional resistance exerted by the boards upon each other. Now, when a solid beam is being bent, there is an exactly similar tendency for the horizontal layers to slide over each other; and, instead of a frictional resistance, there exists shearing stress on all horizontal sections of the beam.

In the pile of boards the amount of slipping is different at different places between any two boards, being greatest near the supports and zero midway between them. Also, in any cross-section the slippage is least between the upper two and lower two boards, and is greatest between the middle two. These facts indicate that the shearing unit-stress on horizontal sections of a solid beam is greatest in the neutral surface at the supports.

It can be proved that at any place in a beam the shearing unit-stresses on a horizontal and on a vertical section are equal.

Fig. 41.

Fig. 42.

It follows that the horizontal shearing unit-stress is greatest at the neutral axis of the section for which the external shear (V) is a maximum. Wood being very weak in shear along the grain, timber beams sometimes fail under shear, the "rupture" being

two horizontal cracks along the neutral surface somewhat as represented in Fig. 42. It is therefore necessary, when dealing with timber beams, to give due attention to their strength as determined by the working strength of the material in shear along the grain.

Example. A wooden beam 3 × 10 inches in cross-section rests on end supports and sustains a uniform load of 4,000 pounds. Compute the greatest horizontal unit-stress in the beam.

The maximum shear equals one-half the load (see Table B, page 55), or 2,000 pounds. Hence, by equation 7, since A = 3 × 10 = 30 square inches,

$$S_s = \frac{2{,}000}{30} = 66\frac{2}{3} \text{ pounds per square inch.}$$

This is the average shearing unit-stress on the cross-sections near the supports; and the greatest value equals

$$\frac{3}{2} \times 66\frac{2}{3} = 100 \text{ pounds per square inch.}$$

According to the foregoing, this is also the value of the greatest horizontal shearing unit-stress. (If of white pine, for example, the beam would not be regarded as safe, since the ultimate shearing strength along the grain of selected pine is only about 400 pounds per square inch.)

72. Design of Timber Beams. In any case we may proceed as follows:—(1) Determine the dimensions of the cross-section of the beam from a consideration of the fibre stresses as explained in Art. 66. (2) With dimensions thus determined, compute the value of the greatest shearing unit-stress from the formula,

$$\text{Greatest shearing unit-stress} = \frac{3}{2}\,V \div ab,$$

where V denotes the maximum external shear in the beam, and b and a the breadth and depth of the cross-section.

If the value of the greatest shearing unit-stress so computed does not exceed the working strength in shear along the grain, then the dimensions are large enough; but if it exceeds that value, then a or b, or both, should be increased until $\frac{3}{2}$ V ÷ ab is less than the working strength. Because timber beams are very often "season checked" (cracked) along the neutral surface, it is advis-

able to take the working strength of wooden beams, in shear along the grain, quite low. One-twentieth of the working fibre strength has been recommended* for all pine beams.

If the working strength in shear is taken equal to one-twentieth the working fibre strength, then it can be shown that,

1. For a beam on end supports loaded in the middle, the safe load depends on the shearing or fibre strength according as the ratio of length to depth ($l - a$) is less or greater than 10.

2. For a beam on end supports uniformly loaded, the safe load depends on the shearing or fibre strength according as $l - a$ is less or greater than 20.

Examples. 1. It is required to design a timber beam to sustain loads as represented in Fig. 11, the working fibre strength being 550 pounds and the working shearing strength 50 pounds per square inch.

The maximum bending moment (see example for practice 3, Art. 43; and example for practice 2, Art. 44) equals practically 7,000 foot-pounds or, 7,000 × 12 = 84,000 inch-pounds. Hence, according to equation 6‴,

$$\frac{\mathrm{I}}{c} = \frac{84{,}000}{550} = 152.7 \text{ inches}^3.$$

Since for a rectangle

$$\frac{\mathrm{I}}{c} = \frac{1}{6}\,ba^2,$$

$$\frac{1}{6}\,ba^2 = 152.7, \text{ or } ba^2 = 916.2.$$

Now, if we let $b = 4$, then $a^2 = 229$;

or, $a = 15.1$ (practically 16) inches.

If, again, we let $b = 6$, then $a^2 = 152.7$;

or $a = 12.4$ (practically 14) inches.

Either of these sizes will answer so far as fibre stress is concerned, but there is more "timber" in the second.

The maximum external shear in the beam equals 1,556 pounds, neglecting the weight of the beam (see example 3, Art. 37; and example 2, Art. 38). Therefore, for a 4 × 16-inch beam,

* See "Materials of Construction."—JOHNSON. Page 55.

$$\text{Greatest shearing unit-stress} = \frac{3}{2} \times \frac{1,556}{4 \times 16}$$
$$= 36.5 \text{ pounds per square inch;}$$

and for a 6×14-inch beam, it equals

$$\frac{3}{2} \times \frac{1,556}{6 \times 14} = 27.7 \text{ pounds per square inch.}$$

Since these values are less than the working strength in shear, either size of beam is safe as regards shear.

If it is desired to allow for weight of beam, one of the sizes should be selected. First, its weight should be computed, then the new reactions, and then the unit-fibre stress may be computed as in Art. 64, and the greatest shearing unit-stress as in the foregoing. If these values are within the working values, then the size is large enough to sustain safely the load and the weight of the beam.

2. What is the safe load for a white pine beam 9 feet long and 2×12 inches in cross-section, if the beam rests on end supports and the load is at the middle of the beam, the working fibre strength being 1,000 pounds and the shearing strength 50 pounds per square inch.

The ratio of the length to the depth is less than 10; hence the safe load depends on the shearing strength of the material Calling the load P, the maximum external shear (see Table B, page 53) equals $\frac{1}{2}$ P, and the formula for greatest shearing unit stress becomes

$$50 = \frac{3}{2} \times \frac{\frac{1}{2}\,P}{2 \times 12}; \text{ or } P = 1,600 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. What size of wooden beam can safely sustain loads as in Fig. 12, with shearing and fibre working strength equal to 50 and 1,000 pounds per square inch respectively?

Ans. 6×12 inches

2. What is the safe load for a wooden beam 4×14 inches, and 18 feet long, if the beam rests on end supports and the load is uniformly distributed, with working strengths as in example 1?

Ans. 3,730 pounds

73. Kinds of Loads and Beams. We shall now discuss the strength of beams under **longitudinal** forces (acting parallel to the beam) and **transverse loads.** The longitudinal forces are supposed to be applied at the ends of the beams and along the axis* of the beam in each case. We consider only beams resting on end supports.

The transverse forces produce **bending** or **flexure,** and the longitudinal or end forces, if pulls, produce **tension** in the beam; if pushes, they produce **compression.** Hence the cases to be considered may be called "Combined Flexure and Tension" and "Combined Flexure and Compression."

74. Flexure and Tension. Let Fig. 43, *a*, represent a beam subjected to the transverse loads L_1, L_2 and L_3, and to two equal end pulls P and P. The reactions R_1 and R_2 are due to the transverse loads and can be computed by the methods of moments just as though there were no end pulls. To find the stresses at any cross-section, we determine those due to the transverse forces (L_1, L_2, L_3, R_1 and R_2) and those due to the longitudinal; then combine these stresses to get the total effect of all the applied forces.

The stress due to the transverse forces consists of a shearing stress and a fibre stress; it will be called the **flexural stress.** The fibre stress is compressive above and tensile below. Let M denote the value of the bending moment at the section considered; c_1 and c_2 the distances from the neutral axis to the highest and the lowest fibre in the section; and S_1 and S_2 the corresponding unit-fibre stresses due to the transverse loads. Then

$$S_1 = \frac{Mc_1}{I}; \text{ and } S_2 = \frac{Mc_2}{I}.$$

The stress due to the end pulls is a simple tension, and it equals P; this is sometimes called the **direct stress.** Let S_0 denote the unit-tension due to P, and A the area of the cross-section; then

$$S_0 = \frac{P}{A}.$$

Both systems of loads to the left of a section between L_1 and

* NOTE. By "axis of a beam" is meant the line through the centers of gravity of all the cross-sections.

L_2 are represented in Fig. 43, b; also the stresses caused by them at that section. Clearly the effect of the end pulls is to increase the tensile stress (on the lower fibres) and to decrease the compressive stress (on the upper fibres) due to the flexure. Let S_c denote the total (resultant) unit-stress on the upper fibre, and S_t that on the lower fibre, due to all the forces acting on the beam. In combining the stresses there are two cases to consider:

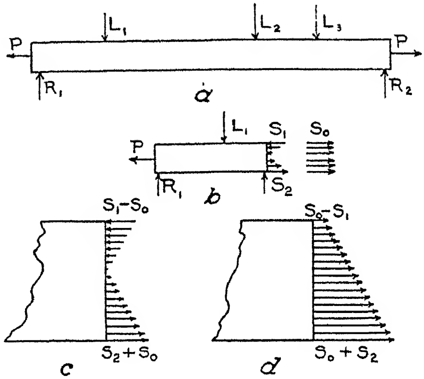


Fig. 43.

(1) The flexural compressive unit-stress on the upper fibre is greater than the direct unit-stress; that is, S_1 is greater than S_0. The resultant stress on the upper fibre is

$$S_c = S_1 - S_0 \text{ (compressive)};$$

and that on the lower fibre is

$$S_t = S_2 + S_0 \text{ (tensile)}.$$

The combined stress is as represented in Fig. 43, c, part tensile and part compressive.

(2) The flexural compressive unit-stress is less than the direct unit-stress; that is, S_1 is less than S_0. Then the combined unit-stress on the upper fibre is

$$S_c = S_0 - S_1 \quad \text{(tensile)};$$

and that on the lower fibre is

$$S_t = S_2 + S_0 \quad \text{(tensile)}.$$

The combined stress is represented by Fig. 43, d, and is all tensile.

Example. A steel bar 2 × 6 inches, and 12 feet long, is subjected to end pulls of 45,000 pounds. It is supported at each end, and sustains, as a beam, a uniform load of 6,000 pounds. It is required to compute the combined unit-fibre stresses.

Evidently the dangerous section is at the middle, and M = $\frac{1}{8}Wl$; that is,

$$M = \frac{1}{8} \times 6{,}000 \times 12 = 9{,}000 \text{ foot-pounds,}$$

or $$9{,}000 \times 12 = 108{,}000 \text{ inch-pounds.}$$

The bar being placed with the six-inch side vertical,

$$c_1 = c_2 = 3 \text{ inches, and}$$

$$I = \frac{1}{12} \times 2 \times 6^3 = 36 \text{ inches}^4. \quad \text{(See Art. 52.)}$$

Hence $$S_1 = S_2 = \frac{108{,}000 \times 3}{36} = 9{,}000 \text{ pounds per square inch.}$$

Since $$A = 2 \times 6 = 12 \text{ square inches,}$$

$$S_0 = \frac{45{,}000}{12} = 3{,}750 \text{ pounds per square inch.}$$

The greatest value of the combined compressive stress is

$$S_1 - S_0 = 9{,}000 - 3{,}750 = 5{,}250 \text{ pounds per square inch,}$$

and it occurs on the upper fibres of the middle section. The greatest value of the combined tensile stress is

$$S_2 + S_0 = 9{,}000 + 3{,}750 = 12{,}750 \text{ pounds per square inch,}$$

and it occurs on the lowest fibres of the middle section.

EXAMPLE FOR PRACTICE.

Change the load in the preceding illustration to one of 6,000 pounds placed in the middle, and then solve.

$$\text{Ans.} \begin{cases} S_c = 14{,}250 \text{ pounds per square inch.} \\ S_t = 21{,}750 \quad `` \quad `` \quad `` \quad `` \end{cases}$$

75. Flexure and Compression. Imagine the arrowheads on P reversed; then Fig. 43, *a*, will represent a beam under combined flexural and compressive stresses. The flexural unit-stresses are computed as in the preceding article. The direct stress is a compression equal to P, and the unit-stress due to P is computed as in the preceding article. Evidently the effect of P is to increase the compressive stress and decrease the tensile stress due to the flexure. In combining, we have two cases as before:

(1) The flexural tensile unit-stress is greater than the direct unit-stress; that is, S_2 is greater than S_0. Then the combined unit-stress on the lower fibre is

$$S_t = S_2 - S_o \text{ (tensile)};$$

and that on the upper fibre is

$$S_c = S_1 + S_o \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *a*, and is part tensile and part compressive.

(2) The flexural unit-stress on the lower fibre is less than the direct unit-stress; that is, S_2 is less than S_o. Then the combined unit-stress on the lower fibre is

$$S_t = S_o - S_2 \text{ (compressive)};$$

and that on the upper fibre is

$$S_c = S_o + S_1 \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *b*, and is all compressive.

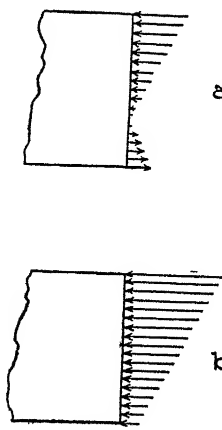


Fig. 44.

Example. A piece of timber 6 × 6 inches, and 10 feet long, is subjected to end pushes of 9,000 pounds. It is supported in a horizontal position at its ends, and sustains a middle load of 400 pounds. Compute the combined fibre stresses.

Evidently the dangerous section is at the middle, and $M = \frac{1}{4} Pl$; that is,

$$M = \frac{1}{4} \times 400 \times 10 = 1{,}000 \text{ foot-pounds},$$

or $$1{,}000 \times 12 = 12{,}000 \text{ inch-pounds}.$$

Since $c_1 = c_2 = 3$ inches, and

$$I = \frac{1}{12} ba^3 = \frac{1}{12} \times 6 \times 6^3 = 108 \text{ inches}^4,$$

$$S_1 = S_2 = \frac{12{,}000 \times 3}{108} = 333\frac{1}{3} \text{ pounds per square inch.}$$

Since $A = 6 \times 6 = 36$ square inches,

$$S_o = \frac{9{,}000}{36} = 250 \text{ pounds per square inch.}$$

Hence the greatest value of the combined compressive stress is

$$S_o + S_1 = 333\frac{1}{3} + 250 = 583\frac{1}{3} \text{ pounds per square inch.}$$

It occurs on the upper fibres of the middle section. The greatest value of the combined tensile stress is

$$S_2 - S_o = 333\frac{1}{3} - 250 = 83\frac{1}{3} \text{ pounds per square inch.}$$

It occurs on the lowest fibres of the middle section.

EXAMPLE FOR PRACTICE.

Change the load of the preceding illustration to a uniform load and solve.

Ans. $\begin{cases} S_c = 417 \text{ pounds per square inch.} \\ S_t = 83 \quad " \quad " \quad " \quad " \end{cases}$ (compression).

76. Combined Flexural and Direct Stress by More Exact Formulas. The results in the preceding articles are only approximately correct. Imagine the beam represented in Fig. 45, *a*, to be first loaded with the transverse loads alone. They cause the beam to bend more or less, and produce certain flexural stresses at each section of the beam. Now, if end pulls are applied they tend to straighten the beam and hence diminish the flexural stresses. This effect of the end pulls was omitted in the discussion of Art. 74, and the results there given are therefore only approximate, the value of the greatest combined fibre unit-stress (S_t) being too large. On the other hand, if the end forces are pushes, they increase the bending, and therefore increase the flexural fibre stresses already caused by the transverse forces (see Fig. 45, *b*). The results indicated in Art. 75 must therefore in this case also be regarded as only approximate, the value of the greatest unit-fibre stress (S_c) being too small.

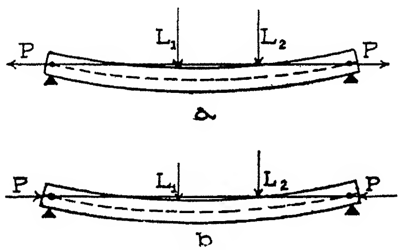


Fig. 45.

For beams loaded in the middle or with a uniform load, the following formulas, which take into account the flexural effect of the end forces, may be used:

M denotes bending moment at the middle section of the beam;

I denotes the moment of inertia of the middle section with respect to the neutral axis;

S_1, S_2, c_1 and c_2 have the same meanings as in Arts. 74 and 75, but refer always to the middle section;

l denotes length of the beam;

E is a number depending on the stiffness of the material, the average values of which are, for timber, 1,500,000; and for structural steel 30,000,000.*

$$S_1 = \frac{Mc_1}{I \pm \frac{Pl^2}{10E}}, \text{ and } S_2 = \frac{Mc_2}{I \pm \frac{Pl^2}{10E}}. \quad (8)$$

The plus sign is to be used when the end forces P are pulls, and the minus sign when they are pushes.

It must be remembered that S_1 and S_2 are flexural unit-stresses. The combination of these and the direct unit-stress is made exactly as in articles 74 and 75.

Examples. 1. It is required to apply the formulas of this article to the example of article 74.

As explained in the example referred to, M = 108,000 inch-pounds; $c_1 = c_2 = 3$ inches; and I = 36 inches^4.
Now, since l = 12 feet = 144 inches,

$$S_1 = S_2 = \frac{108{,}000 \times 3}{36 + \frac{45{,}000 \times 144^2}{10 \times 30{,}000{,}000}} = \frac{324{,}000}{36 + 3.11} = 8{,}284 \text{ pounds}$$

per square inch, as compared with 9,000 pounds per square inch, the result reached by the use of the approximate formula.
As before, S_0 = 3,750 pounds per square inch; hence

S_c = 8,284 – 3,750 = 4,534 pounds per square inch;
and S_t = 8,284 + 3,750 = 12,034 " " " "

2. It is required to apply the formulas of this article to the example of article 75.

As explained in that example,

M = 12,000 inch-pounds;

$c_1 = c_2 = 3$ inches, and I = 108 inches^4.

Now, since l = 120 inches,

$$S_1 = S_2 = \frac{12{,}000 \times 3}{108 - \frac{9{,}000 \times 120^2}{10 \times 1{,}500{,}000}} = \frac{36{,}000}{108 - 8.64} = 362 \text{ pounds}$$

* NOTE. This quantity "E" is more fully explained in Article 95.

per square inch, as compared with $333\frac{1}{3}$ pounds per square inch, the result reached by use of the approximate method.

As before, $S_o = 250$ pounds per square inch; hence

$S_c = 362 + 250 = 612$ pounds per square inch; and

$S_t = 362 - 250 = 112$ " " " " .

EXAMPLES FOR PRACTICE.

1. Solve the example for practice of Art. 74 by the formulas of this article.

Ans. $\begin{cases} S_c = 12{,}820 \text{ pounds per square inch.} \\ S_t = 20{,}320 \text{ " " " "} \end{cases}$

2. Solve the example for practice of Art. 75 by the formulas of this article.

Ans. $\begin{cases} S_c = 430 \text{ pounds per square inch.} \\ S_t = 70 \text{ " " " " (compression).} \end{cases}$

STRENGTH OF COLUMNS.

A stick of timber, a bar of iron, etc., when used to sustain end loads which act lengthwise of the pieces, are called **columns, posts,** or **struts** if they are so long that they would bend before breaking. When they are so short that they would not bend before breaking, they are called **short blocks,** and their compressive strengths are computed by means of equation 1. The strengths of columns cannot, however, be so simply determined, and we now proceed to explain the method of computing them.

77. End Conditions. The strength of a column depends in part on the way in which its ends bear, or are joined to other parts of a structure, that is, on its "end conditions." There are practically but three kinds of end conditions, namely:

1. "Hinge" or "pin" ends,
2. "Flat" or "square" ends, and
3. "Fixed" ends

(1) When a column is fastened to its support at one end by means of a pin about which the column could rotate if the other end were free, it is said to be "hinged" or "pinned" at the former end. Bridge posts or columns are often hinged at the ends.

(2) A column either end of which is flat and perpendicular to its axis and bears on other parts of the structure at that surface, is said to be "flat" or "square" at that end.

(3) Columns are sometimes riveted near their ends directly to other parts of the structure and do not bear directly on their ends; such are called "fixed ended." A column which bears on its flat ends is often fastened near the ends to other parts of the structure, and such an end is also said to be "fixed." The fixing of an end of a column stiffens and therefore strengthens it more or less, but the strength of a column with fixed ends is computed as though its ends were flat. Accordingly we have, so far as strength is concerned, the following classes of columns:

78. Classes of Columns. (1) Both ends hinged or pinned; (2) one end hinged and one flat; (3) both ends flat.

Other things being the same, columns of these three classes are unequal in strength. Columns of the first class are the weakest, and those of the third class are the strongest.

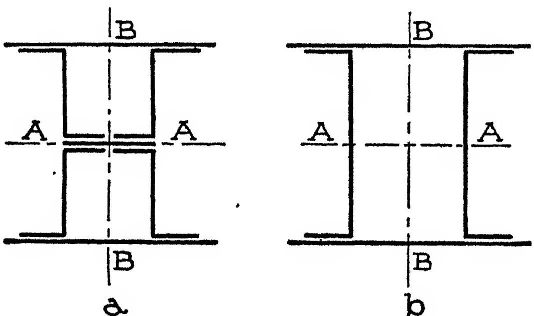


Fig. 46.

79. Cross-sections of Columns. Wooden columns are usually solid, square, rectangular, or round in section; but sometimes they are "built up" hollow. Cast-iron columns are practically always made hollow, and rectangular or round in section. Steel columns are made of single rolled shapes—angles, zees, channels, etc.; but the larger ones are usually "built up" of several shapes. Fig. 46, *a*, for example, represents a cross-section of a "Z-bar" column; and Fig. 46, *b*, that of a "channel" column.

80. Radius of Gyration. There is a quantity appearing in almost all formulas for the strength of columns, which is called "radius of gyration." It depends on the form and extent of the cross-section of the column, and may be defined as follows:

The radius of gyration of any plane figure (as the section of a column) with respect to any line, is such a length that the square of this length multiplied by the area of the figure equals the moment of inertia of the figure with respect to the given line.

Thus, if A denotes the area of a figure; I, its moment of inertia with respect to some line; and r, the radius of gyration with respect to that line; then

$$r^2 A = I; \text{ or } r = \sqrt{I \div A}. \qquad (9)$$

In the column formulas, the radius of gyration always refers to an axis through the center of gravity of the cross-section, and usually to that axis with respect to which the radius of gyration (and moment of inertia) is least. (For an exception, see example 3, Art. 83.) Hence the radius of gyration in this connection is often called for brevity the "least radius of gyration," or simply the "least radius."

Examples. 1. Show that the value of the radius of gyration given for the square in Table A, page 52, is correct.

The moment of inertia of the square with respect to the axis is $\frac{1}{12}a^4$. Since $A = a^2$, then, by formula 9 above,

$$r = \sqrt{\frac{1}{12}a^4 \div a^2} = \sqrt{\frac{1}{12}a^2} = a\sqrt{\frac{1}{12}}.$$

2. Prove that the value of the radius of gyration given for the hollow square in Table A, page 54, is correct.

The value of the moment of inertia of the square with respect to the axis is $\frac{1}{12}(a^4 - a_1^4)$. Since $A = a^2 - a_1^2$,

$$r = \sqrt{\frac{\frac{1}{12}(a^4 - a_1^4)}{a^2 - a_1^2}} = \sqrt{\frac{1}{12}(a^2 + a_1^2)}.$$

EXAMPLE FOR PRACTICE.

Prove that the values of the radii of gyration of the other figures given in Table A, page 52, are correct. The axis in each case is indicated by the line through the center of gravity.

81. Radius of Gyration of Built-up Sections. The radius of gyration of a built-up section is computed similarly to that of any other figure. First, we have to compute the moment of inertia of

the section, as explained in Art. 54; and then we use formula 9, as in the examples of the preceding article.

Example. It is required to compute the radius of gyration of the section represented in Fig. 30 (page 52) with respect to the axis AA.

In example 1, Art. 54, it is shown that the moment of inertia of the section with respect to the axis AA is 429 inches^4. The area of the whole section is

$$2 \times 6.03 + 7 = 19.06;$$

hence the radius of gyration r is

$$r = \sqrt{\frac{429}{19.06}} = 4.74 \text{ inches.}$$

EXAMPLE FOR PRACTICE.

Compute the radii of gyration of the section represented in Fig. 31, a, with respect to the axes AA and BB. (See examples for practice 1 and 2, Art. 54.)

Ans. $\begin{cases} 2.87 \text{ inches.} \\ 2.09 \quad " \end{cases}$

82. Kinds of Column Loads. When the loads applied to a column are such that their resultant acts through the center of gravity of the top section and along the axis of the column, the column is said to be **centrally loaded.** When the resultant of the loads does not act through the center of gravity of the top section, the column is said to be **eccentrically loaded.** All the following formulas refer to columns centrally loaded.

83. Rankine's Column Formula. When a perfectly straight column is centrally loaded, then, if the column does not bend and if it is homogeneous, the stress on every cross-section is a uniform compression. If P denotes the load and A the area of the cross-section, the value of the unit-compression is P ÷ A.

On account of lack of straightness or lack of uniformity in material, or failure to secure exact central application of the load, the load P has what is known as an "arm" or "leverage" and bends the column more or less. There is therefore in such a column a bending or flexural stress in addition to the direct compressive stress above mentioned; this bending stress is compressive

on the concave side and tensile on the convex. The value of the stress per unit-area (unit-stress) on the fibre at the concave side, according to equation 6, is $Mc \div I$, where M denotes the bending moment at the section (due to the load on the column), c the distance from the neutral axis to the concave side, and I the moment of inertia of the cross-section with respect to the neutral axis. (Notice that this axis is perpendicular to the plane in which the column bends.)

The upper set of arrows (Fig. 47) represents the direct compressive stress; and the second set the bending stress if the load is not excessive, so that the stresses are within the elastic limit of the material. The third set represents the combined stress that actually exists on the cross-section. The greatest combined unit-stress evidently occurs on the fibre at the concave side and where the deflection of the column is greatest. The stress is compressive, and its value S per unit-area is given by the formula,

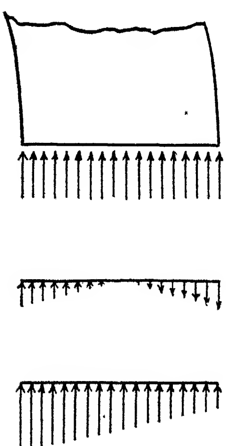


Fig. 47.

$$S = \frac{P}{A} + \frac{Mc}{I}.$$

Now, the bending moment at the place of greatest deflection equals the product of the load P and its arm (that is, the deflection). Calling the deflection d, we have $M = Pd$; and this value of M, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{I}.$$

Let r denote the radius of gyration of the cross-section with respec to the neutral axis. Then $I = Ar^2$ (see equation 9); and this value, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{Ar^2} = \frac{P}{A}\left(1 + \frac{dc}{r^2}\right).$$

According to the theory of the stiffness of beams on end supports, deflections vary directly as the square of the length l, and inversely as the distance c from the neutral axis to the remotest fibre of the cross-section. Assuming that the deflections of columns

follow the same laws, we may write $d = k\,(l^2 \div c)$, where k is some constant depending on the material of the column and on the end conditions. Substituting this value for d in the last equation, we find that

$$\left.\begin{aligned} S &= \frac{P}{A}\left(1 + k\frac{l^2}{r^2}\right); \\ \frac{P}{A} &= \frac{S}{1 + k\dfrac{l^2}{r^2}}; \\ \text{and} \qquad P &= \frac{SA}{1 + k\dfrac{l^2}{r^2}}. \end{aligned}\right\} \qquad \textbf{(10)}$$

Each of these (usually the last) is known as "Rankine's formula."

For *mild-steel* columns a certain large steel company uses $S = 50{,}000$ pounds per square inch, and the following values of k:

1. Columns with two pin ends, $k = 1 \div 18{,}000$.
2. " " one flat and one pin end, $k = 1 \div 24{,}000$.
3. " " both ends flat, $k = 1 \div 36{,}000$.

With these values of S and k, P of the formula means the **ultimate load**, that is, the load causing failure. The safe load equals P divided by the selected factor of safety—a factor of 4 for steady loads, and 5 for moving loads, being recommended by the company referred to. The same unit is to be used for l and r.

Cast-iron columns are practically always made hollow with comparatively thin walls, and are usually circular or rectangular in cross-section. The following modifications of Rankine's formula are sometimes used:

$$\left.\begin{aligned} &\text{For circular sections,} \qquad \frac{P}{A} = \frac{80{,}000}{1 + \dfrac{l^2}{800\,d^2}} \\ &\text{For rectangular sections,} \qquad \frac{P}{A} = \frac{80{,}000}{1 + \dfrac{l^2}{1{,}000\,d^2}} \end{aligned}\right\} \qquad \textbf{(10')}$$

In these formulas d denotes the outside diameter of the circular sections or the length of the lesser side of the rectangular sections. The same unit is to be used for l and d.

Examples. 1. A 40-pound 10-inch steel I-beam 8 feet long is used as a flat-ended column. Its load being 100,000 pounds, what is its factor of safety?

Obviously the column tends to bend in a plane perpendicular to its web. Hence the radius of gyration to be used is the one

with respect to that central axis of the cross-section which is in the web, that is, axis $2-2$ (see figure accompanying table, page 72). The moment of inertia of the section with respect to that axis, according to the table, is 9.50 inches^4; and since the area of the section is 11.76 square inches,

$$r^2 = \frac{9.50}{11.76} = 0.808.$$

Now, $l = 8$ feet $= 96$ inches; and since $k = 1 \div 36{,}000$, and $S = 50{,}000$, the breaking load for this column, according to Rankine's formula, is

$$P = \frac{50{,}000 \times 11.76}{1 + \dfrac{96^2}{36{,}000 \times 0.808}} = 446{,}790 \text{ pounds.}$$

Since the factor of safety equals the ratio of the breaking load to the actual load on the column, the factor of safety in this case is

$$\frac{446{,}790}{100{,}000} = 4.5 \text{ (approx.).}$$

2. What is the safe load for a cast-iron column 10 feet long with square ends and a hollow rectangular section, the outside dimensions being 5×8 inches; the inner, 4×7 inches; and the factor of safety, 6?

In this case $l = 10$ feet $= 120$ inches; $A = 5 \times 8 - 4 \times 7 = 12$ square inches; and $d = 5$ inches. Hence, according to formula 10′, for rectangular sections, the breaking load is

$$P = \frac{80{,}000 \times 12}{1 + \dfrac{120^2}{1{,}000 \times 5^2}} = 610{,}000 \text{ pounds.}$$

Since the safe load equals the breaking load divided by the factor of safety, in this case the safe load equals

$$\frac{610{,}000}{6} = 101{,}700 \text{ pounds.}$$

3. A channel column (see Fig. 46, b) is pin-ended, the pins being perpendicular to the webs of the channel (represented by AA in the figure), and its length is 16 feet (distance between axes

of the pins). If the sectional area is 23.5 square inches, and the moment of inertia with respect to AA is 386 inches^4 and with respect to BB 214 inches^4, what is the safe load with a factor of safety of 4 ?

The column is liable to bend in one of two ways, namely, in the plane perpendicular to the axes of the two pins, or in the plane containing those axes.

(1) For bending in the first plane, the strength of the column is to be computed from the formula for a pin-ended column. Hence, for this case, $r^2 = 386 \div 23.5 = 16$; and the breaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \dfrac{(16 \times 12)^2}{18{,}000 \times 16}} = 1{,}041{,}600 \text{ pounds.}$$

The safe load for this case equals $\dfrac{1{,}041{,}600}{4} = 260{,}400$ pounds.

(2) If the supports of the pins are rigid, then the pins stiffen the column as to bending in the plane of their axes, and the strength of the column for bending in that plane should be computed from the formula for the strength of columns with flat ends. Hence, $r^2 = 214 \div 23.5 = 9.11$, and thebreaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \dfrac{(16 \times 12)^2}{36{,}000 \times 9.11}} = 1{,}056{,}000 \text{ pounds.}$$

The safe load for this case equals $\dfrac{1{,}056{,}000}{4} = 264{,}000$ pounds.

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a column with flat ends sustaining a load of 100,000 pounds. What is its factor of safety?

Ans. 4.1

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle, 9 inches outside and 7 inches inside diameter, what is the factor of safety?

Ans. 8.9

3. A steel Z-bar column (see Fig. 46, a) is 24 feet long and has square ends; the least radius of gyration of its cross-section is

3.1 inches; and the area of the cross-section is 24.5 square inches. What is the safe load for the column with a factor of safety of 4?

Ans. 247,000 pounds.

4. A cast-iron column 13 feet long has a hollow circular cross-section 7 inches outside and $5\frac{1}{2}$ inches inside diameter. What is its safe load with a factor of safety of 6?

Ans. 121,142 pounds.

5. Compute the safe load for a 40-pound 12-inch steel I-beam used as a column with flat ends, its length being 17 feet. Use a factor of safety of 5.

Ans. 52,470 pounds.

84. Graphical Representation of Column Formulas. Column (and most other engineering) formulas can be represented graphically. To represent Rankine's formula for flat-ended mild-steel columns,

$$\frac{P}{A} = \frac{50,000}{1 + \frac{(l \div r)^2}{36,000}},$$

we first substitute different values of $l \div r$ in the formula, and solve for $P - A$. Thus we find, when

$$l - r = 40, \quad P - A = 47,900;$$
$$l - r = 80, \quad P - A = 42,500;$$
$$l - r = 120, \quad P - A = 35,750;$$
$$\text{etc.,} \qquad \text{etc.}$$

Now, if these values of $l - r$ be laid off by some scale on a line from O, Fig. 48, and the corresponding values of $P \div A$ be laid

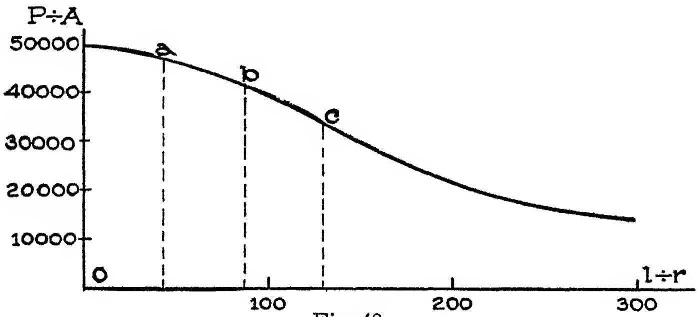


Fig. 48.

off vertically from the points on the line, we get a series of points as a, b, c, etc.; and a smooth curve through the points a, b, c,

etc., represents the formula. Such a curve, besides representing the formula to one's eye, can be used for finding the value of $P \div A$ for any value of $l \div r$; or the value of $l \div r$ for any value of $P \div A$. The use herein made is in explaining other column formulas in succeeding articles.

85. Combination Column Formulas. Many columns have been tested to destruction in order to discover in a practical way the laws relating to the strength of columns of different kinds. The results of such tests can be most satisfactorily represented graphically by plotting a point in a diagram for each test. Thus, suppose that a column whose $l \div r$ was 80 failed under a load of 276,000 pounds, and that the area of its cross-section was 7.12 square inches. This test would be represented by laying off Oa, Fig. 49, equal to 80, according to some scale; and then ab equal to $276{,}000 \div 7.12$ ($P \div A$), according to some other convenient scale. The point b would then represent the result of this particular test. All the dots in the figure represent the way in which the results of a series of tests appear when plotted.

It will be observed at once that the dots do not fall upon any one curve, as the curve of Rankine's formula. Straight lines and

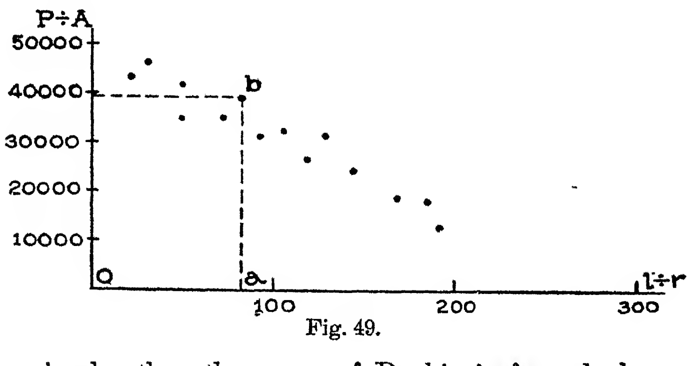


Fig. 49.

curves simpler than the curve of Rankine's formula have been fitted to represent the average positions of the dots as determined by actual tests, and the formulas corresponding to such lines have been deduced as column formulas. These are explained in the following articles.

86. Straight-Line and Euler Formulas. It occurred to Mr. T. H. Johnson that most of the dots corresponding to ordinary

lengths of columns agree with a straight line just as well as with a curve. He therefore, in 1886, made a number of such plats or diagrams as Fig. 49, fitted straight lines to them, and deduced the formula corresponding to each line. These have become known as "straight-line formulas," and their general form is as follows:

$$\frac{P}{A} = S - m\frac{l}{r}, \qquad (11)$$

P, A, l, and r having meanings as in Rankine's formula (Art. 83), and S and m being constants whose values according to Johnson are given in Table E below.

For the slender columns, another formula (Euler's, long since deduced) was used by Johnson. Its general form is—

$$\frac{P}{A} = \frac{n}{(l \div r)^2}, \qquad (12)$$

n being a constant whose values, according to Johnson, are given in the following table:

TABLE E.

Data for Mild-Steel Columns.

	S	m	Limit ($l \div r$)	n
Hinged ends....	52,500	220	160	444,000,000
Flat ends........	52,500	180	195	666,000,000

The numbers in the fourth column of the table mark the point of division between columns of ordinary length and slender columns. For the former kind, the straight-line formula applies; and for the second, Euler's. That is, if the ratio $l \div r$ for a steel column with hinged end, for example, is less than 160, we must use the straight-line formula to compute its safe load, factor of safety, etc ; but if the ratio is greater than 160, we must use Euler's formula.

For *cast-iron columns* with flat ends, S = 34,000, and m = 88; and since they should never be used "slender," there is no use of Euler's formula for cast-iron columns.

The line AB, Fig. 50, represents Johnson's straight-line formula; and BC, Euler's formula. It will be noticed that the two lines are tangent; the point of tangency corresponds to the "limiting value" $l \div r$, as indicated in the table.

Examples. 1. A 40-pound 10-inch steel I-beam column 8

feet long sustains a load of 100,000 pounds, and the ends are fl[at]. Compute its factor of safety according to the methods of th[is] article.

The first thing to do is to compute the ratio $l \div r$ for the column, to ascertain whether the straight-line formula or Euler's

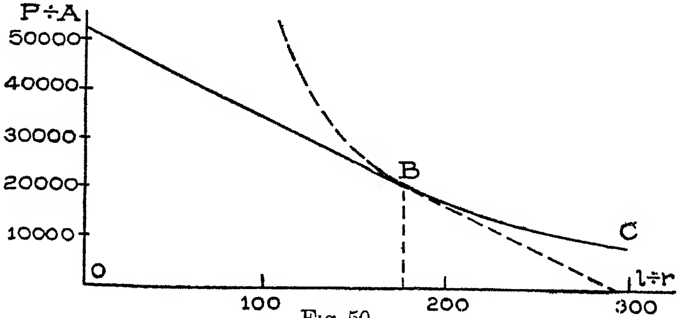


Fig. 50.

formula should be used. From Table C, on page 70, we find that the moment of inertia of the column about the neutral-axis of its cross-section is 9.50 inches^4, and the area of the section is 11.76 square inches. Hence

$$r^2 = \frac{9.50}{11.76} = 0.81; \text{ or } r = 0.9 \text{ inch.}$$

Since l = 8 feet = 96 inches,

$$\frac{l}{r} = \frac{96}{0.9} = 106\frac{2}{3}$$

This value of $l \div r$ is less than the limiting value (195) indicated by the table for steel columns with flat ends (Table E, p. 97), and we should therefore use the straight-line formula; hence

$$\frac{P}{11.76} = 52{,}500 - 180 \times 106\frac{2}{3};$$

or, $$P = 11.76\,(52{,}500 - 180 \times 106\frac{2}{3}) = 391{,}600 \text{ pounds.}$$

This is the breaking load for the column according to the straight-line formula; hence the factor of safety is

$$\frac{391{,}600}{100{,}000} = 3.9$$

2. Suppose that the length of the column described in the preceding example were 16 feet. What would its factor of safety be?

Since $l = 16$ feet $= 192$ inches; and, as before, $r = 0.9$ inch, $l \div r = 213\frac{1}{3}$. This value is greater than the limiting value (195) indicated by Table E (p. 97) for flat-ended steel columns; hence Euler's formula is to be used. Thus

$$\frac{P}{11.76} = \frac{666{,}000{,}000}{(213\frac{1}{3})^2};$$

or,

$$P = \frac{11.76 \times 666{,}000{,}000}{(213\frac{1}{3})^2} = 172{,}100 \text{ pounds.}$$

This is the breaking load; hence the factor of safety is

$$\frac{172{,}100}{100{,}000} = 1.7$$

3. What is the safe load for a cast-iron column 10 feet long with square ends and hollow rectangular section, the outside dimensions being 5×8 inches and the inside 4×7 inches, with a factor of safety of 6?

Substituting in the formula for the radius of gyration given in Table A, page 52, we get

$$r = \sqrt{\frac{8 \times 5^3 - 7 \times 4^3}{12\ (8 \times 5 - 7 \times 4)}} = 1.96 \text{ inches.}$$

Since $l = 10$ feet $= 120$ inches,

$$\frac{l}{r} = \frac{120}{1.96} = 61.22$$

According to the straight-line formula for cast iron, A being equal to 12 square inches,

$$\frac{P}{12} = 34{,}000 - 88 \times 61.22;$$

or,

$$P = 12\ (34{,}000 - 88 \times 61.22) = 343{,}360 \text{ pounds.}$$

This being the breaking load, the safe load is

$$\frac{343{,}360}{6} = 57{,}227 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute the factor of safety by the formulas of this article.

Ans. 3.5

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle of 9 inches outside and 7 inches inside diameter, compute the factor of safety by the straight-line formula.

Ans. 4.8

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of the cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. 219,000 pounds.

4. A hollow cast-iron column 13 feet long has a circular cross-section, and is 7 inches outside and $5\frac{1}{2}$ inches inside in diameter. Compute its safe load by the formulas of this article, using a factor of safety of 6.

Ans. 68,300 pounds.

5. Compute by the methods of this article the safe load for a 40-pound 12-inch steel I-beam used as a column with flat ends, if the length is 17 feet and the factor of safety 5.

Ans. 35,050 pounds.

87. Parabola-Euler Formulas. As better fitting the results of tests of the strength of columns of "ordinary lengths," Prof. J. B. Johnson proposed (1892) to use parabolas instead of straight lines. The general form of the "parabola formula" is

$$\frac{P}{A} = S - m\left(\frac{l}{r}\right)^2, \qquad (13)$$

P, A, l and r having the same meanings as in Rankine's formula, Art. 83; and S and m denoting constants whose values, according to Professor Johnson, are given in Table F below.

Like the straight-line formula, the parabola formula should not be used for slender columns, but the following (Euler's) is applicable:

$$\frac{P}{A}=\frac{n}{(l-r)^2}, \qquad (14)$$

the values of n (Johnson) being given in the following table:

TABLE F.

Data for Mild Steel Columns.

	S	m	Limit $(l-r)$	n
Hinged ends......	42,000	0.97	150	456,000,000
Flat ends.	42,000	0.62	190	712,000,000

The point of division between columns of ordinary length and slender columns is given in the fourth column of the table. That is, if the ratio $l \div r$ for a column with hinged ends, for example, is less than 150, the parabola formula should be used to compute the safe load, factor of safety, etc.; but if the ratio is greater than 150, then Euler's formula should be used.

The line AB, Fig. 51, represents the parabola formula; and the line BC, Euler's formula. The two lines are tangent, and the point of tangency corresponds to the "limiting value" $l \div r$ of the table.

For *wooden columns* square in cross-section, it is convenient to replace r by d, the latter denoting the length of the sides of the square. The formula becomes

$$\frac{P}{A}=S-m\left(\frac{l}{d}\right)^2,$$

S and m for flat-ended columns of various kinds of wood having the following values according to Professor Johnson:

For	White pine,	S=2,500,	$m=0.6$;
"	Short-leaf yellow pine,	S=3,300,	$m=0.7$;
"	Long-leaf yellow pine,	S=4,000,	$m=0.8$,
"	White oak,	S=3,500,	$m=0.8$.

The preceding formula applies to any wooden column whose ratio, $l \div d$, is less than 60, within which limit columns of practice are included.

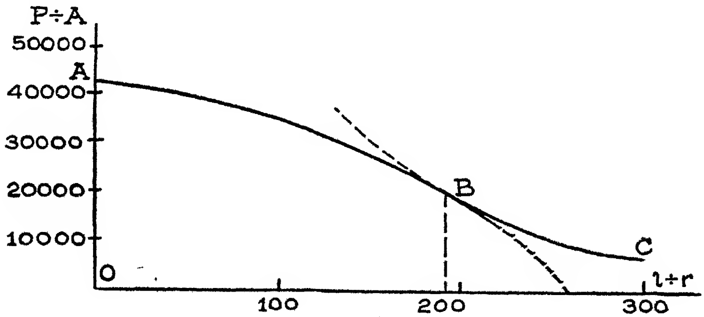


Fig. 51.

Examples. 1. A 40-pound 10-inch steel I-beam column

8 feet long sustains a load of 100,000 pounds, and its ends are flat. Compute its factor of safety according to the methods of this article.

The first thing to do is to compute the ratio $l - r$ for the column, to ascertain whether the parabola formula or Euler's formula should be used. As shown in example 1 of the preceding article, $l \div r = 106\frac{2}{3}$. This ratio being less than the limiting value, 190, of the table, we should use the parabola formula. Hence, since the area of the cross-section is 11.76 square inches (see Table C, page 70),

$$\frac{P}{11.76} = 42{,}000 - 0.62\ (106\tfrac{2}{3})^2;$$

or, $$P = 11.76\ [42{,}000 - 0.62\ (106\tfrac{2}{3})^2] = 410{,}970 \text{ pounds.}$$

This is the breaking load according to the parabola formula; hence the factor of safety is

$$\frac{410{,}970}{100{,}000} = 4.1$$

2. A white pine column 10×10 inches in cross-section and 18 feet long sustains a load of 40,000 pounds. What is its factor of safety?

The length is 18 feet or 216 inches; hence the ratio $l \div d = 21.6$, and the parabola formula is to be applied.
Now, since $A = 10 \times 10 = 100$ square inches,

$$\frac{P}{100} = 2{,}500 - 0.6 \times 21.6^2;$$

or, $$P = 100\ (2{,}500 - 0.6 \times 21.6^2) = 222{,}000 \text{ pounds.}$$

This being the breaking load according to the parabola formula, the factor of safety is

$$\frac{222{,}000}{40{,}000} = 5.5$$

3. What is the safe load for a long-leaf yellow pine column 12×12 inches square and 30 feet long, the factor of safety being 5?

The length being 30 feet or 360 inches,

$$\frac{l}{d} = \frac{360}{12} = 30;$$

hence the parabola formula should be used. Since A $= 12 \times 12 = 144$ square inches,

$$\frac{P}{144} = 4{,}000 - 0.8 \times 30^2;$$

or, $$P = 144\ (4{,}000 - 0.8 \times 30^2) = 472{,}320 \text{ pounds.}$$

This being the breaking load according to the parabola formula, the safe load is

$$\frac{472{,}320}{5} = 94{,}465 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute its factor of safety by the formulas of this article.

Ans. 3.8

2. A white oak column 15 feet long sustains a load of 30,000 pounds. Its section being 8×8 inches, compute the factor of safety by the parabola formula.

Ans. 6.6

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of its cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. **224,500** pounds.

4. A short-leaf yellow pine column 14×14 inches in section is 20 feet long. What load can it sustain, with a factor of safety of 6?

Ans. **101,100** pounds.

88. "Broken Straight-Line" Formula. A large steel company computes the strength of its flat-ended steel columns by two formulas represented by two straight lines AB and BC, Fig. 52. The formulas are

$$\frac{P}{A} = 48{,}000,$$

and $$\frac{P}{A} = 68{,}400 - 228\ \frac{l}{r},$$

P, A, l, and r having the same meanings as in Art. 83.

The point B corresponds very nearly to the ratio $l \div r = 90$. Hence, for columns for which the ratio $l \div r$ is less than 90, the first formula applies; and for columns for which the ratio is greater than 90, the second one applies. The point C corresponds to the ratio $l \div r = 200$, and the second formula does not apply to a column for which $l \div r$ is greater than that limit.

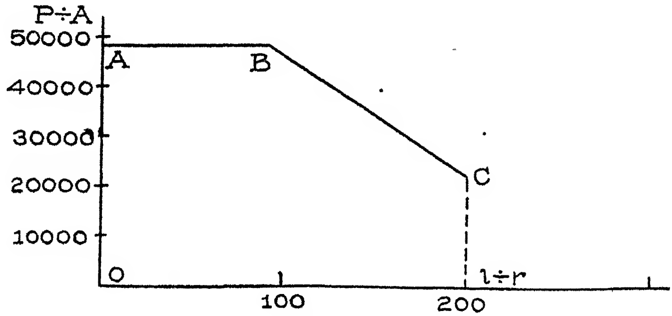


Fig. 52.

The ratio $l \div r$ for steel columns of practice rarely exceeds 150, and is usually less than 100.

Fig. 53 is a combination of Figs. 49, 50, 51 and 52, and represents graphically a comparison of the Rankine, straight-line, Euler, parabola-Euler, and broken straight-line formulas for flat-ended mild-steel columns, It well illustrates the fact that our knowledge of the strength of columns is not so exact as that, for example, of the strength of beams.

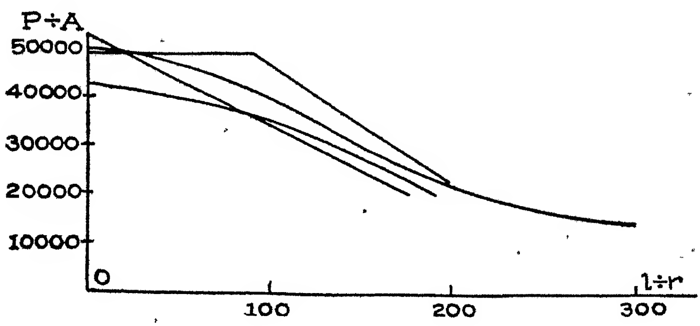


Fig. 53.

89. Design of Columns. All the preceding examples relating to columns were on either (1) computing the factor of safety

of a given loaded column, or (2) computing the safe load for a given column. A more important problem is to design a column to sustain a given load under given conditions. A complete discussion of this problem is given in a later paper on design. We show here merely how to compute the *dimensions* of the cross-section of the column after the *form* of the cross-section has been decided upon.

In only a few cases can the dimensions be computed directly (see example 1 following), but usually, when a column formula is applied to a certain case, there will be two unknown quantities in it, A and r or d. Such cases can best be solved by trial (see examples 2 and 3 below).

Example. 1. What is the proper size of white pine column to sustain a load of 80,000 pounds with a factor of safety of 5, when the length of the column is 22 feet?

We use the parabola formula (equation 13). Since the safe load is 80,000 pounds and the factor of safety is 5, the breaking load P is

$$80{,}000 \times 5 = 400{,}000 \text{ pounds.}$$

The unknown side of the (square) cross-section being denoted by d, the area A is d^2. Hence, substituting in the formula, since $l = 22$ feet $= 264$ inches, we have

$$\frac{400{,}000}{d^2} = 2{,}500 - 0.6\,\frac{264^2}{d^2}.$$

Multiplying both sides by d^2 gives

$$400{,}000 = 2{,}500\,d^2 - 0.6 \times 264^2,$$

or $$2{,}500\,d^2 = 400{,}000 + 0.6 \times 264^2 = 441{,}817.6.$$

Hence $$d^2 = 176.73, \text{ or } d = 13.3 \text{ inches.}$$

2. What size of cast-iron column is needed to sustain a load of 100,000 pounds with a factor of safety of 10, the length of the column being 14 feet?

We shall suppose that it has been decided to make the cross-section circular, and shall compute by Rankine's formula modified for cast-iron columns (equation 10′). The breaking load for the column would be

$$100{,}000 \times 10 = 1{,}000{,}000 \text{ pounds.}$$

The length is 14 feet or 168 inches; hence the formula becomes

$$\frac{1{,}000{,}000}{A} = \frac{80{,}000}{1 + \dfrac{168^2}{800d^2}};$$

or, reducing by dividing both sides of the equation by 10,000, and then clearing of fractions, we have

$$100\left[1 + \frac{168^2}{800d^2}\right] = 8A.$$

There are two unknown quantities in this equation, d and A, and we cannot solve directly for them. Probably the best way to proceed is to assume or guess at a practical value of d, then solve for A, and finally compute the thickness or inner diameter. Thus, let us try d equal to 7 inches, first solving the equation for A as far as possible. Dividing both sides by 8 we have

$$A = \frac{100}{8}\left[1 + \frac{168^2}{800d^2}\right],$$

and, combining,

$$A = 12.5 + \frac{441}{d^2}.$$

Now, substituting 7 for d, we have

$$A = 12.5 + \frac{441}{49} = 21.5 \text{ square inches.}$$

The area of a hollow circle whose outer and inner diameters are d and d_1 respectively, is $0.7854\ (d^2 - d_1^2)$. Hence, to find the inner diameter of the column, we substitute 7 for d in the last expression, equate it to the value of A just found, and solve for d_1. Thus,

$$0.7854\ (49 - d_1^2) = 21.5\cdot$$

hence

$$49 - d_1^2 = \frac{21.5}{0.7854} = 27.37;$$

and $$d_1^2 = 49 - 27.37 = 21.63 \text{ or } d_1 = 4.65.$$

This value of d makes the thickness equal to

$$\tfrac{1}{2}\ (7 - 4.65) = 1.175 \text{ inches,}$$

which is safe. It might be advisable in an actual case to try d equal to 8 repeating the computation.*

EXAMPLE FOR PRACTICE.

1. What size of white oak column is needed to sustain a load of 45,000 pounds with a factor of safety of 6, the length of the column being 12 feet.

Ans. $d = 9.05$, practically a 10×10-inch section

STRENGTH OF SHAFTS.

A **shaft** is a part of a machine or system of machines, and is used to transmit power by virtue of its torsional strength, or resistance to twisting. Shafts are almost always made of metal and are usually circular in cross-section, being sometimes made hollow.

90. Twisting Moment. Let AF, Fig. 54, represent a shaft with four pulleys on it. Suppose that D is the driving pulley and that B, C and E are pulleys from which power is taken off to drive machines. The portions of the shafts between the pulleys

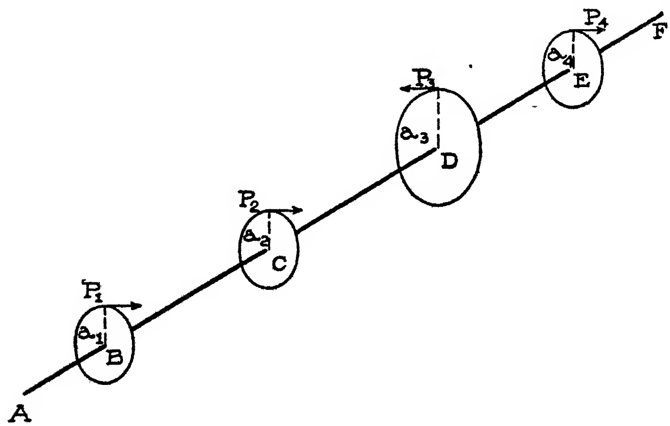


Fig. 54.

are twisted when it is transmitting power; and by the twisting moment at any cross-section of the shaft is meant the algebraic sum of the moments of all the forces acting on the shaft on either

*NOTE. The structural steel handbooks contain extensive tables by means of which the design of columns of steel or cast iron is much facilitated. The difficulties encountered in the use of formulæ are well illustrated in this example.

side of the section, the moments being taken with respect to the axis of the shaft. Thus, if the forces acting on the shaft (at the pulleys) are P_1, P_2, P_3, and P_4 as shown, and if the arms of the forces or radii of the pulleys are a_1, a_2, a_3, and a_4 respectively, then the twisting moment at any section in

BC is $P_1 a_1$,
CD is $P_1 a_1 + P_2 a_2$,
DE is $P_1 a_1 + P_2 a_2 - P_3 a_3$.

Like bending moments, twisting moments are usually expressed in inch-pounds.

Example. Let $a_1 = a_2 = a_4 = 15$ inches, $a_3 = 30$ inches, $P_1 = 400$ pounds, $P_2 = 500$ pounds, $P_3 = 750$ pounds, and $P_4 = 600$ pounds.* What is the value of the greatest twisting moment in the shaft?

At any section between the first and second pulleys, the twisting moment is

$$400 \times 15 = 6{,}000 \text{ inch-pounds;}$$

at any section between the second and third it is

$$400 \times 15 + 500 \times 15 = 13{,}500 \text{ inch-pounds; and}$$

at any section between the third and fourth it is

$$400 \times 15 + 500 \times 15 - 750 \times 30 = -9{,}000 \text{ inch-pounds.}$$

Hence the greatest value is 13,500 inch-pounds.

91. Torsional Stress. The stresses in a twisted shaft are called "torsional" stresses. The torsional stress on a cross-section of a shaft is a shearing stress, as in the case illustrated by Fig. 55, which represents a flange coupling in a shaft. Were it not for the bolts, one flange would slip over the other when either part of the shaft is turned; but the bolts prevent the slipping. Obviously there is a tendency to shear the bolts off unless they are screwed up very tight; that is, the material of the bolts is subjected to shearing stress.

Just so, at any section of the solid shaft there is a tendency for one part to slip past the other, and to prevent the slipping or

* Note. These numbers were so chosen that the moment of P (driving moment) equals the sum of the moments of the other forces. This is always the case in a shaft rotating at constant speed; that is, the power given the shaft equals the power taken off.

shearing of the shaft, there arise shearing stresses at all parts of the cross-section. The shearing stress on the cross-section of a shaft is not a uniform stress, its value per unit-area being zero at the center of the section, and increasing toward the circumference. In circular sections, solid or hollow, the shearing stress per unit-area (unit-stress) varies directly as the distance from the center of the section, provided the elastic limit is not exceeded. Thus, if the shearing unit-stress at the circumference of a section is

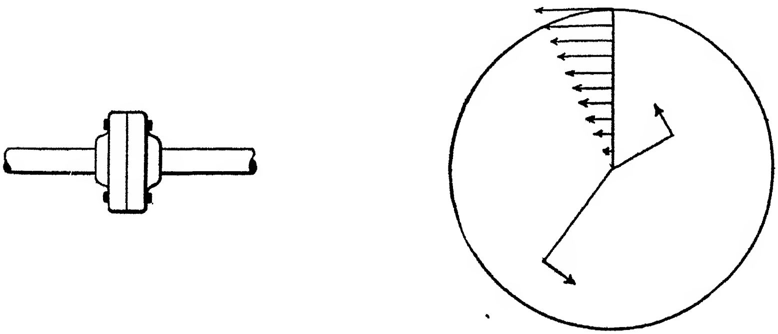


Fig. 55.

1,000 pounds per square inch, and the diameter of the shaft is 2 inches, then, at $\frac{1}{2}$ inch from the center, the unit-stress is 500 pounds per square inch; and at $\frac{1}{4}$ inch from the center it is 250 pounds per square inch. In Fig. 55 the arrows indicate the values and the directions of the shearing stresses on very small portions of the cross-section of a shaft there represented.

92. Resisting Moment. By "resisting moment" at a section of a shaft is meant the sum of the moments of the shearing stresses on the cross-section about the axis of the shaft.

Let S_s denote the value of the shearing stress per unit-area (unit-stress) at the outer points of a section of a shaft; d the diameter of the section (outside diameter if the shaft is hollow); and d_1 the inside diameter. Then it can be shown that the resisting moment is:

For a solid section, $0.1963\ S_s\ d^3$;

For a hollow section, $\dfrac{0.1963\ S_s\ (d^4 - d_1^4)}{d}$.

93. Formula for the Strength of a Shaft. As in the case

of beams, the resisting moment equals the twisting moment at any section. If T be used to denote twisting moment, then we have the formulas:

$$\left.\begin{aligned}&\text{For solid circular shafts, } 0.1963\, S_s\, d^3 = T;\\&\text{For hollow circular shafts, } \frac{0.1963\, S_s\,(d^4 - d_1^4)}{d} = T.\end{aligned}\right\} \quad \textbf{(15)}$$

In any portion of a shaft of constant diameter, the unit-shearing stress S_s is greatest where the twisting moment is greatest. Hence, to compute the greatest unit-shearing stress in a shaft, we first determine the value of the greatest twisting moment, substitute its value in the first or second equation above, as the case may be, and solve for S_s. It is customary to express T in inch-pounds and the diameter in inches, S_s then being in pounds per square inch.

Examples. 1. Compute the value of the greatest shearing unit-stress in the portion of the shaft between the first and second pulleys represented in Fig. 54, assuming values of the forces and pulley radii as given in the example of article 90. Suppose also that the shaft is solid, its diameter being 2 inches.

The twisting moment T at any section of the portion between the first and second pulleys is 6,000 inch-pounds, as shown in the example referred to. Hence, substituting in the first of the two formulas 15 above, we have

$$0.1963\, S_s \times 2^3 = 6{,}000;$$

or, $$S_s = \frac{6{,}000}{0.1963 \times 8} = 3{,}820 \text{ pounds per square inch.}$$

This is the value of the unit-stress at the outside portions of all sections between the first and second pulleys.

2. A hollow shaft is circular in cross-section, and its outer and inner diameters are 16 and 8 inches respectively. If the working strength of the material in shear is 10,000 pounds per square inch, what twisting moment can the shaft safely sustain?

The problem requires that we merely substitute the values of S_s, d, and d_1 in the second of the above formulas 15, and solve for T. Thus,

$$T = \frac{0.1963 \times 10{,}000\,(16^4 - 8^4)}{16} = 7{,}537{,}920 \text{ inch-pounds.}$$

EXAMPLES FOR PRACTICE.

1. Compute the greatest value of the shearing unit-stress in the shaft represented in Fig. 54, using the values of the forces and pulley radii given in the example of article 90, the diameter of the shaft being 2 inches.

Ans. 8,595 pounds per square inch

2. A solid shaft is circular in cross-section and is 9.6 inches in diameter. If the working strength of the material in shear is 10,000 pounds per square inch, how large a twisting moment can the shaft safely sustain? (The area of the cross-section is practically the same as that of the hollow shaft of example 2 preceding.)

Ans. 1,736,736 inch-pounds.

94. Formula for the Power Which a Shaft Can Transmit. The power that a shaft can safely transmit depends on the shearing working strength of the material of the shaft, on the size of the cross-section, and on the speed at which the shaft rotates.

Let H denote the amount of horse-power; S_s the shearing working strength in pounds per square inch; d the diameter (outside diameter if the shaft is hollow) in inches; d_1 the inside diameter in inches if the shaft is hollow; and n the number of revolutions of the shaft per minute. Then the relation between power transmitted, unit-stress, etc., is:

$$\left.\begin{aligned} &\text{For solid shafts,}\quad H = \frac{S_s d^3 n}{321{,}000};\\ &\text{For hollow shafts,}\quad H = \frac{S_s (d^4 - d_1^4)\, n}{321{,}000\, d}. \end{aligned}\right\} \qquad (16)$$

Examples. 1. What horse-power can a hollow shaft 16 inches and 8 inches in diameter safely transmit at 50 revolutions per minute, if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the second of the two formulas 16 above, and reduce. Thus,

$$H = \frac{10{,}000\,(16^4 - 8^4)\,50}{321{,}000 \times 16} = 6{,}000 \text{ horse-power (nearly).}$$

2. What size of solid shaft is needed to transmit 6,000 horse-power at 50 revolutions per minute if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the first of the two formulas 16, and solve for d. Thus,

$$6{,}000 = \frac{10{,}000 \times d^4}{321{,}000} \times \frac{50}{};$$

therefore $$d^3 = \frac{6{,}000 \times 321{,}000}{10{,}000 \times 50} = 3{,}852;$$

or, $$d = \sqrt[3]{3{,}852} = 15.68 \text{ inches.}$$

(A solid shaft of this diameter contains over 25% more material than the hollow shaft of example 1 preceding. There is therefore considerable saving of material in the hollow shaft.)

3. A solid shaft 4 inches in diameter transmits 200 horse-power while rotating at 200 revolutions per minute. What is the greatest shearing unit-stress in the shaft?

We have merely to substitute in the first of the equations 16, and solve for S_s. Thus,

$$200 = \frac{S_s \times 4^3 \times 200}{321{,}000};$$

or, $$S = \frac{200 \times 321{,}000}{4^3 \times 200} = 5{,}016 \text{ pounds per square inch.}$$

EXAMPLES FOR PRACTICE.

1. What horse-power can a solid shaft 9.6 inches in diameter safely transmit at 50 revolutions per minute, if its shearing working strength is 10,000 pounds per square inch?

Ans. 1,378 horse-power.

2. What size of solid shaft is required to transmit 500 horse-power at 150 revolutions per minute, the shearing working strength of the material being 8,000 pounds per square inch.

Ans. 5.1 inches.

3. A hollow shaft whose outer diameter is 14 and inner 6.7 inches transmits 5,000 horse-power at 60 revolutions per minute. What is the value of the greatest shearing unit-stress in the shaft?

Ans. 10,273 pounds per square inch.

STIFFNESS OF RODS, BEAMS, AND SHAFTS.

The preceding discussions have related to the *strength* of

materials. We shall now consider principally the *elongation of rods*, *deflection of beams*, and *twist of shafts*.

95. Coefficient of Elasticity. According to Hooke's Law (Art. 9, p. 7), the elongations of a rod subjected to an increasing pull are proportional to the pull, provided that the stresses due to the pull do not exceed the elastic limit of the material. Within the elastic limit, then, the ratio of the pull and the elongation is constant; hence the ratio of the unit-stress (due to the pull) to the unit-elongation is also constant. This last-named ratio is called "coefficient of elasticity." If E denotes this coefficient, S the unit-stress, and s the unit-deformation, then

$$E = \frac{S}{s}. \qquad (17)$$

Coefficients of elasticity are usually expressed in pounds per square inch.

The preceding remarks, definition, and formula apply also to a case of compression, provided that the material being compressed does not bend, but simply shortens in the direction of the compressing forces. The following table gives the *average* values of the coefficient of elasticity for various materials of construction:

TABLE G.

Coefficients of Elasticity.

Material.	Average Coefficient of Elasticity.
Steel	30,000,000 pounds per square inch.
Wrought iron. ...	27,500,000 " " " "
Cast iron...	15,000,000 " " " "
Timber..........	1,800,000 " " " "

The coefficients of elasticity for steel and wrought iron, for different grades of those materials, are remarkably constant; but for different grades of cast iron the coefficients range from about 10,000,000 to 30,000,000 pounds per square inch. Naturally the coefficient has not the same value for the different kinds of wood; for the principal woods it ranges from 1,600,000 (for spruce) to 2,100,000 (for white oak).

Formula 17 can be put in a form more convenient for use, as follows:

Let P denote the force producing the deformation; A the area of the cross-section of the piece on which P acts; l the length of the piece; and D the deformation (elongation or shortening).

Then

$$S = P \div A \text{ (see equation 1),}$$

and

$$s = D \div l \text{ (see equation 2).}$$

Hence, substituting these values in equation 17, we have

$$E = \frac{Pl}{AD}; \text{ or } D = \frac{Pl}{AE}. \qquad (17')$$

The first of these two equations is used for computing the value of the coefficient of elasticity from measurements of a "test," and the second for computing the elongation or shortening of a given rod or bar for which the coefficient is known.

Examples. 1. It is required to compute the coefficient of elasticity of the material the record of a test of which is given on page 9.

Since the unit-stress S and unit-elongation s are already computed in that table, we can use equation 17 instead of the first of equations 17′. The elastic limit being between 40,000 and 45,000 pounds per square inch, we may use any value of the unit-stress less than that, and the corresponding unit-elongation.

Thus, with the first values given,

$$E = \frac{5{,}000}{0.00017} = 29{,}400{,}000.$$

With the second,

$$E = \frac{10{,}000}{0.00035} = 28{,}600{,}000.$$

This lack of constancy in the value of E as computed from different loads in a test of a given material, is in part due to errors in measuring the deformation, a measurement difficult to make. The value of the coefficient adopted from such a test, is the average of all the values of E which can be computed from the record.

2. How much will a pull of 5,000 pounds stretch a round steel rod 10 feet long and 1 inch in diameter?

We use the second of the two formulas 17′. Since $A = 0.7854 \times 1^2 = 0.7854$ square inches, $l = 120$ inches, and $E = 30{,}000{,}000$ pounds per square inch, the stretch is:

$$D = \frac{5{,}000 \times 120}{0.7854 \times 30{,}000{,}000} = 0.0254 \text{ inch.}$$

EXAMPLES FOR PRACTICE.

1. What is the coefficient of elasticity of a material if a pull of 20,000 pounds will stretch a rod 1 inch in diameter and 4 feet long 0.045 inch?

Ans. 27,000,000 pounds per square inch.

2. How much will a pull of 15,000 pounds elongate a round cast-iron rod 10 feet long and 1 inch in diameter?

Ans. 0.152 inch.

96. Temperature Stresses. In the case of most materials, when a bar or rod is heated, it lengthens; and when cooled, it shortens if it is free to do so. The **coefficient of linear expansion** of a material is the ratio which the elongation caused in a rod or bar of the material by a change of one degree in temperature bears to the length of the rod or bar. Its values for Fahrenheit degrees are about as follows:

For Steel,	0.0000065.
For Wrought iron,	.0000067.
For Cast iron,	.0000062.

Let K be used to denote this coefficient; t a change of temperature, in degrees Fahrenheit; l the length of a rod or bar; and D the change in length due to the change of temperature. Then

$$D = K\,t\,l. \qquad (18)$$

D and l are expressed in the same unit.

If a rod or bar is confined or restrained so that it cannot change its length when it is heated or cooled, then any change in its temperature produces a stress in the rod; such are called **temperature stresses.**

Examples. 1. A steel rod connects two solid walls and is screwed up so that the unit-stress in it is 10,000 pounds per square inch. Its temperature falls 10 degrees, and it is observed that the walls have not been drawn together. What is the temperature stress produced by the change of temperature, and what is the actual unit-stress in the rod at the new temperature?

Let l denote the length of the rod. Then the change in length which would occur if the rod were free, is given by formula 18, above, thus:

$$D = 0.0000065 \times 10 \times l = 0.000065\,l.$$

Now, since the rod could not shorten, it has a greater than normal length at the new temperature; that is, the fall in temperature has produced an effect equivalent to an elongation in the rod amounting to D, and hence a tensile stress. This tensile stress can be computed from the elongation D by means of formula 17. Thus,

$$S = E\,s;$$

and since s, the unit-elongation, equals

$$\frac{D}{l} = \frac{.0000065\,l}{l} = .0000065,$$

$S = 30{,}000{,}000 \times .0000065 = 1950$ pounds per square inch. This is the value of the temperature stress; and the new unit-stress equals

$$10{,}000 + 1950 = 11{,}950 \text{ pounds per square inch.}$$

Notice that the unit temperature stresses are independent of the length of the rod and the area of its cross-section.

2. Suppose that the change of temperature in the preceding example is a rise instead of a fall. What are the values of the temperature stress due to the change, and of the new unit-stress in the rod?

The temperature stress is the same as in example 1, that is, 1,950 pounds per square inch; but the rise in temperature releases, as it were, the stress in the rod due to its being screwed up, and the final unit stress is

$$10{,}000 - 1{,}950 = 8{,}050 \text{ pounds per square inch.}$$

EXAMPLE FOR PRACTICE.

1. The ends of a wrought-iron rod 1 inch in diameter are fastened to two heavy bodies which are to be drawn together, the temperature of the rod being 200 degrees when fastened to the objects. A fall of 120 degrees is observed not to move them. What is the temperature stress, and what is the pull exerted by the rod on each object?

Ans. $\begin{cases} \text{Temperature stress, 22,110 pounds per square inch.} \\ \text{Pull, 17,365 pounds.} \end{cases}$

97. Deflection of Beams. Sometimes it is desirable to know how much a given beam will deflect under a given load, or to design

a beam which will not deflect more than a certain amount under a given load. In Table B, page 53, Part I, are given formulas for deflection in certain cases of beams and different kinds of loading.

In those formulas, d denotes deflection; I the moment of inertia of the cross-section of the beam with respect to the neutral axis, as in equation 6, and E the coefficient of elasticity of the material of the beam (for values, see Art. 95).

In each case, the load should be expressed in pounds, the length in inches, and the moment of inertia in biquadratic inches; then the deflection will be in inches.

According to the formulas for d, the deflection of a beam varies inversely as the coefficient of its material (E) and the moment of inertia of its cross-section (I); also, in the first four and last two cases of the table, the deflection varies directly as the cube of the length (l^3).

Example. What deflection is caused by a uniform load of 6,400 pounds (including weight of the beam) in a wooden beam on end supports, which is 12 feet long and 6 × 12 inches in cross-section? (This is the safe load for the beam; see example 1, Art. 65.)

The formula for this case (see Table B, page 53) is

$$d = \frac{5 \text{ W} l^3}{384 \text{ EI}}.$$

Here W = 6,400 pounds; l = 144 inches; E = 1,800,000 pounds per square inch; and

$$\text{I} = \frac{1}{12} ba^3 = \frac{1}{12} 6 \times 12^3 = 864 \text{ inches}^4.$$

Hence the deflection is

$$d = \frac{5 \times 6{,}400 \times 144^3}{384 \times 1{,}800{,}000 \times 864} = 0.16 \text{ inch.}$$

EXAMPLES FOR PRACTICE.

1. Compute the deflection of a timber built-in cantilever 8 × 8 inches which projects 8 feet from the wall and bears an end load of 900 pounds. (This is the safe load for the cantilever, see example 1, Art. 65.)

Ans. 0.43 inch.

2. Compute the deflection caused by a uniform load of 40,000

pounds on a 42-pound 15-inch steel I-beam which is 16 feet long and rests on end supports.

Ans. 0.28 inch.

98. Twist of Shafts. Let Fig. 57 represent a portion of a shaft, and suppose that the part represented lies wholly between

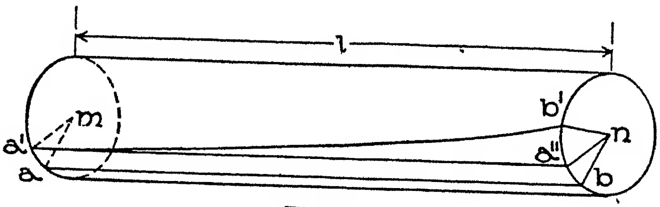


Fig. 57.

two adjacent pulleys on a shaft to which twisting forces are applied (see Fig. 54). Imagine two radii ma and nb in the ends of the portion, they being parallel as shown when the shaft is not twisted. After the shaft is twisted they will not be parallel, ma having moved to ma', and nb to nb'. The angle between the two lines in their twisted positions (ma' and nb') is called the **angle of twist**, or **angle of torsion**, for the length l. If $a'a''$ is parallel to ab, then the angle $a''nb'$ equals the angle of torsion.

If the stresses in the portion of the shaft considered do not exceed the elastic limit, and if the twisting moment is the same for all sections of the portion, then the angle of torsion a (in degrees) can be computed from the following:

For solid circular shafts,

$$a = \frac{584\ Tl}{E^1\ d^4} = \frac{36{,}800{,}000\ Hl}{E^1\ d^4 n}.$$

For hollow circular shafts,

$$a = \frac{584\ Tld}{E^1(d^4 - d_1^4)} = \frac{36{,}800{,}000\ Hl}{E^1\ (d^4 - d_1^4)\ n}. \qquad \textbf{(19)}$$

Here T, l, d, d_1, H, and n have the same meanings as in Arts. 93 and 94, and should be expressed in the units there used. The letter E^1 stands for a quantity called **coefficient of elasticity for shear**; it is analogous to the coefficient of elasticity for tension and compression (E), Art. 95. The values of E^1 for a few materials average about as follows (roughly $E^1 = \frac{2}{5}$ E):

For Steel,	11,000,000	pounds per square inch.
For Wrought iron,	10,000,000	" " " "
For Cast iron,	6,000,000	" " " "

Example. What is the value of the angle of torsion of a steel shaft 60 feet long when transmitting 6,000 horse-power at 50 revolutions per minute, if the shaft is hollow and its outer and inner diameters are 16 and 8 inches respectively?

Here $l = 720$ inches; hence, substituting in the appropriate formula (19), we find that

$$a = \frac{36,800,000 \times 6,000 \times 720}{11,000,000 \times (16^4 - 8^4)\ 50} = 4.7 \text{ degrees.}$$

EXAMPLE FOR PRACTICE.

Suppose that the first two pulleys in Fig. 54 are 12 feet apart; that the diameter of the shaft is 2 inches; and that $P_1 = 400$ pounds, and $a_1 = 15$ inches. If the shaft is of wrought iron, what is the value of the angle of torsion for the portion between the first two pulleys?

Ans. 3.15 degrees.

99. Non-elastic Deformation. The preceding formulas for elongation, deflection, and twist hold only so long as the greatest unit-stress does not exceed the elastic limit. There is no theory, and no formula, for non-elastic deformations, those corresponding to stresses which exceed the elastic limit. It is well known, however, that non-elastic deformations are not proportional to the forces producing them, but increase much faster than the loads. The value of the ultimate elongation of a rod or bar (that is, the amount of elongation at rupture), is quite well known for many materials. This elongation, for eight-inch specimens of various materials (see Art. 16), is:

For Cast iron, about 1 per cent.
For Wrought iron (plates), 12 – 15 per cent.
For " " (bars), 20 – 25 " ".
For Structural steel, 22 – 26 " ".

Specimens of ductile materials (such as wrought iron and structural steel), when pulled to destruction, **neck down**, that is, diminish very considerably in cross-section at some place along the length of the specimen. The decrease in cross-sectional area

is known as **reduction of area,** and its value for wrought iron and steel may be as much as 50 per cent.

RIVETED JOINTS.

100. Kinds of Joints. A **lap joint** is one in which the plates or bars joined overlap each other, as in Fig. 58, *a*. A **butt joint** is one in which the plates or bars that are joined butt against each other, as in Fig. 58, *b*. The thin side plates on butt joints

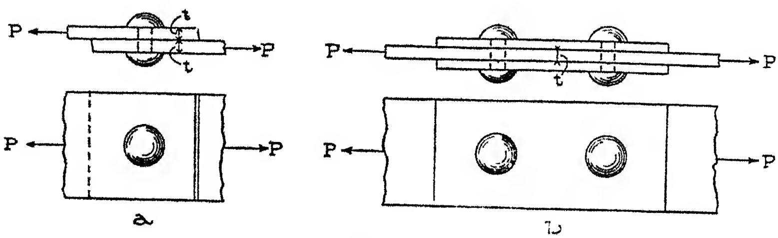


Fig. 58.

are called **cover-plates**; the thickness of each is always made not less than one-half the thickness of the **main plates,** that is, the plates or bars that are joined. Sometimes butt joints are made with only one cover-plate; in such a case the thickness of the cover-plate is made not less than that of the main plate.

When wide bars or plates are riveted together, the rivets are placed in rows, always parallel to the "seam" and sometimes also perpendicular to the seam; but when we speak of a row of rivets, we mean a row parallel to the seam. A lap joint with a single row of rivets is said to be **single-riveted**; and one with two rows of rivets is said to be **double-riveted.** A butt joint with two rows of rivets (one on each side of the joint) is called "single-riveted," and one with four rows (two on each side) is said to be "double-riveted."

The distance between the centers of consecutive holes in a row of rivets is called **pitch.**

101. Shearing Strength, or Shearing Value, of a Rivet. When a lap joint is subjected to tension (that is, when P, Fig. 58, *a*, is a pull), and when the joint is subjected to compression (when P is a push), there is a tendency to cut or shear each rivet along the surface between the two plates. In butt joints with two cover-

plates, there is a tendency to cut or shear each rivet on two surfaces (see Fig. 58, *b*). Therefore the rivets in the lap joint are said to be in **single shear**; and those in the butt joint (two covers) are said to be in **double shear.**

The "shearing value" of a rivet means the resistance which it can safely offer to forces tending to shear it on its cross-section. This value depends on the area of the cross-section and on the working strength of the material. Let d denote the diameter of the cross-section, and S_s the shearing working strength. Then, since the area of the cross-section equals $0.7854\ d^2$, the shearing strength of one rivet is:

For single shear,	$0.7854\ d^2\ S_s$.
For double shear,	$1.5708\ d^2\ S_s$.

102. Bearing Strength, or Bearing Value, of a Plate. When a joint is subjected to tension or compression, each rivet presses against a part of the sides of the holes through which it passes. By "bearing value" of a plate (in this connection) is meant the pressure, exerted by a rivet against the side of a hole in the plate, which the plate can safely stand. This value depends on the thickness of the plate, on the diameter of the rivet, and on the compressive working strength of the plate. Exactly how it depends on these three qualities is not known; but the bearing value is always computed from the expression $t\ d\ S_c$, wherein t denotes the thickness of the plate; d, the diameter of the rivet or hole; and S_c, the working strength of the plate.

103. Frictional Strength of a Joint. When a joint is subjected to tension or compression, there is a tendency to slippage between the faces of the plates of the joint. This tendency is overcome wholly or in part by frictional resistance between the plates. The frictional resistance in a well-made joint may be very large, for rivets are put into a joint hot, and are **headed** or **capped** before being cooled. In cooling they contract, drawing the plates of the joint tightly against each other, and producing a great pressure between them, which gives the joint a correspondingly large frictional strength. It is the opinion of some that all well-made joints perform their service by means of their frictional strength; that is to say, the rivets act only by pressing the plates together and are not under shearing stress, nor

are the plates under compression at the sides of their holes. The "frictional strength" of a joint, however, is usually regarded as uncertain, and generally no allowance is made for friction in computations on the strength of riveted joints.

104. Tensile and Compressive Strength of Riveted Plates. The holes punched or drilled in a plate or bar weaken its tensile strength, and to compute that strength it is necessary to allow for the holes. By **net section**, in this connection, is meant the smallest cross-section of the plate or bar; this is always a section along a line of rivet holes.

If, as in the foregoing article, t denotes the thickness of the plates joined; d, the diameter of the holes; n_1, the number of rivets in a row; and w, the width of the plate or bar; then the net section $= (w - n_1 d)\ t$.

Let S_t denote the tensile working strength of the plate; then the strength of the unriveted plate is wtS_t, and the reduced tensile strength is $(w - n_1 d)\ t\ S_t$.

The compressive strength of a plate is also lessened by the presence of holes; but when they are again filled up, as in a joint, the metal is replaced, as it were, and the compressive strength of the plate is restored. No allowance is therefore made for holes in figuring the compressive strength of a plate.

105. Computation of the Strength of a Joint. The strength of a joint is determined by either (1) the shearing value of the rivets; (2) the bearing value of the plate; or (3) the tensile strength of the riveted plate if the joint is in tension. Let P_s denote the strength of the joint as computed from the shearing values of the rivets; P_c, that computed from the bearing value of the plates; and P_t, the tensile strength of the riveted plates. Then, as before explained,

$$\left.\begin{array}{l} P_t = (w - n_1 d)\ tS_t; \\ P_s = n_2\ 0.7854\ d^2 S_s; \text{ and} \\ P_c = n_3 t d S_c; \end{array}\right\} \qquad (20)$$

n_2 denoting the total number of rivets in the joint; and n_3 denoting the total number of rivets in a lap joint, and one-half the number of rivets in a butt joint.

Examples. 1. Two half-inch plates $7\frac{1}{2}$ inches wide are con-

nected by a single lap joint double-riveted, six rivets in two rows. If the diameter of the rivets is $\frac{3}{4}$ inch, and the working strengths are as follows: $S_t = 12{,}000$, $S_s = 7{,}500$, and $S_c = 15{,}000$ pounds per square inch, what is the safe tension which the joint can transmit?

Here $n_1 = 3$, $n_2 = 6$, and $n_3 = 6$; hence

$$P_t = (7\frac{1}{2} - 3 \times \frac{3}{4}) \times \frac{1}{2} \times 12{,}000 = 31{,}500 \text{ pounds;}$$

$$P_s = 6 \times 0.7854 \times (\frac{3}{4})^2 \times 7{,}500 = 19{,}880 \text{ pounds;}$$

$$P_c = 6 \times \frac{1}{2} \times \frac{3}{4} \times 15{,}000 = 33{,}750 \text{ pounds.}$$

Since P_s is the least of these three values, the strength of the joint depends on the shearing value of its rivets, and it equals 19,880 pounds.

2. Suppose that the plates described in the preceding example are joined by means of a butt joint (two cover-plates), and 12 rivets are used, being spaced as before. What is the safe tension which the joint can bear?

Here $n_1 = 3$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_t = 31{,}500; \text{ and } P_c = 33{,}750 \text{ pounds; but}$$

$$P_s = 12 \times 0.7854 \times (\frac{3}{4})^2 \times 7{,}500 = 39{,}760 \text{ pounds.}$$

The strength equals 31,500 pounds, and the joint is stronger than the first.

3. Suppose that in the preceding example the rivets are arranged in rows of two. What is the tensile strength of the joint?

Here $n_1 = 2$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_s = 39{,}760; \text{ and } P_c = 33{,}750 \text{ pounds; but}$$

$$P_t = (7\frac{1}{2} - 2 \times \frac{3}{4}) \frac{1}{2} \times 12{,}000 = 36{,}000 \text{ pounds.}$$

The strength equals 33,750 pounds, and this joint is stronger than either of the first two.

EXAMPLES FOR PRACTICE.

Note. Use working strengths as in example 1, above.

$S_t = 12{,}000$, $S_s = 7{,}500$, and $S_c = 15{,}000$ pounds per square inch.

1. Two half-inch plates 5 inches wide are connected by a lap joint, with two $\frac{3}{4}$-inch rivets in a row. What is the safe strength of the joint?

Ans. 6,625 pounds.

2. Solve the preceding example supposing that four $\frac{3}{4}$-inch rivets are used, in two rows.

Ans. 13,250 pounds.

3. Solve example 1 supposing that three 1-inch rivets are used, placed in a row lengthwise of the joint.

Ans. 17,670 pounds.

4. Two half-inch plates 5 inches wide are connected by a butt joint (two cover-plates), and four $\frac{3}{4}$-inch rivets are used, in two rows. What is the strength of the joint?

Ans. 11,250 pounds.

106. Efficiency of a Joint. The ratio of the strength of a joint to that of the solid plate is called the "efficiency of the joint." If ultimate strengths are used in computing the ratio, then the efficiency is called **ultimate efficiency;** and if working strengths are used, then it is called **working efficiency.** In the following, we refer to the latter. An efficiency is sometimes expressed as a per cent. To express it thus, multiply the ratio *strength of joint* ÷ *strength of solid plate*, by 100.

Example. It is required to compute the efficiencies of the joints described in the examples worked out in the preceding article.

In each case the plate is $\frac{1}{2}$ inch thick and $7\frac{1}{2}$ inches wide; hence the tensile working strength of the solid plate is

$$7\frac{1}{2} \times \frac{1}{2} \times 12{,}000 = 45{,}000 \text{ pounds.}$$

Therefore the efficiencies of the joints are:

$$(1) \quad \frac{19{,}880}{45{,}000} = 0.44, \text{ or } 44 \text{ per cent;}$$

$$(2) \quad \frac{31{,}500}{45{,}000} = 0.70, \text{ or } 70 \text{ per cent;}$$

$$(3) \quad \frac{33{,}750}{45{,}000} = 0.75, \text{ or } 75 \text{ per cent.}$$

REVIEW QUESTIONS

REVIEW QUESTIONS

ON THE SUBJECT OF

BRIDGE ENGINEERING

PART I

---

1. Write a short history of early bridges.

2. Define: Truss, bridge truss, truss bridge, girders, and girder bridges.

3. Draw an outline of a through bridge, and also an outline of a deck bridge.

4. Make an outline diagram of a truss, and write the names of the various parts on the respective members.

5. Make an outline diagram of a Warren, Howe, Pratt, bowstring, and Baltimore truss.

6. Compute the weight of steel in a 130-foot highway bridge whose trusses are 16 feet center to center, given $W = 34 + 22b + 0.16bl + 0.7l$.

7. Compute the weight of steel in a deck plate-girder span of 100 feet. Loading, E 50. Given $W = 124.0 + 12.0l$.

8. What are *equivalent uniform loads?*

9. What is Cooper's E 40 loading?

10. Prove that the stress in a diagonal of a horizontal chord truss with a simple web system is $V \sec \phi$.

11. Prove that the chord stress is $M \div h$, where M is the moment at the point, and h is the height of the truss.

12. Prove that the load must be on the segment of the span to the right of the section to produce the maximum positive shear.

13. Compute the maximum positive and negative live-load shears in a 13-panel Howe truss, the live panel load being 40 000 pounds.

REVIEW QUESTIONS

ON THE SUBJECT OF

BRIDGE ENGINEERING

PART II

---

1. Write an essay of 200 words on the economic considerations governing the decision to build and the decision as to what kind of bridge to employ.

2. What determines the height and width of railroad truss bridges?

3. Draw a clearance diagram for a bridge on a straight track, and state what allowance should be made if the bridge is on a curve.

4. Describe a stress sheet, and tell what should be on it.

5. Make a sketch of a cross-section of a deck plate-girder, showing the cross-ties, guard-rails, and rails in place.

6. Make a sketch showing how tracks on curves are constructed.

7. What is the span under coping, the span center to center of bearings, and the span over all?

8. Design a tie for Cooper's E 50 loading.

9. If the end shear of a plate-girder is 394 500 pounds, design the web section, it being 108 inches deep.

10. If the dead-load moment is 8 489 000 pound-inches and the live-load moment is 30 610 000 pound-inches, design the flange, if the distance back to back of flange angle, is 7 feet $6\frac{1}{2}$ inches, it being assumed that the web does not take any bending moment.

11. If, in the girder of Question 10, above, the web were 90 by $\frac{7}{16}$ inch, design the flange, considering $\frac{1}{8}$ of the gross area of the web as effective flange area.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS

PART I

---

1. When a $\frac{3}{4}$-inch round rod sustains a pull of 10,000 pounds, what is the value of the unit-tensile stress in the rod?

2. What do you understand by Hooke's Law?

3. What are the dimensions of a square white pine post, needed to support a steady load of 6,500 pounds with a factor of safety of 8?

4. How large a force is required to punch a 1-inch hole through a $\frac{3}{4}$-inch plate of wrought iron, if the ultimate shearing strength of the material is 40,000 pounds per square inch?

5. Compare the ultimate strengths of wood along and across the grain; also the ultimate tensile and compressive strengths of cast iron.

6. Make a sketch of a beam 20 feet long resting on end supports, and represent loads of 6,000, 3,000, 1,000, and 4,000 pounds at points 2, 5, 11, and 16 feet from the left end, respectively. What is the value and sign of the moment of each of these loads about the middle of the beam? Also about the left end?

7. A beam 15 feet long is supported at two points, 2 feet from the right end, and 3 feet from the left end. If the beam sustains a uniform load of 400 pounds per foot, what are the values of the reactions?

8. Compute the values of the external shear and bending moment for the loaded beam described in Question 6, at sections 1, 4, 10, and 15 feet from the left end.

9. Draw shear and moment diagrams to scale for the beam described in Question 7.

10. Suppose a T-bar 2 inches deep, has a flange 3 inches wide and is $\frac{1}{4}$ inch thick throughout. Locate the center of gravity by computation.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS

PART II

---

1. A cantilever beam 6 feet in length projects from a wall and sustains an end load of 300 pounds. The cross-section being as in Fig. 38, find the greatest tensile and compressive unit-stresses and state where they occur.

2. An I-beam weighing 30 pounds per foot, rests on end supports 25 feet apart. Its section modulus is 20.4 inches, and its working strength 16,000 pounds per square inch. Calculate weight of the beam.

3. A wooden beam 15 feet long, 4×14 inches in cross-section sustains a load of 4,000 pounds 5 feet from one end, and 2,000 pounds at the middle. Compute the greatest unit shearing stress.

4. What do you know about radius of gyration? Give an example.

5. Find the factor of safety of a 24-inch 80-pound steel I-beam 15 feet long, used as a flat-ended column to sustain a load of 150,000 pounds. Note—Use "Rankine's Formula".

6. A steel Z-bar is 20 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches, and its area of cross-section is 24.5 square inches. Calculate the safe load with a factor of safety of 6. Note—Use "Rankine's Formula".

7. Make sketches of the following:

Lap joint single-riveted;
Lap joint double-riveted;
Butt joint single-riveted;
Butt joint double-riveted.

INDEX

INDEX

The page numbers of this volume will be found at the bottom of the pages; the numbers at the top refer only to the section.

Note—For page numbers see foot of pages.

Note—For page numbers see foot of pages.